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The Mortensen Rule and Efficient Coordination Unemployment*

Benoit Julien, John Kennes and Ian King†

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Abstract

We apply the efficiency axioms of Mortensen (1982) to a set of matching games involving coordination frictions between buyers. These games include markets with finite numbers of buyers and sellers and markets with infinite numbers of heterogenous buyers and homogenous sellers. We show that the Mortensen rule, but not the Hosios rule, gives constrained efficient allocations. We also show that the Mortensen rule is implemented by a simple auction.

1 Introduction

Matching games are often used to illustrate a social dilemma: The actions of each player affect the probability they will be responsible for future events involving others, and the way in which the capital value of each event is divided between the players involved determines the incentives for such actions. Mortensen (1982) proposes a novel way to think about the optimal surplus division in these social situations. He argues that the surplus of a match should be rewarded to the agent that initiates the match.

The present paper has three goals. The first goal is to show that the Mortensen rule can be applied to an interesting class of matching games involving coordination frictions between buyers (see Burdett, et al., 2001, Julien et al., 2000 and Shimer, 1999, 2004). The second goal is to show that an alternative rule, known as the Hosios rule (see Hosios 1990), cannot be applied to these same matching games. The final goal is to link the Mortensen rule to a simple auction.

2 Finite markets and homogeneous agents

The market consists of a finite number of M identical buyers and a finite number of N identical sellers. Each seller has one identical good for sale. The seller's reservation value of the good is zero and each buyer's valuation of the good is $y > 0$. Buyers are randomly allocated to the sellers in the following way: buyers

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can visit only one seller and the probability that a buyer visits any particular seller is given by $\pi = 1/N$. The expected number of matches is given by

$$x(N, M) = N(1 - (1 - 1/N)^M) \quad (1)$$

This "matching function" has decreasing returns to scale but, in the limit where M and N are large, has constant returns to scale.

The random assignment of buyers to sellers yields a set of local markets, each with one seller and a stochastic number of buyers. Let $\theta(m)$ denote the probability that the local market of a *seller* has m buyers. It is easy to show that $\theta(0) = ((N-1)/N)^M$ and $\theta(1) = (M/N)((N-1)/N)^{M-1}$. Let $\xi(m)$ denote the conditional probability that any particular *buyer* is one of m buyers in a local market. The probability the buyer is alone at his or her chosen local market is given by $\xi(1) = ((N-1)/N)^{M-1}$.

The surplus of a match between a buyer and a seller in a local market is equal to the value of the match minus the disagreement points of the seller and the buyer. Each local market contains one seller S, a set of identical buyers $B = (B_1, B_2, .., B_m)$ if $m \geq 1$, and no buyers if $m = 0$. The surplus of a match between the seller and any particular buyer B_i is given by

$$\Lambda_i(S, B) = V(S, B_i) - d_s(B) - d_i(B) \quad (2)$$

where $V(S, B_i)$ is the total value of the match, $d_s(B)$ is the disagreement point of the seller, and $d_i(B)$ is the disagreement point of the buyer. The total valuation of a match is given by $V(S, B_i) = y$. The disagreement point of the *buyer* is equal to zero, because once inside the the local market, the buyer can trade only with the seller. The disagreement point of the *seller* is given by $\max V(S, B_{-i})$ - the maximum total value of the good to the seller and the set of other buyers.[1]

The Mortensen rule is given by two basic axioms.

- **Axiom 1.** The pair of local market participants with the highest value $V(S, B_i)$ formed a match if the surplus $\Lambda_i(S, B)$ is positive (local efficiency).
- **Axiom 2.** The surplus of the match $\Lambda_i(S, B)$ is rewarded to the initiator of the match (Mortensen rule).

Axiom 1 of the Mortensen rule is common to Nash's (1953) solution concept. Axiom 2 presumes that the identity of the match initiator is known - which we take to be the buyer in games of coordination frictions.

The Mortensen rule is equivalent to an auction by the seller. In particular, the seller obtains a winning bid equal to the second highest valuation of the good. Thus the seller earns a price y if there are multiple buyers in his/her local market and a price of zero otherwise. Suppose that the number of buyers is determined by free entry with each additional buyer to the market paying a cost, k. We find[2]

[1] This disagreement point implicitly assumes a particular stage game where the seller can always extract the maximum total value of the good to the seller and the remaining set of other buyers that are committed to this local market. See Groes and Tranæs (1999) for one possible version of such a game.

[2] A similar result holds if the supply of buyers is fixed and the number of sellers is determined by free entry with each additional seller to the market paying a cost, k'.

Proposition 1 *In a finite sized market with homogeneous agents and prices set by the Mortensen rule, the marginal private benefit of an extra buyer equals the marginal social benefit of an extra buyer.*

Proof. The marginal private benefit of the last buyer is given by the expected payoff, $\xi(1)y - k$. The marginal social benefit of the last buyer is given by the extra matches created less the additional cost of an extra buyer. It is easy to verify that $x(N, M)y - x(N, M-1)y - k = \xi(1)y - k$ ■

The Hosios rule cannot be applied to this matching game because the matching function in a finite sized market does not display constant returns.[3]

3 Large markets with heterogeneity

Consider a simple matching game with a large number of homogenous firms and heterogeneous workers. That is we send M and N to infinity but keep the ratio constant. In analyzing this game, we first cast firms in the role of sellers and workers in the role of buyers. We then consider the alternative with workers in the role of sellers and firms in the role of buyers. In either case, we assume a buyer can choose to visit only one seller.

Let M denote the number of identical unmatched firms (i.e vacancies) and let $N = \phi M$ denote the number of unmatched workers. Let n_2 denote the fraction of workers that have high human capital, n_1 denote the fraction of workers that have low human capital and n_0 denote the fraction of workers that have no human capital at all. If workers can choose only one firm to visit and are randomly allocated to firms, the aggregate quantity of job matches involving workers with some amount of human capital is given by

$$x(M, n_1 N, n_2 N) = M(1 - e^{-n_2\phi}) + M e^{-n_2\phi}(1 - e^{-n_1\phi}) \tag{3}$$

where $M(1 - e^{-n_2\phi})$ is the number of matches between jobs and workers with high human capital and $Me^{-n_2\phi}(1 - e^{-n_1\phi})$ is the number of matches between jobs and workers with low human capital. We are implicitly assuming, at this point, that high quality workers are chosen over low quality workers.

Workers invest in human capital by choosing education. In particular, let s denote a worker's education choice and let $h(s)$ denote this worker's level of human capital, which is also the output of the worker when matched to a firm. For simplicity, assume that workers can choose either high human capital, h_2, low human capital, h_1, or no human capital at all. These choices are related by the following education opportunity set:

$$h(s) = \begin{cases} h_2 & \text{if } s = s_2 \\ h_1 & \text{if } s = s_1 \\ 0 & \text{if } s = 0 \end{cases} \tag{4}$$

where $h_2 > h_1 > 0, h_1 > s_1$ and $h_2 > s_2$. We assume *positive but diminishing returns* to education: $h_2 - s_2 > h_1 - s_1$ and $h_1/s_1 > h_2/s_2$.

[3] However, the Hosios rule can be applied in the limiting case with many buyers and sellers In this case, Hosios (1990) shows that a simple price posting game implements his solution. See also Kultti (1999).

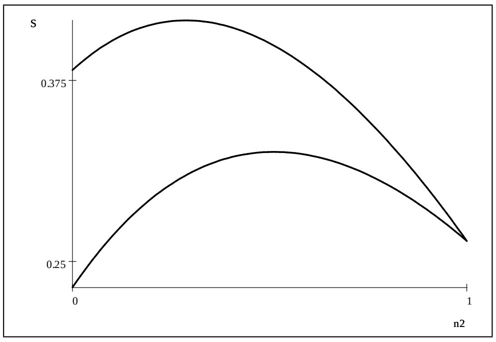

Figure 1: Social welfare with heterogeneous human capital investments

3.1 The planner's solution

A social planner maximizes the total output of the economy less the workers' investments in education as follows:

$$S = \max_{n_1,n_2} \left\{ M(1-e^{-n_2\phi})h_2 + Me^{-n_2\phi}(1-e^{-n_1\phi})h_1 - n_2Ns_2 - n_1Ns_1 \right\} \quad (5)$$

$$\text{s.t. } 1-n_1-n_2 \geq 0 \text{ and } n_1,n_2 \geq 0.$$

Proposition 2 *A social planner chooses heterogeneous human capital if the education opportunity sets of workers display positive but diminishing marginal returns.*

Proof. Define $X \equiv e^{-n_2\phi}e^{-n_1\phi}h_1 - s_1$ and $Y \equiv e^{-n_2\phi}e^{-n_1\phi}h_2 + e^{-n_2\phi}(1-e^{-n_1\phi})(h_2-h_1) - s_2$. The solution to the social planning problem is

$$n_1,n_2 = \begin{cases} 1,0 & if \quad X>Y \\ 0,1 & if \quad X<Y \\ n_1, 1-n_1 & if \quad X=Y \end{cases}$$

It is easy to verify that $n_1 > 0$ and $n_2 > 0$ if the education opportunity set has positive but diminishing returns. ■

Figure 1 illustrates the interior solution of the social planning problem for two parameterizations of the education opportunity sets of workers ($\phi = 1, h_2 = 2, h_1 = 1, s_2 = 1,\ s_1 \in \{.25, .4\}$). The vertical axis is social welfare and the horizontal axis is the fraction of workers choosing high human capital. The top line gives social welfare when low human capital is cheap ($s_1 = .25$) and the lower line gives social welfare when low human capital is more expensive ($s_1 = .4$). Raising the cost of low human capital encourages the social planner to make more workers invest in high human capital. In both cases, the social planner maximizes social welfare by a heterogeneous assignment of human capital.[4]

The intuition behind proposition 2 is that expensive high human capital gives high returns in any match, but also creates the largest costs associated

[4] Moen (1999) shows that a distribution of investment levels is efficient in a similar model. See Julien et. al. (2004) for a model with a distribution of investment levels by firms and on-the-job search by workers.

with coordination frictions, which exist whenever two workers with high human capital approach one job vacancy. These events creates a large loss in social welfare because only one worker can be hired. Thus there is a trade-off between ensuring that workers invest in the highest human capital possible and ensuring that cost of coordination frictions are as small as possible.

3.2 The decentralized economy

Local markets in the decentralized economy have heterogeneous buyers. In particular, each local market consists of one seller (the firm), m_1 workers with low human capital and m_2 workers with high human capital. Let $\xi(m_2+m_1|1)$ denote the probability that a buyer with low human capital is in a local market with m_2+m_1 buyers of jobs. Random matching implies that the probability a worker is alone in this submarket is given by $\xi(1|1)=e^{-\phi}$. Let $\xi(m_2,m_1|2)$ denote the conditional probability that a worker with high human capital is in a local market with m_2 workers with high human capital and m_1 workers with low human capital. Random matching also implies $\xi(1,0|2)=e^{-\phi}$. Moreover, the probability a seller with high human capital is in a local market consisting of sellers with low human capital but no other sellers of high human capital is given by $\xi(1,m_1\geq 1|2)=e^{-n_2\phi}(1-e^{-n_1\phi})$

The surplus of a match between a buyer and a seller in a local market is equal to the value of the match minus the disagreement points of both the seller and the buyer. Each local market contains one seller S and a set of buyers with low human capital, $L=\{(B_1,B_2,..,B_{m_1})$ if $m_1\geq 1, 0$ otherwise$\}$, and a set of buyers with high human capital $H=\{(B_{m_1+1},B_{m_1+1},..,B_{m_1+m_2})$ if $m_2\geq 1, 0$ otherwise$\}$. In a local market defined by L and H, The surplus of a match between the seller and any particular buyer B_i is given by $\Lambda_i(S,B)=V(S,B_i)-d_s(L,H)-d_i(L,H)$ where $V(S,B_i)$ is the total value of the match, $d_s(L,H)$ is the disagreement point of the seller, and $d_i(L,H)$ is the disagreement point of the buyer. The total value of a match is given by

$$V(S,B_i)=\begin{cases} h_1 & \text{if } i\leq m_1 \\ h_2 & \text{if } m_1+1\leq i\leq m_1+m_2 \end{cases} \quad \forall\, i\in\{1,...,m_1+m_2\} \tag{6}$$

The disagreement point of the buyer is zero and the disagreement point of the seller is $\max V(S,B_{-i})$ - the maximum total valuation of the good to the seller and the set of other buyers.

The Mortensen axioms are applied to this matching game as follows. By axiom 1, the match is always created between the highest valuation worker and the firm in each local market. By axiom 2, the surplus of the match is rewarded to the initiator - in this case, the worker who chooses which job to visit. Therefore, the expected payoff $\Pi(s_1)$ of a worker investing in low human capital is given by $\Pi(s_1)=e^{-\phi}h_1-s_1$. The expected payoff $\Pi(s_2)$ of a worker investing in high human capital also includes a positive return if the second best candidate has low human capital. Thus $\Pi(s_2)=e^{-\phi}h_2+e^{-n_2\phi}(1-e^{-n_1\phi})(h_2-h_1)-s_2$.

Proposition 3 *If wages are determined by the Mortensen rule, the decentralized economy is equivalent to the socially planned economy that maximizes aggregate output.*

Proof. The payoffs $\Pi(s_1)$ and $\Pi(s_2)$ are equal to X and Y in Proposition 2. Therefore, the equilibrium of the decentralized economy is equivalent to the solution to the social planning problem. ■

Of course, the Hosios rule cannot be applied to this matching game, because the equilibrium matching function, $x(M, n_1N, n_2N)$, has three arguments. Therefore, the elasticity conditions of Hosios (1990) cannot be used to determine an efficient sharing rule.

3.3 What makes a seller?

Drawing on work by Acemoglu (2001) and Moen (1997), Ljungvist and Sargent (2004) report that efficient matching is possible, in the Hosios sense, only if heterogenous agents are allocated to submarkets. Therefore, it is instructive to reverse the role of sellers and buyers in the previous matching game such that two distinct submarkets now operate for the two types of sellers - where sellers in this case are now workers. In this case, efficient matching demands that either all workers invest in high human capital, or all workers invest in low human capital. If $\phi = 1$, the maximum value of social welfare, S^r, is given by:

$$S^r = \max\{(1 - e^{-1})h_2 - s_2, (1 - e^{-1})h_1 - s_1\} \tag{7}$$

How does this level of welfare compare to the game without submarkets? Consider figure 1 where we reported S for $\phi = 1, h_2 = 2, h_1 = 1, s_2 = 1$, $s_1 \in \{.25, .4\}$. If $s_1 = .25$, then S^r is given by the corresponding value of S at $n_2 = 0$. Likewise, if $s_1 = .4$, then S^r is given by the corresponding value of S associated at $n_2 = 1$. In both case, these outcomes represent a drastic reduction in welfare when compared to the game analyzed previously without submarkets. Therefore, even though the Hosios rule can be applied to the modified matching game with submarkets to achieve S^r, this outcome is inefficient compared to a corresponding game without submarkets.

This game does not illustrate that the Hosios rule leads to inefficiency. Instead, the game illustrates that the rules of a matching game can often be modified in such a way that the Hosios rule is made applicable. Here, we have simply modified the rule concerning who approaches whom. Moreover, in this particular game, if such rules are in the choice set of the social planner, the planner would obviously not structure it to correspond to one in which the Hosios rule is applicable.

Why does switching the role of buyers and sellers change welfare? On the one hand, investment dispersion is not socially advantageous on the sellers' side, because we have assumed that each buyer can visit only one seller. On the other hand, investment dispersion is socially advantageous on the buyers' side, because sellers can entertain multiple offers. In this case, the market allows a subtle form of friction reduction in which high cost investments are rarely made redundant and low cost investments are readily made available.

4 Conclusion

This paper developed a simple axiomatic characterization of efficiency in games with coordination unemployment. The approach follows Mortensen (1982) who uses it in a different context. We described two matching games where the

Mortensen rule, but not the Hosios rule, gives efficiency. We also showed that partitioning buyers and sellers into submarkets does not present a solution to the problem of social efficiency. Therefore, the axiomatic methods developed by Moen (1997), which are also used in Sargent and Ljunqvist (2004), are not applicable to this class of matching games.

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