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Another Tale of Two-Sided markets

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Abstract

This note generalizes the frequently used Hotelling model for two-sided markets. We demonstrate an invariance theorem: advertisement levels neither depend on the media price nor on the location of the media firm. An increase in advertising revenues does not change location but only the media price. In conclusion, a Hotelling model of two-sided markets is equivalent to one of one-sided markets.

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1 Introduction

Recent years have seen a huge increase in the literature on two-sided markets (e.g., Armstrong, 2006, and Rochet and Tirole, 2003, 2006). Inspired by media markets, many papers have used a Hotelling model of location, media prices and advertising. However, most of these papers make very specific assumptions about competition for advertising and about consumer heterogeneity. In particular, it is typically assumed that consumers are uniformly distributed along the Hotelling line.¹ This tends to oversimplify the analysis

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¹See, for instance, Anderson and Coate (2005), Crampes, Haritchabalet and Jullien (2005), Gabszewicz, Laussel and Sonnac (2001, 2002) and Peitz and Valletti (2004).

of location decisions, characteristically resulting in maximum or minimum differentiation, depending on the set-up of the model.

This note tries to make progress on our understanding of media firms' location by relaxing on the assumption that consumers are uniformly distributed. Moreover, we do not make any specific assumption about the type of competition in the advertisement market. Media firms can compete by prices or by ad space, and we allow for both single-homing and multi-homing.

Within this set-up we show that competition on locations does not necessarily result in maximum or minimum differentiation. More surprisingly, we find that a generalized Hotelling model of two-sided markets is not much different from that of one-sided markets if profit functions are well-behaved and if solutions are interior. Our invariance theorem shows that advertisement policies can be determined independently from media prices and location decisions. Advertisement revenue works like a reduction in marginal cost of the media product, and the decision on prices and location is similar to Anderson *et al* (1997). The following section develops these results, and the final section concludes.

2 The model

We employ a Hotelling model with two competing media firms, $i = 1, 2$. Media firm i charges price p_i and is located at x_i . Without loss of generality, we assume that $x_2 \geq x_1$. The media firms also sell advertising space to producers, and the resulting advertising level is given by a_i . The media consumers may either have a negative or a positive attitude towards ads, and the net utility level of a consumer located at x who buys media product i is given by $U = v - p_i - t(x - x_i)^2 - d(a_i)$. With this specification the consumers perceive ads as a bad if $d(a_i) < 0$ and as a good if $d(a_i) > 0$.² The constant $v > 0$ is assumed to be sufficiently large to ensure market coverage.

Denoting the consumer who is indifferent between buying media products 1 and 2 by \tilde{x} , we find

²See Depken II and Wilson (2004) and Sonnac (2000) for a discussion of whether magazine/newspaper readers consider advertising as a good or a bad.

$$\tilde{x} = \frac{1}{2} \left(x_1 + x_2 + \frac{p_2 - p_1 + d(a_2) - d(a_1)}{t(x_2 - x_1)} \right). \quad (1)$$

Consumers located to the left of \tilde{x} buy media product 1, while consumers to the right of \tilde{x} buy media product 2.

The consumers are continuously distributed on $-\infty \leq a < b \leq \infty$, and the cumulative distribution is denoted by $F(x)$. We normalize the population size to one, and the density function $f(x) = F'(x)$ is assumed to be log-concave on $[a, b]$ and twice differentiable. The marginal costs of producing the media product equal c , and for simplicity we let the marginal costs of inserting ads be zero, so that the profit functions of the two media firms read as

$$\begin{aligned} \Pi_1 &= F(\tilde{x})(p_1 - c + A_1(\cdot)), \\ \Pi_2 &= (1 - F(\tilde{x}))(p_2 - c + A_2(\cdot)), \end{aligned} \quad (2)$$

where A_i is advertising revenue per consumer. As usual in the literature, aggregate ad revenues depend linearly on the number of media consumers. Otherwise, the model is very general. We allow for both single-homing and multi-homing, and assume that ad revenues per consumer depend on the strategic variables s_1 and s_2 , such that $A_i = A_i(s_1, s_2)$. Advertisement levels are also a function of these strategic variables, such that $a_i = a_i(s_1, s_2)$. In a simple Cournot setting we have $s_i = a_i$. But the model also allows for price competition on the ad-market, *i.e.* it can accommodate competition in strategic substitutes as well as strategic complements.

In the following we consider a two-stage game, where the media firms choose locations before they simultaneously compete for consumers and advertising revenue (setting p_i and s_i , respectively). We assume that the profit functions (2) are quasi-concave in p_i and s_i , and that solutions are interior.³ Thereby, we can use the first-order conditions to determine optimal prices and advertising strategies.

³In one-sided markets, log-concavity of $f(x)$ guarantees that the sufficient conditions w.r.t. p_i are fulfilled (see Anderson *et al*, 1997). However, this condition is not sufficient in the setting here.

As for prices we find that

$$\begin{aligned}\frac{\partial \Pi_1}{\partial p_1} &= F(\tilde{x}) + (p_1 - c + A_1)f(\tilde{x})\frac{\partial \tilde{x}}{\partial p_1} = 0, \\ \frac{\partial \Pi_2}{\partial p_2} &= [1 - F(\tilde{x})] + (p_2 - c + A_2)f(\tilde{x})\left(-\frac{\partial \tilde{x}}{\partial p_2}\right) = 0.\end{aligned}\quad (3)$$

From equation (1), we derive

$$\frac{\partial \tilde{x}}{\partial p_1} = -\frac{1}{2t(x_2 - x_1)} \quad \text{and} \quad \frac{\partial \tilde{x}}{\partial p_2} = \frac{1}{2t(x_2 - x_1)}.\quad (4)$$

The first-order conditions for advertisement strategies are given by

$$\begin{aligned}\frac{\partial \Pi_1}{\partial s_1} &= F(\tilde{x})\frac{\partial A_1}{\partial s_1} + (p_1 - c + A_1)f(\tilde{x})\left(\frac{\partial \tilde{x}}{\partial a_1}\frac{\partial a_1}{\partial s_1} + \frac{\partial \tilde{x}}{\partial a_2}\frac{\partial a_2}{\partial s_1}\right) = 0, \\ \frac{\partial \Pi_2}{\partial s_2} &= [1 - F(\tilde{x})]\frac{\partial A_2}{\partial s_2} - (p_2 - c + A_2)f(\tilde{x})\left(\frac{\partial \tilde{x}}{\partial a_2}\frac{\partial a_2}{\partial s_2} + \frac{\partial \tilde{x}}{\partial a_1}\frac{\partial a_1}{\partial s_2}\right) = 0.\end{aligned}\quad (5)$$

Note carefully that each media firm takes into account the fact that any change in its ad policy may also affect the resulting ad level of the other media firm.⁴ We can now show:

Proposition 1 *Advertisement levels depend only on the marginal disutility of adverts and not on the media price, the location of the media firms or the size of the market.*

Proof: By inserting for $(p_i - c + A_i)f(\tilde{x})$ from (3) into (5) we have

$$\begin{aligned}\frac{\partial \Pi_1}{\partial s_1} &= F(\tilde{x})\left[\frac{\partial A_1}{\partial s_1} - \left(\frac{\partial \tilde{x}}{\partial p_1}\right)^{-1}\left(\frac{\partial \tilde{x}}{\partial a_1}\frac{\partial a_1}{\partial s_1} + \frac{\partial \tilde{x}}{\partial a_2}\frac{\partial a_2}{\partial s_1}\right)\right], \\ \frac{\partial \Pi_2}{\partial s_2} &= [1 - F(\tilde{x})]\left[\frac{\partial A_2}{\partial s_2} + \left(\frac{\partial \tilde{x}}{\partial p_2}\right)^{-1}\left(\frac{\partial \tilde{x}}{\partial a_2}\frac{\partial a_2}{\partial s_2} + \frac{\partial \tilde{x}}{\partial a_1}\frac{\partial a_1}{\partial s_2}\right)\right].\end{aligned}\quad (6)$$

Equations (1) and (4) further yield (for $i \neq j$)

$$\frac{\partial \tilde{x}}{\partial a_i}\frac{\partial a_i}{\partial s_i} + \frac{\partial \tilde{x}}{\partial a_j}\frac{\partial a_j}{\partial s_i} = \frac{\partial \tilde{x}}{\partial p_i}\left(d'(a_i)\frac{\partial a_i}{\partial s_i} - d'(a_j)\frac{\partial a_j}{\partial s_i}\right).\quad (7)$$

⁴If media firms are monopolists in their ad markets, $\partial a_2/\partial s_1 = \partial a_1/\partial s_2 = 0$, but this will not be the case if they compete in the ad market.

In equilibrium, $\partial\Pi_1/\partial s_1 = \partial\Pi_2/\partial s_2 = 0$. Equations (6) and (7) thus imply

$$\begin{aligned}\frac{\partial A_1}{\partial s_1} - d'(a_1)\frac{\partial a_1}{\partial s_1} + d'(a_2)\frac{\partial a_2}{\partial s_1} &= 0, \\ \frac{\partial A_2}{\partial s_2} - d'(a_2)\frac{\partial a_2}{\partial s_2} + d'(a_1)\frac{\partial a_1}{\partial s_2} &= 0.\end{aligned}\tag{8}$$

Expression (8) implicitly determines the advertising level as a function of the marginal disutility of ads and the ad revenue function. Even though the media firm with the larger market share has the higher total revenue from ads, the ad revenue per consumer is thus independent of the market size and the media price. Interestingly, Proposition 1 holds even if - for whatever reason - the ad-revenue functions differ across media firms. \square

Proposition 1 demonstrates that only the ad revenue functions and the (dis-)utility of ads determine equilibrium ad levels per consumer in a generalized Hotelling model of two-sided markets. The distribution plays no role for this exercise.

Assume that the ad-revenue functions are symmetric, and let the common equilibrium advertisement revenue per media consumer be denoted by \widehat{A} . Using (3) and (4), we find that

$$\begin{aligned}p_1 &= 2t(x_2 - x_1)\frac{F(\tilde{x})}{f(\tilde{x})} + c - \widehat{A}, \\ p_2 &= 2t(x_2 - x_1)\frac{1 - F(\tilde{x})}{f(\tilde{x})} + c - \widehat{A},\end{aligned}\tag{9}$$

which is an expression similar to the general Hotelling model of one-sided markets of Anderson *et al* (1997). Furthermore,

$$p_2 - p_1 = 2t(x_2 - x_1)\frac{1 - 2F(\tilde{x})}{f(\tilde{x})},\tag{10}$$

which shows that $p_2 > p_1$ if $F(\tilde{x}) < 1/2$ and vice versa. Expression (10) shows that the media firm with the larger market share charges the higher media price. The intuition can most easily be seen from equation (3): the marginal gain from increasing the media price is proportional to the market

size. The marginal loss, on the other hand, is independent of the market size (but larger the smaller the distance between the media firms and the lower the transport costs). Note also from (9) that media prices are decreasing in ad revenue per media consumer, \widehat{A} . The reason for this is that an increase in advertisement revenues per viewer is like a reduction in marginal cost.

As in Anderson *et al* (1997) we can now write profits as a function of locations only:⁵

$$\begin{aligned}\widehat{\Pi}_1 &= 2t(x_2 - x_1) \frac{F(\tilde{x})^2}{f(\tilde{x})}, \\ \widehat{\Pi}_2 &= 2t(x_2 - x_1) \frac{(1 - F(\tilde{x}))^2}{f(\tilde{x})}.\end{aligned}\tag{11}$$

Let y denote the median consumer such that $F(y) = 0.5$. We are now able to demonstrate

Proposition 2 *If profit functions (11) are quasi-concave, firm 1 has a higher market share than firm 2 if $f'(y) < 0$, and a smaller market share if $f'(y) > 0$.*

Proof: We can write the location as an implicit function (see (1)):

$$g(\cdot) = \frac{x_1 + x_2}{2} + \frac{1 - 2F(\tilde{x})}{f(\tilde{x})} = 0$$

because $a_1 = a_2$ and thus $d(a_2) - d(a_1) = 0$. Partial differentiation yields

$$g_{\tilde{x}} = -\frac{3f^2 + f'(1 - 2F)}{f^2}, g_{x_1} = g_{x_2} = \frac{1}{2} \Rightarrow \frac{\partial \tilde{x}}{\partial x_1} = \frac{\partial \tilde{x}}{\partial x_2} = \frac{f^2}{6f^2 + 2f'(1 - 2F)}.$$

Marginal profits with respect to locations can consequently be written as:

$$\begin{aligned}\frac{\partial \widehat{\Pi}_1}{\partial x_1} &= -\frac{2tF^2}{f} + \frac{\partial x}{\partial x_1} \frac{2t(x_2 - x_1)F(2f^2 - f'F)}{f^2}, \\ \frac{\partial \widehat{\Pi}_2}{\partial x_2} &= \frac{2t(1 - F)^2}{f} - \frac{\partial x}{\partial x_2} \frac{2t(x_2 - x_1)(1 - F)(2f^2 + f'(1 - F))}{f^2}.\end{aligned}\tag{12}$$

⁵For uniqueness and existence in the location game, see Assumptions 1 and 2 in Anderson *et al* (1997).

Logconcavity of $f(x)$ implies $\partial\tilde{x}/\partial x_1 = \partial\tilde{x}/\partial x_2 > 0$ (see Anderson *et al* (1997), p. 107) and $2f^2 - f'F > 0, 2f^2 - f'(1 - F) > 0$. An interior solution to (12) thus satisfies $x_1^* > a$ and $x_2^* < b$. Let us evaluate the marginal profits if both firms choose locations such that the median consumer is the indifferent consumer, *i.e.* if $\tilde{x} = y$. Define

$$D \equiv 2t(x_2 - x_1)\frac{\partial\tilde{x}}{\partial x_i} > 0, \Phi \equiv -\frac{t}{2f(y)} + D.$$

Since $\partial\tilde{x}/\partial x_1 = \partial\tilde{x}/\partial x_2$, marginal profits for $\tilde{x} = y$ are equal to

$$\begin{aligned} \frac{\partial\hat{\Pi}_1}{\partial x_1}(\tilde{x} = y) &= \Phi - \frac{f'(y)D}{2f^2}, \\ \frac{\partial\hat{\Pi}_2}{\partial x_2}(\tilde{x} = y) &= -\Phi - \frac{f'(y)D}{2f^2}. \end{aligned} \tag{13}$$

Suppose that firm 1 has chosen x_1 such that its profits are maximized and firm 2 has set x_2 such that $\tilde{x} = y$ holds. From (13), it follows

$$\frac{\partial\hat{\Pi}_1}{\partial x_1}(\tilde{x} = y) = 0 \Rightarrow \frac{\partial\hat{\Pi}_2}{\partial x_2}(\tilde{x} = y) = -\frac{f'(y)D}{f^2}.$$

Hence, firm 2's marginal profits are positive if $f'(y) < 0$, and negative if $f'(y) > 0$. Consequently, firm 2 will increase x_2 if $f'(y) < 0$, thereby increasing firm 1's market share, and vice versa. \square

Proposition 2 shows that asymmetric distributions lead to asymmetric market sizes. If $f'(y)$ is negative (positive), firm 2, located on the right, will have a lower (higher) market share. The reason is that the location decision affects the behavior of the marginal consumer only. If $f'(y)$ is negative (positive), the distribution is skewed at the median consumer such that firm 2 gains by moving to the right (left).

3 Concluding remarks

Our note has demonstrated that a generalized Hotelling model of two-sided markets behaves like a standard Hotelling model in which ad revenues just reduce marginal production costs. It is therefore not the principle of maximum

or minimum differentiation which decouples the ad policy from the pricing decision, but the very nature of the Hotelling model. This result relies on two assumptions. First, overall revenues depend linearly on the number of consumers. If they do not, the invariance result does not hold. However, any alternative assumption has to explain why an additional media consumer has a different impact on marginal ad revenue than the existing ones. Second, we have assumed interior solutions for media prices. Obviously, results will change if ad revenues are so large that at least one media firm would like to sell its product for free.⁶

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⁶More precisely, the qualitative results may change with zero consumer payments if the media firms cannot subsidize their consumers. Note, however, that newspapers may contain discount vouchers, and that TV and radio stations may organize raffles such that the expected consumer price is negative. In this case, our invariance result still holds true.

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