Cross-Country Growth and Convergence: 
A Semi-Parametric Analysis

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Abstract

This paper shows that convergence occurs among countries with very low and very high initial 
incomes, indicating that convergence clubs characterize the cross-country growth process. The 
results provide evidence for non-linear convergence and are consistent with some new growth 
thories.

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1. Introduction

Much of the empirical research on cross-country growth has concentrated on whether per capita incomes converge once structural differences across countries have been accounted for. The notion of (conditional) convergence emerges from the Solow (1956) growth model and, following Barro (1991), the most common approach to identifying convergence is to regress per capita income growth on initial income and other conditioning variables. A negative coefficient on initial income is interpreted as evidence of poor countries growing faster than richer ones. A key assumption in this approach is that cross-country growth is linear, something which holds under the Solow model (Durlauf and Johnson, 1995). In contrast, in some new growth theories cross-country growth behavior is typically non-linear, being characterized by multiple steady states (see, for example, Azariadis and Drazen, 1990). Durlauf and Johnson (1995) examine such possible non-linearity by estimating growth equations for four groups of countries, chosen according to initial conditions, and find that implied rates of convergence differ markedly between the four groups thus suggesting non-linearity between initial income and income growth.\(^1\) Here, we examine the possible non-linearity more precisely by allowing the effect of initial income on income growth to vary across the entire distribution of initial income rather than assuming homogeneity within pre-specified groups.

2. Empirical Analysis

In traditional work on conditional convergence it is customary to estimate the following linear equation:

\[
Y = \alpha + \beta X + \delta Z + \eta
\]  

\(^1\)This suggests multiple steady states. Quah (1996) also finds evidence for multiple steady states in his work on intra-distribution dynamics. For a review and extensive discussion of this aspect of the literature see Durlauf and Quah (1999).
where $Y$ represents growth in real GDP per capita over a fixed period, $X$ is level of real per capita GDP at the start of the period, $Z$ is a vector of variables that control for structural differences across countries, $u$ is a disturbance term and $\beta$ and $\delta$ are coefficients to be estimated. A negative and significant coefficient on $X$ is interpreted as evidence of convergence. As a first step, we estimate (1) using five-year panel data for 103 countries covering the period 1960-1990 generated from the Penn World Tables 5.6 and World Bank publications. The variables in $Z$ are fraction of real GDP devoted to investment, growth in population, primary school enrolment rate, total exports and imports as a fraction of real GDP, and time and country specific dummies. The $\beta$ values from the OLS estimations using five and ten year growth rates for $Y$ were both significantly negative at the one per cent level. However, while these results suggest convergence, a Ramsay (1969) RESET test decisively rejected the null hypothesis that $X$ contained no significant higher order terms in both cases, suggesting that the impact of $X$ on $Y$ may be non-linear and that the finding of overall convergence is perhaps erroneous.

An attractive approach to further investigating the possible non-linearity of the relationship between $Y$ and $X$ in (1) more precisely, while also allowing for the (linear) effect of the conditioning variables $Z$, follows the semi-parametric methodology proposed by Robinson (1988) using the Kernel regression estimator. Accordingly, consider the following equation to be estimated:

$$Y = \alpha + g(X) + \delta Z + u$$ (2)
where \( g() \) is a smooth and continuous, possibly non-linear, function of \( X \), \( Z \) is a vector of variables assumed to have a linear effect on \( Y \), and \( \nu \) is a random error term. Estimation of \( g(X) \) can be made by:\(^6\):

\[
g^\wedge(X) = m_y^\wedge(X) - \delta^\wedge m_z^\wedge(X) \tag{3}
\]

where \( m_y^\wedge(X) \) and \( m_z^\wedge(X) \) are the (non-parametric) Nadaraya-Watson estimates\(^7\) of \( \text{E}(Y \mid X) \) and \( \text{E}(X \mid X) \), such that, for a given continuous, bounded, and real shape function, \( K_h() \) integrating to one with a smoothing parameter \( h \), \( m_y^\wedge(X) \) (and similarly \( m_z^\wedge(X) \)) is defined as:

\[
m_y^\wedge(x) = n^{-1} \sum_{i=1}^{n} K_h(x - X_i)Y_i
\]

\[
m_z^\wedge(x) = n^{-1} \sum_{i=1}^{n} K_h(x - X_i)
\]

and \( \delta^\wedge \) is the OLS estimator of:

\[
Y - m_y^\wedge(Z) = \delta (X - m_z^\wedge(Z)) + \nu \tag{5}
\]

The appeal of the estimator (3) lies in its very flexible approach to non-linearity by allowing the relationship between \( Y \) and \( X \) to vary over all values of \( X \) after purging the effects of other explanatory variables. Specifically, this technique corresponds to estimating the regression function at a particular point by locally fitting constants to the data via weighted least squares, where those observations closer to the chosen point have more influence on the regression estimate than those further away as determined by the choice of \( h \). An additional advantage of this estimator is that it avoids any parametric

\(^6\) The fact that \( \delta \) is in part estimated using OLS makes this a semi- rather than non-parametric estimator.

\(^7\) See Nadaraya (1964) and Watson (1964).
assumptions regarding the conditional mean function $m(x)$, and thus about its functional form or error structure. For all estimations we use a Gaussian kernel for $K_h$ and the optimal smoothing parameter suggested by Fox (1990).

Given the semi-parametric nature of the estimate of $g(X)$ it cannot be subjected to the kind of standard statistical tests (such as an F-test or a t-test) that economists have grown so accustomed to in parametric regressions. However, it is possible to calculate upper and lower point-wise confidence intervals, as suggested by Härdle (1990). Specifically, we calculated bands at the 1st and 99th percentiles along the range of initial income and at every fifth percentile in between. Choosing points according to the distribution of observations also allows one to gauge how the density of the sample affects the approximation bias, since these are inversely related.¹⁸

Figure 1: Semi-Parametric Estimate and Point-wise Confidence Bands

Five-Year Growth Rates

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¹⁸ It should be noted that the accuracy of the estimate of $g(X)$ at $X$ is positively related to the density of other observations around that point. Furthermore, the approximation bias is larger at the boundaries (see, Wand and Jones, 1995).
The graph of \( g(X) \) for five-year growth rates along with the confidence bands is shown in Figure 1. Of particular interest is the general pattern that describes the marginal impact of \( X \) on \( Y \) as initial income increases (moving from left to right along the range of \( X \)).\(^9\) As can be seen, the relationship between \( Y \) and \( X \) is not constant over the range of values of \( X \).\(^{10}\) For small values of \( X \) the relationship is downward sloping, suggesting convergence, although it must be noted that these low-income estimates are based on relatively few observations, as can be seen from the horizontal distance between the surrounding confidence bands.\(^{11}\) For much of the mid-section the slope of the curve is positive, implying that in this range there is divergence. It is also noteworthy that the divergence range contains the mean maximum of the log of initial income (7.55) of those countries that may be classified as ‘developing’ and the mean minimum (8.44) of the ‘developed’ countries. The final part of the estimate of \( g(X) \) suggests convergence (the slope becomes negative again) except towards the end of the distribution, which is based on only around one per cent of the sample. We also estimated \( g(X) \) allowing for a longer time horizon for convergence to occur by using ten-year growth rates. As can be seen from Figure 2, the general pattern remains except in the region of very high values of \( X \) where the upturn has disappeared.

3. Concluding Remarks

In the traditional empirical convergence literature a negative coefficient on initial income in a cross-country growth regression is interpreted as evidence of poor countries growing faster than richer ones. A key assumption in this work is that the relationship between initial income and income growth is linear. In other words, the marginal impact

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\(^9\) Since the dependent variable used in (3) is generated by predicted values its range does not necessarily correspond in range to that of the observed \( Y \)’s.

\(^{10}\) The pattern is very similar when the estimation is done without country fixed effects.

\(^{11}\) The horizontal distance provides an insight into the relative number of neighboring points (close-by points more heavily weighted than distant ones) since the points are chosen according to the distribution of observations. The vertical distance indicates the accuracy of the estimate.
of a change in initial income on income growth is assumed to be constant across the range of initial income. Using a semi-parametric methodology to estimate a conditional convergence equation, this paper provides further evidence to refute the notion of parameter constancy in cross-country growth equations. We find that convergence is not widespread, occurring among countries with very low and very high initial income levels. There is no evidence of convergence among countries with intermediate values of initial income. The finding of non-linearity lends credence to the idea that convergence clubs characterize the cross-country growth process and that there is a clustering of countries in economic growth performance.
References


