A Contest Model of Liberalizing Government Procurements

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Abstract

This paper models government procurements as a contest among domestic firms, and – in case of liberalization – domestic and foreign firms. Liberalizing procurements reduces wasteful domestic lobbying but also increases the likelihood that a foreign firm will capture the rent. We show that the domestic welfare change is not monotonic in the foreign firms’ abilities. Domestic welfare increases only if the gross surplus generated by foreign firms is sufficiently large. Furthermore, we show that, from the global welfare point of view, domestically optimal liberalization policies can be either excessive or too restrictive.

**JEL-Classification:** F12, F13, H57.

Keywords: Trade liberalization, trade in services, government procurements.
1 Introduction

While liberalization of trade in goods has been progressing for decades, liberalizing trade in services seems to face substantial reluctance, in particular when the services relate to government contracts. Governmental procurements are quite substantial in modern economies: the OECD estimates that public procurement by governments and by state-owned enterprises amounts to about 15% of GDP in OECD countries (see OECD, 2005). There is a lot of evidence indicating a strict preference for domestic agents or at least more protection for public procurement agencies than for private firms (see for example Trionfetti, 2000). The WTO had established a Working Group on Transparency in Government Procurement in order to enhance transparency in procurement decisions and prepare an international agreement. However, members could not agree on the launch of negotiations in 2003 and referred this issue back to the WTO General Council. The Council decided in 2004 that this issue should not be taken any further and should not form part of the Doha Round Programme. This decision basically allows a country to set up its own rules and procedures unless they are in conflict with supranational provisions, such as rules set up by the European Union, or bilateral agreements with other countries.

Hence, it is fair to assume that procurement rules are still based on national interest. At the same time, it is also well known that bidding for a governmental contract does not resemble an auction in the usual sense, because government officials take into account many additional criteria beyond the bids. For example, a recent survey shows that firms often have to offer a substantial share of the contract value as additional illicit payments to secure a procurement contract (see Transparency International, 2007, p. 320.) These observations suggest that governmental procurement decisions may be strongly influenced by partisanship or even corruption. Although this practice is to be contained by the OECD Antibribery Convention, this convention seems to fail to a large extent. Auriol (2006) quotes a World Bank estimate that about USD 200 billion are spent on public procurement bribery per year, which amounts to about 3.5 % of public procurement spending. She
also demonstrates that the deadweight loss is substantial.

Given this background, we take the view in this paper that government procurement can be modelled as an imperfectly discriminating contest in which firms lobby for winning (Tullock, 1980 1991, Rowley et al., 1988, Hillman and Riley, 1989, Nitzan, 1994). The winner may run the project and will realize a surplus from the project. The size of this surplus, however, may depend on the type of the winning firm. In particular, foreign firms may have different abilities to run the project compared to domestic firms. We will explore conditions under which the inclusion of foreign firms will improve domestic welfare. We also include an analysis of global welfare effects as we would like to learn whether a bilateral liberalization of government procurement will lead to excessive or insufficient lobbying. For this purpose, we use an asymmetric contest model in which domestic and foreign firms have different abilities.\footnote{Asymmetry in abilities among contestants were dealt with in Nti (2002) and Stein (1999). For a general paper on asymmetric contests, see Cornes and Hartley (2005).} Concerning domestic welfare, we note two important points. First, rents that accrue to domestic contestants form part of social welfare, while those that accrue to foreign firms are not. Second, the real domestic resources used in lobbying by domestic contestants are socially wasteful, but real domestic resources used by foreign contestants are domestic exports. We determine conditions under which a country is better off by allowing foreign contestants to participate.

Although the literature on procurement discrimination is vast, this paper is to our knowledge the first which discusses government procurement policies in an asymmetric contest model.\footnote{For an overview of the literature, see for example Evenett and Hoekman (2005).} McAfee and McMillan (1989) show in an auction model that excluding foreign firms may enhance competition among domestic firms and can thus be welfare-improving. Branco (1994) and Vagstad (1995) extend the Brander-Spencer (1981) analysis to procurement and show that the profit-shifting incentive may call for the exclusion of for-
eign firms. In our setting, however, domestic and foreign firms do not compete by bids on a level-playing field. Instead, foreign and domestic firms are asymmetric from the outset, and lobbying by each firm uses up real resources and enhances its probability of winning.

The remainder of the paper is organized as follows. Section 2 explains the contest model, Section 3 discusses the domestic welfare effect of liberalizing government procurements and Section 4 deals with the global welfare effects. Section 5 concludes. The details of our proofs are relegated to the appendix.

2 The model

We assume two countries, a domestic country and a foreign country which will be denoted by a star. A project is available in the domestic country. Let $n(n^*)$ denote the number of domestic (foreign) contestants for this project. Domestic and foreign contestants differ in terms of their abilities: a winning domestic (foreign) firm will generate a gross surplus $V (V^*)$ from the project. We take both $n$ and $n^*$ as exogenous. To focus on the interesting case where foreign firms have interest in participation, we make the following assumption:

**Assumption 1** $V^* \geq (n - 1)V/n$.

As will be seen below, Assumption 1 implies that foreign firms will actively lobby. Let $s_i$ ($s_i^*$) denotes individual domestic (respectively, foreign) lobbying, and $S = \sum s_i$ (respectively, $S^* = \sum s_i^*$) denotes aggregate domestic (foreign) lobbying. The probability that the domestic (foreign) firm $i$ wins the contest is $s_i/(S+S^*)$ (respectively, $s_i^*/(S+S^*)$). The expected profit of each domestic (foreign) contestant, denoted by $\Pi$ ($\Pi^*$), is the probability of winning the contest times the surplus net of taxes, minus the lobbying effort:

$$\Pi = \frac{s_i}{S+S^*}(1-t)V - s_i,$$

$$\Pi^* = \frac{s_i^*}{S+S^*}(1-t)V^* - s_i^*.$$
The tax rate $t$ is exogenous and equal to the corporate profit tax rate in the country under consideration. Assume $0 < t < 1$. Profits are taxed at their source. The first-order conditions and symmetry within each group of firms lead to

\[
\begin{align*}
    s_i &= \max \left(0, \frac{(n + n^* - 1)(1 - t)VV^*(n^*V - (n^* - 1)V^*)}{(n^*V + nV^*)^2} \right) \text{ if } s_i > 0, \\
    s_i &= \frac{(n - 1)(1 - t)V}{n^2} \text{ if } s_i^* = 0, \quad (2) \\
    s_i^* &= \frac{(n + n^* - 1)(1 - t)VV^*(nV^* - (n - 1)V)}{(n^*V + nV^*)^2} \text{ if } s_i > 0, \\
    s_i^* &= \frac{(n^* - 1)(1 - t)V^*}{n^{*2}} \text{ if } s_i = 0.
\end{align*}
\]

Foreign lobbying – if allowed – is always positive due to Assumption 1, but domestic lobbying may not be profitable.

Liberalization is defined as a move from autarky (i.e., when only domestic firms are allowed to contest) to a regime where all $n^*$ foreign firms are allowed to contest. We distinguish two cases, which depend on parameter values. In case 1, where the foreign surplus $V^*$ is very large, foreign contestants will lobby very strongly and thus render unprofitable any lobbying effort of domestic contestants. According to (2), this will happen if $V^* \geq \tilde{V} \equiv n^*V/(n^* - 1)$. In case 2, $V^* < \tilde{V}$, and consequently domestic lobbying will continue to be profitable after liberalization. We call $\tilde{V}$ the foreign-dominance threshold level of gross surplus of foreign firms, beyond which domestic firms will not lobby.

**Definition 1** The foreign-dominance threshold level of gross surplus of foreign firms is

\[
\tilde{V} \equiv n^*V/(n^* - 1).
\]

Note that the greater is the number $n^*$, the lower is the threshold level $\tilde{V}$. The foreign firms are said to be dominant if $V^* \geq \tilde{V}$.

Domestic welfare is equal to the sum of expected net profits of domestic firms and expected tax revenues. Without liberalization, $n^* = s_i^* = S^* = 0,$
and this will serve as our benchmark. As tax revenues cancel out in this case, the domestic welfare under autarky is the difference between the domestic gross surplus and aggregate domestic lobbying. Denote this by $W^{AU}$ where the superscript $AU$ means autarky, or pre-liberalization.

$$W^{AU} = V - S = V - \frac{(n-1)(1-t)V}{n} = \frac{(1 + (n-1)t)V}{n}. \tag{3}$$

### 3 Optimal Domestic Policies

Let us now calculate domestic welfare under liberalization. Case 1 (the case of dominant foreign firms) is the easiest to deal with, because domestic lobbying is zero. Foreign lobbying is only wasteful for the foreign country. If foreign firms use domestic resources for lobbying, such resources are exports of services from the domestic to the foreign country. Accordingly, without domestic lobbying, domestic welfare is equal to the tax revenues, i.e., $tV^*$. Comparing $tV^*$ with welfare under autarky, $W^{AU}$ in equation (3), leads to

**Proposition 1** Assume dominant foreign firms, i.e., $V^* \geq \hat{V}$. Then trade liberalization eliminates domestic lobbying. This improves domestic welfare if and only if

$$V^* > \frac{1 + t(n-1)}{nt}V \equiv V' \equiv \frac{W^{AU}}{t}. \tag{4}$$

Given $V^* \geq \hat{V}$, the condition $V^* > V'$ holds if and only if the tax rate is sufficiently high:

$$t \geq \frac{n^* - 1}{n + (n^* - 1)}. \tag{5}$$

Proposition 1 demonstrates that dominant foreign contestants will improve domestic welfare, if they are substantially outnumbered by potential domestic contestants. The welfare gain comes from both the elimination of wasteful domestic lobbying and the tax revenue from the foreign firm. Note that a high tax rate also implies low level of lobbying under autarky, hence the benefits from liberalization do not mainly come from avoiding wasteful lobbying but from taxing the high gross surplus of a foreign firm.
In Case 2, where foreign firms are not dominant, domestic lobbying and foreign lobbying co-exist under liberalization. The foreign gross surplus is smaller than the threshold $\tilde{V}$, and liberalization has two opposite effects on a country’s welfare. First, it improves welfare because wasteful domestic lobbying is reduced (but not eliminated) and replaced by foreign lobbying. Second, it reduces domestic (expected) welfare as a foreign contestant may win the contest, resulting in the surplus net of taxes being transferred abroad. Appendix A.1 deals with details on the welfare gains when both domestic and foreign firms actively lobby. Let $\Delta$ denote the welfare change due to liberalization. It is shown in Appendix A.1 that, for $V^*$ in the non-empty range $[(n-1)V/n, n^*V/(n^*-1)]$ (so that both groups of firms actively lobby), $\Delta$ can be expressed as a cubic function of $V^*$:

$$\Delta(V^*) = A(V^*)\frac{\Omega(V^*)}{n^2t}.$$  

(6)

with

$$\Omega(V^*) = V^{*2} - \frac{2n(1-t) - n^*n + n^2t}{n^2t}V^{*}V - \frac{n^*(1-t) + nn^*}{n^2t}V^2.$$  

(7)

$$A(V^*) = \frac{n^*(nV^* - (n-1)V)}{n(n^*V + nV^*)^2} > 0 \text{ for } V^* > \frac{(n-1)V}{n}.$$  

(8)

Our next proposition shows that the foreign surplus must be sufficiently large as to make liberalization worthwhile.

**Proposition 2** Assume that $\frac{(n-1)V}{n} < V^* < \frac{n^*V}{n^*-1} \equiv \tilde{V}$, so that both groups of firms lobby.

1. Liberalization results in a negative domestic welfare change if foreign contestants produce the same surplus ($V^* = V^*$).

2. Liberalization results in a negative domestic welfare change if the threshold level $\tilde{V}$ is below $V^*$.

3. Assume $\tilde{V} > V^*$. Then there exists a critical surplus level $\bar{V}^*$, where $V < \bar{V}^* < \tilde{V}$ and
(a) liberalization results in a negative domestic welfare change if $V^* \in \left( \frac{(n-1)V}{n}, \bar{V}^* \right]$ and

(b) liberalization results in a positive domestic welfare change if $V^* \in \left[ \bar{V}^*, \tilde{V} \right]$. 

Proof: see Appendix A.1. □

The intuition for part 1 is simple: domestic welfare declines because the reduced waste of lobbying resources from lower domestic lobbying is small relative to the risk of rent being shifted to foreign firms. The other two parts demonstrate that welfare declines with liberalization when domestic and foreign contestants co-exist and $V^*$ is not very large, but a sufficiently large $V^*$ will guarantee a positive welfare effect.

These results can be illustrated best by two numerical examples. These two examples use the same parameter values except for the number of foreign contestants. In the first (respectively, second) simulation, we set $n^* = 30$ (respectively, $n^* = 5$) so that the threshold level $\tilde{V}$ is below (respectively, above) $V'$, i.e., part 2 (respectively, part 3) of Proposition 2 applies. Due to our parameter specification, foreign contestants become active if $V^*$ is larger than 95. The value of $V'$ is equal to 111.67 in both simulations.

![Figure 1: Domestic welfare change for $\tilde{V} < V'$](image)

Figure 1 refers to part 2 of Proposition 2 as $\tilde{V} = 103.45 < V'$. In this case, coexistence of both domestic and foreign contestants (i.e., when $V^*$ is

3Both simulations assume $n = 20, V = 100$ and $t = 0.3$; $n^* = 30$ implies $\tilde{V} < V'$ (see Figure 1) and $n^* = 5$ implies $\tilde{V} > V'$ (see Figure 2).
in the range $(95, 103.45)$ will unambiguously deteriorate domestic welfare relative to the pre-liberalization level. The change in welfare is U-shaped and non-monotonic, however. If the foreign surplus is larger than $\tilde{V}$, foreign firms dominate and domestic welfare increases linearly with $V^*$, but the welfare effect of liberalization is still negative until $V^*$ reaches $V'$. (When $V^* > V'$, Proposition 1 applies: the gain from liberalization is positive.)

Figure 2 refers to part 3 of Proposition 2 as $\tilde{V} = 125 > V'$. For $V^* > 125$, foreign firms are dominant, and in this case liberalization will unambiguously improve domestic welfare. However, domestic welfare improves also for foreign surplus levels in the range $(115, 125)$ where both domestic and foreign firms lobby. Once again, the welfare gain is U-shaped in the coexistence range, and liberalization leads to domestic welfare losses for low levels of $V^*$, i.e., for $V^*$ in the range $(95, 115)$.

What are the implications for domestic liberalization policies? Obviously, a country will agree to accept foreign contestants only if the foreign surplus is sufficiently large. If the foreign-dominance threshold level is low, foreign and domestic contestants will co-exist only if the foreign surplus is close to the domestic surplus. In this case, liberalizing trade does not make sense as the reduction in domestic lobbying efforts is only moderate but the expected loss in expected domestic profits is substantial.

**Corollary 1** 1. If $V^* \leq V$, the domestic country does not want to liberalize trade.
2. If $V^* > V$, the domestic country

(a) does not want to liberalize trade if $V^*$ is too small,

(b) does want to liberalize trade if $V^*$ is sufficiently large.

Proof: This follows from Propositions 1 and 2. □

Note that Corollary 1 determines the preference of the domestic country in terms of a specific procurement only. The country under consideration may have credibly committed to liberalize trade, including governmental procurements. In this case, the potential loss may be overcompensated by potential gains from trade in governmental procurements for which the domestic welfare effect is positive and/or by gains from trade in other areas. Proposition 1 shows that liberalizing governmental procurements may not qualify as a source for gains from trade in general.

4 Optimal Global Policies

In this section, we want to discuss whether and how the optimal domestic policy will differ from the optimal global policy. From the viewpoint of globally optimal policies, both the (wasteful) lobbying efforts of the foreign firms and the expected payoff for foreign firms have to be taken into account. Note carefully that – compared to domestic welfare without trade liberalization – the global expected surplus depends on the winning probabilities unless $V = V^*$. The global expected surplus is the expected surplus of all domestic and foreign firms, i.e.,

$$\frac{ns_i V}{S + S^*} + \frac{n^* s_i^* V^*}{S + S^*}.$$ 

Once again, we have to distinguish between dominant foreign firms and coexistence of domestic and foreign contestants. For the case of dominant foreign firms, global welfare increases (decreases) if

$$\frac{(1 + (n^* - 1)t)V^*}{n^*} > (\prec)\frac{(1 + (n - 1)t)V}{n} \iff V^* > (\prec)\frac{n^* V (1 + n(1 + t))}{n(1 + n^*(1 + t))}.$$ 

(9)
Condition (9) follows from the welfare autarky level (3); in case of dominant foreign firms, global welfare is equal to expression (3) with domestic values replaced by foreign values. Two observations are noteworthy. First, both a negative and a positive welfare change are possible, since the foreign-threshold level which renders foreign firms dominant does not have any bearing on expression (9). In fact, we can easily construct cases under which
\[ \frac{n^*V(1 + n(1 + t))}{n(1 + n^*(1 + t))} > \frac{n^*V}{n^* - 1}. \]
This means that a global welfare loss is possible even when foreign firms dominate. Second, when comparing expression (9) with \( V' \) of Proposition 1 (see eq. (4)), a similar conclusion holds true. We can easily find parameter values such that either
\[ \frac{n^*(1 + n(1 + t))}{n(1 + n^*(1 + t))} > \frac{W^{AU}}{t} \quad \text{or} \quad \frac{n^*(1 + n(1 + t))}{n(1 + n^*(1 + t))} < \frac{W^{AU}}{t} \]
holds true. This leads to the following lemma.

**Lemma 1** In the case of dominant foreign firms, trade liberalization may be globally welfare deteriorating for sufficiently low levels of \( V^* \). Global welfare may decline when domestic welfare increases.

We now turn to the case of co-existence of domestic and foreign firms. Appendix A.2 has all the details for this case, and we find the following result:

**Lemma 2** In the case where both groups of firms are lobbying, global welfare increases (decreases) if
\[ V^* > (<) \frac{(n + 1 - t)V}{n}. \]  
(10)

Proof: See Appendix A.2. □

Condition (10) is not in conflict with the dominance definition (1). Therefore global welfare may decline or increase also in the case where both foreign and domestic contestants lobby. This can be demonstrated by considering the
special case of symmetric foreign and domestic firms, i.e., if \( V = V^* \). Global welfare in this case is given by

\[
\frac{(1 + (N - 1)t)V}{N}
\]

where \( N = n + n^* \) denotes the total number of domestic and foreign firms. Trade liberalization increases the number of contestants, and differentiating global welfare w.r.t. \( N \) yields \(-(1 - t)V/N^2 < 0\). Thus, global welfare unambiguously declines with trade liberalization (as does domestic welfare) if firms are symmetric.

Furthermore, condition (10) is also compatible with the condition of domestic welfare improvement of Proposition 2. We can easily find parameter values that are consistent with both (10) and

\[
\frac{(n + 1 - t)V}{n} \geq V^*
\]

(see eq. (A.5) in Appendix A.1). Hence, even in the case of co-existence of domestic and foreign contestants, global welfare may decline when domestic welfare improves.

Why can global welfare decline when domestic welfare increases at the same time? The intuition is as follows. While domestic welfare does not take the foreign rent into account, it also does not include as a negative term the wasteful foreign lobbying. If the foreign surplus is moderate, the increase in global welfare if a foreign firm were allowed to lobby is relatively small. However, there is a huge increase in foreign lobbying efforts. From the viewpoint of the domestic country, however, the increase in foreign surplus and the decrease in domestic lobbying may be sufficient to overcompensate the expected loss in rents by the increase in expected tax revenues. Our final Proposition summarizes these results.

**Proposition 3** The domestically optimal liberalization policy may be either excessive or too restrictive in case of a moderately large foreign surplus.

Proof: This follows from Lemma 1 and Lemma 2. □

The case of excessive trade liberalization can occur only for a moderate foreign surplus. If the foreign surplus increases, both domestic and global
welfare will eventually improve. Although (wasteful) foreign lobbying efforts increase with foreign surplus, this effect is overcompensated by sufficiently large tax revenues to make domestic welfare improve and by a sufficiently large expected surplus to make global welfare improve.

5 Concluding remarks

This paper has shown that a country will be reluctant to liberalize government procurement if foreign contestants are not able to generate a substantially larger surplus than domestic firms. For low levels of foreign surplus, the gain from reducing wasteful domestic lobbying is more than offset by the increased likelihood that the rent will be captured by a foreign firm. Hence, countries can be expected to be more likely to agree to liberalization of government procurements only if the productivity of foreign firms is substantially larger than that of domestic firms.

We have also compared the optimal domestic policies with the optimal global policies, and the results are mixed. For low levels of surplus generated by foreign firms, we may find that domestic liberalization policies are excessive from the global welfare point of view. While domestic welfare improves, global welfare may go down, because foreign firms excessively lobby for success in the domestic market. Hence, both an excessive or a too restrictive domestic policy may be observed. These effects should be taken into account when liberalizing governmental procurements, in particular between similar countries with similar firms.

Of course, even if the welfare effect of liberalizing government procurements is negative, the overall welfare effect of trade liberalization, including trade of goods and service on private markets, may still be positive and substantially large. Furthermore, changing the rules of the game such that governmental procurement is turned from an imperfectly discriminating contest into a real auction in which partisanship does not play any role will definitely make a positive contribution to domestic and global welfare. While positive gains may be observed in highly integrated areas such as the European Union, some empirical evidence still points to the other direction. If
governmental procurements continue to resemble a Tullock contest, our paper has shown that liberalization does not necessarily qualify as a source for gains from trade.

Appendix

A.1 Domestic liberalization effects

Maximization of (1) w.r.t. $s_i$ and $s^*_i$, respectively, yields individual lobbying levels of

$$s_i = \frac{(n + n^* - 1)(n^*V - (n^* - 1)V^*)(1 - t)VV^*}{(n^*V + nV^*)^2}, \quad (A.1)$$

$$s^*_i = \frac{(n + n^* - 1)(nV^* - (n - 1)V)(1 - t)VV^*}{(n^*V + nV^*)^2}.$$

Since $S = nS_i$ and $S^* = n^*S^*_i$, the probability that a given domestic [respectively, foreign] contestant wins is $(n^*V - (n^* - 1)V^*)/(n^*V + nV^*)$[respectively, $(nV^* - (n - 1)V)/(n^*V + nV^*)$], and the probability that the winner is a domestic [respectively, foreign] contestant is $(n^*V - (n^* - 1)V^*)n^*/(n^*V + nV^*)$[respectively, $(nV^* - (n - 1)V)n/(n^*V + nV^*)$]. Thus aggregate welfare is equal to $B + T - S$, where $B$ is the gross surplus of a domestic firm multiplied by the probability that a domestic firm wins,

$$B \equiv \frac{n^*V - (n^* - 1)V^*}{n^*V + nV^*}nV, \quad (A.2)$$

and $T$ is the tax revenue from a winning foreign firm, multiplied by the probability that a foreign firm wins,

$$T \equiv \frac{(nV^* - (n - 1)V)}{n^*V + nV^*}n^*tV^*, \quad (A.3)$$

and $S$ is the resource costs incurred by domestic firms (wasteful) lobbying. The difference between $B + T - S$ and the autarky welfare level according to (3) yields expression (6). Consider the graph of $\Delta(V^*)$. Now, at $V^* = V$,
\( \Omega < 0, \) hence \( \Delta(V^* = V) < 0. \) This proves part 1 of Proposition 2. Observe that at \( V^* = (n - 1)V/n, \) \( \Delta = 0. \) Furthermore, \( G = 0 \) if \( \Omega = 0. \)

Consider the following cubic equation in \( V^* \)

\[
\Delta(V^*) = A(V^*)\Omega(V^*) = 0
\]

This cubic equation has three roots, one at \( V^* = (n - 1)V/n, \) at which \( A(V^*) = 0. \) The other two roots are solutions of the quadratic equation \( \Omega(V^*) = 0. \)

Let the two roots of the quadratic equation for \( \Omega = 0 \) be denoted by \( V^{*\,(1)} \) and \( V^{*\,(2)} \). According to Vieta’s Theorem, the product of the roots is

\[
V^{*\,(1)}V^{*\,(2)} = -\frac{n^{*}(1 - t) + nn^{*}}{n^2t}V^2 < 0.
\] (A.4)

which shows one root, say \( V^{*\,(1)} \), must be negative and the other root, \( V^{*\,(2)} \), is positive. Since \( \Omega(V^*) \) tends to infinity as \( V^* \) tends to plus or minus infinity, and \( \Omega(V^* = V) < 0, \) it follows that the positive root \( V^{*\,(2)} \) is larger than \( V. \)

Differentiating \( \Delta \) at \( V^* = (n - 1)V/n \) yields

\[
\frac{d\Delta(V^*)}{dV^*} = \frac{(n - 1)V/n}{n^2} = -\frac{n^n(2 - t) + 2n(n - 1)}{n^2} < 0
\]

This shows that the gain from liberalization is negative for low levels of \( V^*. \) Furthermore,

\[
\Omega(V^* = V) = -V^2(2n + n^*)(1 - t) < 0.
\]

Consider two cases:

**Case I**, where \( \check{V} \leq V' \), and

**Case II**, where \( \check{V} > V' \).

Under Case I, where \( \check{V} \leq V' \), our Proposition 1 shows that at \( V^* = \check{V}, \) no domestic firm will lobby, and liberalization reduces welfare, i.e., \( \Delta(V^* = \check{V}) < 0. \) Note that the welfare change is continuous in \( V^*. \) It follows that, in Case I, the root \( V^{*\,(2)} \) must exceed \( \check{V}, \) and therefore is irrelevant. (Recall that for \( V^* > \check{V}, \) only foreign firms lobby). This proves part 2 of Proposition 2.
Next, consider Case II, where $\tilde{V} > V'$. The welfare gain is positive for $V^* = \tilde{V} > V'$, when foreign contestants dominate. According to (A.4), a critical level $\bar{V}^*$ (which is the positive root $V^{*2}$ of the quadratic equation $\Omega(V^*) = 0$) must exist with $\Delta(\bar{V}^*) = 0$.

In fact, we may write

$$\Omega(V^*) = V^{*2} + bV^* + c$$

with

$$b = \frac{(nn^* - n^2t - 2n(1 - t))V}{n^2t},$$

$$c = -\frac{(nn^* + n^*(1 - t))V^2}{n^2t} < 0,$$

and it follows that

$$V^* = \frac{-b + \sqrt{b^2 - 4c}}{2}$$

$$= \frac{V}{2nt} \left(\sqrt{n^* + (2 + (n - 2)t)^2 - 2n^*(3n + 2 - 2t^2) - (n^* - 2) + (n - 2)t}\right) > 0.$$ (A.5)

### A.2 Global liberalization effects

The joint lobbying effort of both types are equal to

$$S_i + S_i^* = ns_i + n^*s_i^* = \frac{(n + n^* - 1)(1 - t)VVn^*}{n^*V + nV^*},$$ (A.6)

where (2) has been used. The probability of a domestic [foreign] firm winning is $s_i/(S_i + S_i^*)[s_i^*/(S_i + S_i^*)]$ which leads to an expected surplus across all firms of

$$\frac{ns_iV + n^*s_i^*V^*}{S_i + S_i^*} = \frac{(n + n^*)VV^* + nn^*(V^* - V)^2}{n^*V + nV^*}.$$ (A.7)

The difference between (A.7) and (A.6) determines global welfare, denoted by $W^G$:

$$W^G(V^*) = \frac{nn^*(V^* - V)^2 + ((n + n^*)t + (1 - t))VV^*}{n^*V + nV^*}.$$ (A.8)
Differentiating w.r.t. $V^*$ yields

$$\frac{dW^G}{dV^*} = n^* (2n^*VV^* + n^2V^*2 + ((n + n^*)t + (1 - t) - n(2n^* + n))V^2) \over (n^*V + nV^*)^2.$$  

(A.9)

Global welfare improves (deteriorates) if $W^G$ is larger (smaller) than $W^{AU}$. Setting $W^G$ equal to $W^{AU}$ yields two solutions, denoted by $V^{*\text{(3)}}$ and $V^{*\text{(4)}}$:

$$V^{*\text{(3)}} = \frac{n - 1}{n}V, V^{*\text{(4)}} = \frac{n + 1 - t}{n}V.$$ 

$V^{*\text{(3)}}$ coincides with the lower bound of Assumption 1. Computing the first derivative of global welfare w.r.t. $V^*$ for $V^* = V^{*\text{(3)}}$ gives

$$\frac{dW^G(V^{*\text{(3)}})}{dV^*} = -n^* ((n + n^* - 1)(2 - t)V^2) \over (n^*V + nV^{*\text{(3)}})^2} < 0.$$  

(A.10)

Expression (A.10) proves that global welfare declines with an increase of $V^*$ for low levels of $V^*$, and improves once $V^*$ has passed $V^{*\text{(4)}}$. This proves part of Lemma 2.

References


