

# Volatility Forecasting in the 90-Day Australian Bank Bill Futures Market

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## Executive Summary

This study employs a comprehensive data set from the 90-Day Australian Bank Bills Futures market to test the predictive power of the theoretically superior implied volatility against historical volatility. Overall, the volatility forecasting results are in line with previous research into futures markets. Implied volatility is a biased forecaster, historical volatility contains no extra information beyond that contained in implied volatility and the market is relatively efficient. The market tested also has two unique characteristics compared to previous research, futures style margining on the option contracts and geographic location. This, combined with the use of both overlapping and non-overlapping data sets, ensures the results are significant.

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## **Introduction**

Volatility is the extent to which the return on an underlying asset fluctuates over a given period of time. It is most commonly calculated as the annualized standard deviation of returns and represents the risk associated with that particular asset. Historically, financial price series have shown great variation in volatility over time. Furthermore, there is significant evidence of volatility ‘clumping’. This means that periods of high volatility tended to occur together, as do periods of low volatility. As volatility represents risk, the phenomenon of clumping is very relevant to market participants. However, on a given day, past volatility itself is not important. What is important, is being able to accurately predict future volatility. This is because volatility is a key component of many financial decisions, including asset allocation, risk management and portfolio selection.

This paper tests the forecasting ability of two distinct measures of volatility, the theoretically-superior implied volatility and historical volatility, in the 90-Day Australian Bank Bill Futures market. The methodology that is used is based on simple regressions and consistent with the general procedure for tests of volatility forecasting. This involves comparing the volatility forecasts, implied volatility (hereafter IV) and historical volatility (hereafter HV), with the subsequent or realized volatility (hereafter RV) that relates to the forecast period. From this, numerous hypotheses can be tested and meaningful conclusions formed.

The rest of the paper is organised as follows. A brief overview of volatility forecasting is first. This is followed by a comprehensive review of previous research and a justification for testing volatility forecasting in the 90-Day Australian Bank Bill Futures market. The data is then described and the methodology of the research set out, including a discussion of the various models for calculating and obtaining volatility forecasts. The statistical test procedures are then discussed in detail and, lastly, the empirical results are presented and conclusions drawn.

## Volatility Forecasting - Preface

Market participants and academics have long been aware of the significance of predicting volatility. Early development of volatility forecasting theory relied on using the historical standard deviation of returns as a measure of an asset's risk. However, it soon became apparent that an asset's volatility was not constant over its lifetime. Thus, it became important to provide accurate measures of volatility that capture this changing effect. This led to the development of numerous theories for calculating volatility including moving averages (MA), exponentially-weighted moving averages (EWMA), autoregressive-conditional heteroskedastic models (ARCH), and generalised-autoregressive heteroskedastic models (GARCH). It is important to realise that all of these models rely on past data. In contrast, IV is the only method of volatility forecasting that is forward looking.

IV represents the underlying asset's volatility in the famed Black-Scholes-Merton option pricing model. It is the only parameter in the model that is not directly observable in the market. When there is an active options market on an asset, the prices of these options are based on the market participants' expectations of future volatility. Therefore, by observing an options' price and using the Black-Scholes-Merton (hereafter BSM) option pricing model it is possible to solve for, or 'back out', the underlying asset's IV. This is the market's unbiased expectation of the asset's volatility for the time period until the option matures, given the assumption that the option pricing model is correct.

IV should be a superior forecaster because it is a traded asset and HV is not. It is traded in the sense that if new information becomes available indicating that an asset's risk has changed, the traders in the options market will react. They may feel that the current IV is not high enough and they will demand more long positions in the options market. This will push up the options prices, thus increasing the IV until it reflects the trader's views. In fact, a straddle (one call and one put) is a direct position on future volatility. If a current IV is deemed to be too low then a long position in a straddle is taken, if it is believed to be too high, the strategy is to short a straddle. Thus, IVs change as traders react to new information. This means that IV has a degree

of forward looking that HV does not. Essentially, IV can be thought of as being derived from a superior information set. If market participants are efficiently processing the new information available to them, they will form expectations of volatility that are, on average, at least as good as historical measures. This is in line with the unbiased expectations hypothesis and, subsequently, the test that IV is a superior forecaster compared to HV has the joint hypothesis of semi-strong form market efficiency. Semi-strong form efficiency means that prices reflect all publicly available information. Conversely, any evidence that HV is superior to IV would indicate an inefficient market (again, assuming the option pricing model is correct). The consequences of inefficient markets are not discussed in detail here, however, a significant amount of finance theory is based on the efficient market hypothesis and the rejection of this in such a vital market as the Australian short-term interest rate market would have significant consequences for its participants. One notable consequence would be that arbitrage opportunities are likely to be available in a market where prices do not reflect all publicly available information.

## Prior Research

Early research into volatility forecasting indicated that IV had superior predictive power compared to HV. Latane and Rendleman (1976) used stock options and calculated weighted implied standard deviations (WISDs), which are essentially an average of IVs from multiple options, differing only in strike price. They concluded that “the WISD is generally a better predictor of future volatility than standard deviation predictors based on historical data.” Clearly, this is evidence in favour of IV. However, Latane and Rendleman (1976) ignored many important considerations such as dividend payments, transaction costs, taxes and timing differences between stock and option markets that could have altered their results. They also found that volatility is not constant over time and that individual stock volatilities tend to move together. A key assumption of the B-S-M option pricing model is that the volatility is constant over the life of the option or changes in a known way and Latane and Rendleman (1976) found evidence against this.

Chiras and Manaster (1978) used Merton’s generalised form of the Black-Scholes model that accounts for dividends and still found IV to be a better predictor of subsequent stock return variance than HV. They also devised a trading strategy to exploit this, finding it provided abnormally high returns. This means that the market was not using IV to forecast volatility and shows how little was known about IV in the early stages of option trading. Using stock options, Schmalensee and Trippi (1978) and Beckers (1981) also find support for IV as a forecaster.

However, subsequent research found contrasting results to the initial findings. Canina and Figlewski (1993)<sup>1</sup> (hereafter CF) argue that Beckers (1981) was the only researcher to identify the importance of timing. The other studies relied on monthly data and did not match RVs with the time to maturity remaining on the option. They argue that many of the papers even analyzed forecasts of volatility over periods before the forecast was made. This would clearly lead to irrelevant results.

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<sup>1</sup> Canina and Figlewski (1993) is perhaps the most well known paper that tests the forecasting ability of various volatility estimates. This is because of the regression methodology it developed. A very similar methodology is adopted for this paper and it is discussed in more detail later.

CF tested for forecasting ability using S&P 500 index options, but their test differed in a significant way. All previous research relied on the assumption that the BSM model was appropriate and accurate. However, CF recognised that this is not the case for American style options, the most common type of options. This is because American options contain an early exercise premium and, thus, the BSM model overstates the IV. Therefore, CF used a binomial model to capture the value of early exercise. They argued that this accounted for the possibility that the market may not price the options with exactly the same model as the researcher does. Using regression tests, CF found that IV is a poor forecaster of subsequent RV and that it did not even incorporate the information present in HV. Further supporting this, Lamoureux and Lastrapes (1993) used individual equity options and rejected the hypothesis that available information cannot be used to improve the market's forecast. In contrast to initial findings, both of these studies clearly find evidence against market efficiency.

However, while CF's paper is highly regarded it does not escape criticism. Corrado and Miller (2005) (hereafter CM) perform the same tests as CF with a data set that includes CF's data. They found that the forecasting ability of IV has improved since 1995 and that it is now superior to HV.

CM split their data into two basic periods, pre-1995 and post-1995. Using the volatility index, VIX, calculated by the Chicago Mercantile Exchange using multiple strike prices, they ran the same regression tests, finding that adding any independent variables beyond implied volatility did not appear to add any more explanatory power the model. This means that that all of the information in historical measures was computed into IV measures. Therefore, they concluded that IV is a better forecaster than HV.

CM also suggested the results obtained by studies such as CF were contaminated by methodological issues and econometric problems. Their major concern was that there are significant biases present when the forecasting ability of IV is tested. The main ones are the errors-in-variables problem and model misspecification bias. Errors-in-variables mean that the observed values of the independent variables include errors. CM's paper tests this by running instrumental variable regressions and comparing them to the OLS regression. They find that the

errors-in-variables problem drops out after 1995. The model misspecification bias occurs when the decision is made about how to calculate HV. A simple standard deviation of returns is used by most studies, while there are arguments that various models which weight past volatilities differently should in fact be used. CM tests this by using various models, finding that there is no benefit gained by using the more complex models compared to more simple methods. This suggests that using ARCH, GARCH and EWMA models is no better than using the simple standard deviation of past returns when testing volatility forecasting.

The two results of CM are significant for this study. Firstly, the fact that the errors-in-variables problem is present pre-1995 is of a concern. This is because the period being tested for this research is 1992 to 2000. However, as discussed in the next section, futures markets are considered to be free of the error-in-variables that CM argued contaminated CF's data. The second main finding of CM is that the standard deviation of past returns is a sufficient measure of HV for volatility forecasting. This means that there are likely to be no significant benefits from using the more complex HV models and, therefore, only the standard deviation of past returns is used as a measure of HV in this study.

## **Forecasting Volatility in Futures Markets**

So far, all of the previous research discussed has focused on forecasting volatility in spot equity markets. While this is undoubtedly an area of huge significance, an important implication of the results from these tests is that they are specific to each data set. One cannot conclusively infer from them the best method for forecasting volatility in any market, as there may be specific market characteristics that cause results to differ. Therefore, because the underlying market being tested in this paper is a futures market, it would be useful to examine previous research that specifically focuses on futures markets.

Szakmary et al. (2003) provided a comprehensive test of 35 futures markets from the U.K. and U.S. The types of markets tested include equities, currency, interest rate and commodity futures markets. In 34 out of the 35 markets tested, the authors find IV to be a superior forecaster compared to HV, with sugar the sole exception.

Szakmary et al. (2003) argued that the reason the results they found are conclusively in favour of market efficiency is due to the nature of futures markets. The key characteristic futures markets have is that both the futures contract and the options contract are traded on the same exchange, whereas this is not the case for equity markets. This minimises transaction costs and means that there are fewer barriers to arbitrage, which is the foundation of the efficiency argument. Also, because they are traded on the same exchange, the two markets close at the same time. Harvey and Whaley (1992) showed that equity markets closed 15 minutes prior to options markets, meaning that the closing price of an asset does not correspond to the closing price of the respective option. Therefore, the IV calculated may not represent the market's unbiased expectation of future volatility. This is known as non-synchronous data and could introduce measurement error to the independent variables. Corrado & Miller (2005) identified this as the errors-in-variables problem and suggested that it could lead to biased results. This has led to the use of instrumental variables to avoid measurement error. However, Szakmary et al. (2003) argued that this problem is overcome when the underlying market being tested is a futures market, meaning the use of instrumental variables is not needed. This is because of the characteristics specific to futures markets and means that they provide the ideal market for testing volatility forecasting. Further justification for the choice of this particular futures market follows.

### **The Importance of the 90-Day Australian Bank Bill Futures market**

The 90-Day Australian Bank Bill Futures contract was launched in 1979 on the Sydney Futures Exchange and was the first interest rate futures contract to be listed outside the U.S. It is much larger than the bank bill market itself, with turnover approximately six times higher and, because of this, it represents Australia's benchmark indicator for short-term interest rates. The reason this market was chosen for this research is that it is a highly liquid market with significant regional importance and it has specific characteristics that make it ideal for testing volatility forecasting.

The short-term interest rate is important because it affects economic activity through a number of channels. Floating rate mortgages and fixed-income securities are often pegged against it, so changes in the rate affect consumer income and spending levels. The rate can also provide the basis for the risk-free rate used in the capital asset pricing model (CAPM). CAPM is commonly used to determine corporate decision making and investment levels. The short-term interest rate also affects the exchange rate and the supply of credit in the economy. All of these individual effects lead to changes in the aggregate demand of the economy and, subsequently, to changes in inflation. This means that the short-term interest rate is a significant tool for implementing monetary policy. This makes being able to accurately forecast the volatility in short-term rates important, both to the public and private sectors.

The 90-Day Australian Bank Bill Futures market also has a number of characteristics that make it ideal for testing volatility forecasting. Firstly, consistent with all tests of volatility forecasting in futures markets, the futures and option contracts are traded on the same exchange with the same opening and closing times. This eliminates the problem of non-synchronous data, rectifying a major problem with studies where the underlying market is a cash market.<sup>2</sup> Secondly, the option contracts have futures style margining. This means that the contracts are marked-to-market at the end of each day, resulting in a series of daily cash flows. The fact that these cash flows are received instantaneously eliminates the risk-free rate of interest from the option pricing model and is important because it means it is never optimal to exercise the option early.<sup>3</sup> If options are exercised early in the market this could bias IVs as expectations of future volatility. This is a key point of difference between this paper and previous research: tests of volatility forecasting have never been undertaken in a market where the options have futures style margining. The result is that the market should be more efficient and, hence, the results more conclusive.

Furthermore, as most previous research has focused on the U.S. and U.K. markets, this paper extends the literature by providing evidence from a new market in a geographically different location. This point alone makes the results significant as it

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<sup>2</sup> Corrado & Miller (2005).

<sup>3</sup> Lieu (1990).

is the first test of volatility forecasting that the author knows of that uses Australian data.

## **Data and Methodology**

### **Description of data**

Two financial price series were used for this research. They are the 90-Day Australian Bank Bill Futures and Options on the 90-Day Australian Bank Bill Futures price series from September 1992 to December 2000. The 90-Day Australian Bank Bill Futures series contains settlement prices for the futures with expiration in March, June, September and December, with term to maturities ranging up to five years. The Options price series contains put and call premiums with term to maturities up to two years. The options expire seven days before the underlying futures contract expires, to allow for the delivery of the futures and prepare for delivery on the futures.

From these two price series, two distinctively different data sets were used. The first is an overlapping data series, where there is an IV, HV, and RV value for every day possible. It is overlapping in the sense that on consecutive days the HV and RV values will be made up of largely the same observation windows of futures returns. The final overlapping data series contains 10,608 observations. The second data set used for this research is three non-overlapping series. This involves using only one set of observations from each futures contract, determined by a specific time to maturity. The times to maturity used are 21, 41, and 62 trading-days and this limited the number of observations to 33 or 32 for each series. These time to maturities are chosen to reflect 30, 60, and 90-calendar day windows respectively and the method is consistent with Jorion (1995), Christensen and Prabhala (1998), and Szakmary et al. (2003) to allow comparisons to be made.

The reason for using the two data series is to add to the significance and comparability of the results. The overlapping series has a very large number of observations that make the regression tests very powerful. However, due to the overlapping nature of the observation windows, there is an inherent level of serial correlation in the HV and RV series. Therefore, following Christensen and Prabhala (1998) and Corrado and Miller (2005), the Newey-West (1987) correction method is used to adjust the standard errors of the regression coefficients to accurately reflect

the serial correlation of varying lengths in the residuals.<sup>4</sup> The non-overlapping series, on the other hand, has the benefit that each observation of RV and HV contains no overlapping futures return data. This means that there is no underlying serial correlation in the data.

Also, by testing both sets of data this study inadvertently tests whether or not there is any difference between the two techniques and allows the results to be comparable with as many previous studies as possible. A formal presentation of the models used to calculate IV, HV and RV follows. For further clarification, a complete example of the methodology is included in the appendix.

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<sup>4</sup> See Newey-West (1987) for details of the method.

## Volatility Models

### Option Implied Volatility

The model that is used to price options with futures style margining is an extension of the Black (1976) model for options on futures contracts and was developed by Asay (1982). It can be thought of as the forward price of Black's model. Formally, the Asay model for the call premium is:

$$C = FN(d_1) - XN(d_2)$$

Where:

$$d_1 = \frac{\ln(F/X) + \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

And

- $F$  = futures price;
- $X$  = exercise price;
- $C$  = call price;
- $t$  = time to maturity, and;
- $\sigma$  = instantaneous volatility.

The put premium is given by:

$$P = XN(-d_2) - FN(-d_1)$$

The Asay (1982) model for pricing options with futures style margining is relatively simpler than the more well-known Black (1976) and Black and Scholes (1973) option pricing models. The big difference is the lack of an interest rate in the formula. This is because the risk-free hedge that is used to derive the option's price can be established without any initial investment funds. There are also no upfront costs and interest is earned on the margin posted. Lieu (1990) showed that when options have futures style margining there is no value to early exercise. This is because it would be more profitable to sell the option, as it will always be valued above its intrinsic values. Therefore, this eliminates a common source of bias in IVs that may have influenced previous results. For these reasons, it would be expected that the Asay (1982) model is accurate in the Australian short-term interest rate market for near-the-money options.

To stress this point, it is important to realise that the results of this research are conditional on the assumption that the Asay model correctly prices the options on 90-Day Australian Bank Bill Futures contracts. While no studies have been done to examine if this is the case for this exact options market, its validity has been tested in a similar market. Brown (1999) showed that the Asay model correctly prices near the money options in the Australian equity futures market. However, she found that the model was not accurate for deep in the money or deep out-of-the-money options as it did not capture the effect of the volatility ‘smile’. The ‘smile’ is a result of the market pricing options with underlying leptokurtic distributions, as opposed to the normal distribution used by the pricing model. Empirical evidence has shown that the distribution of returns does tend to be leptokurtic. Therefore, the smile is more likely a result of the model inadequacy rather than mispricing by market participants.<sup>5</sup>

As mentioned, if the model is not pricing the options accurately then the IV will be a biased forecast. Therefore, as the Asay model is most likely to be accurate for the nearest-to-the-money options, only these are used to calculate the implied volatility. This consists of using the nearest-to-the-money call and the nearest-to-the-money put. For the S&P 100, Harvey and Whaley (1991) showed that the put and call IVs are negatively correlated, so a more efficient estimator is obtained by combining them.<sup>6</sup> In this case, because of the relatively small financing and transaction costs for futures options relative to cash instrument options, the IV’s of the puts and calls are almost identical, however, the IV of the straddle is still used to improve accuracy. Secondly, all options contracts with a term to maturity of less than 10 trading-days were removed because their IVs have been shown to be unstable.<sup>7</sup>

The futures prices are then matched with the corresponding options by date and contract expiration date. Using this matched data, a Newton-Raphson method was employed to compute the IV for each straddle. This is because the option premium is a non-linear function of the volatility, so IV could not be solved for without a numerical estimation method. This gives one IV forecast for each option price.

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<sup>5</sup> Jackworth and Rubenstein (1996).

<sup>6</sup> Corrado and Miller (2005).

<sup>7</sup> Szakmary et al. (2003).

Following Canina and Figlewski (1993), the first step to test market efficiency is to recognize that IV is regarded as a forecast of the market's RV. To formally state this:

$$IV = E_{MKT} [RV]$$

Where,  $E_{MKT}$  is the market's expectation of the RV.

This assumption allows conclusions to draw from the results with regards to market efficiency.

### **Historical Volatility**

The HV model used as the benchmark in this paper is the sample standard deviation of daily futures returns. Previous research has indicated that the more complex models for measuring HV such as ARCH, GARCH and EWMA have added no significant explanatory power to forecasting tests compared to more simple measures.<sup>8</sup> Therefore, this research only uses the historical annualized standard deviation of returns as a measure of HV.

To determine the matching HV forecast for each RV, a historical window that matched the time to maturity of the option was used. This is Figlewski's (1997) optimized unconditional conditional heteroskedastic (OUCH)<sup>9</sup> estimator. Figlewski (1997) found that the accuracy of the forecast was highest when the window, from which the historical measure of volatility was calculated, equalled the time to maturity of the option. Formally, the HV estimate for a particular day was calculated as the annualized standard deviation of returns on the futures contract from the previous  $T_M$  days, where  $T_M$  is the term to maturity of the option contract used to calculate the IV observation on that same day. This only applied to the full overlapping data set, HV. In the non-overlapping series, the HV observation represented the RV observation from the previous futures contract. This ensured consistency with previous research. This meant that for observations in the non-overlapping data series, HV21, HV41, and HV62, the values are simply lags of the relevant RV series, RV21, RV41, and RV62.

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<sup>8</sup> Corrado and Miller (2005) and Szakmary et al. (2003).

<sup>9</sup> Figlewski (1997).

## Realized Volatility

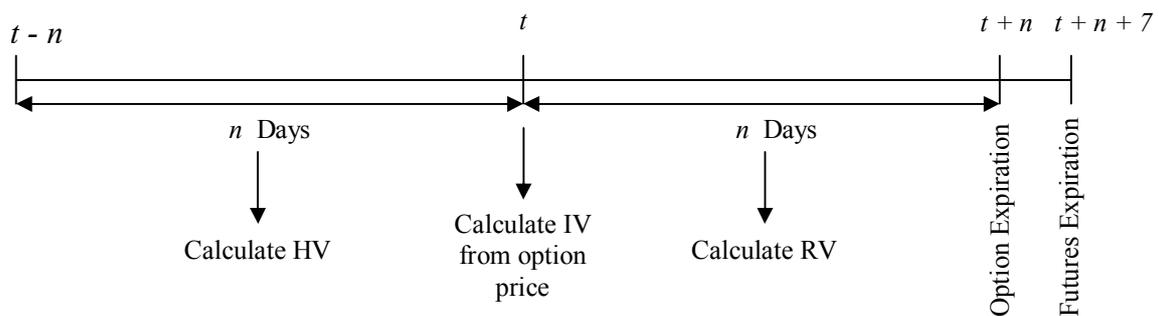
The RV for each day was calculated as the annualized standard deviation of returns of the futures contract from the observation date until the option contract that the IV estimate was obtained from expired. It is essentially the forward looking version of each HV observation. This meant that the observation dates relating to the futures contracts whose options expired after December 2000 had to be removed as it would not be possible to calculate the RV. Formally, following Canina and Figlewski (1993) and Szakmary et al. (2003), the RV was calculated as the annualized standard deviation of the continuously compounded futures returns:

$$RV = \left( \frac{250}{T_M - 1} \sum_{t=1}^{T_M} (R_t - \bar{R})^2 \right)^{1/2}$$

Where,  $T_M$  is the time to maturity in days of the option contract,  
 $R_t = \ln(P_t/P_{t-1})$ , the return on the futures contract, and;  
 $\bar{R}$  is the sample mean of  $R_t$ .

Overall, the general methodology is outlined below:

**DIAGRAM 1**  
Volatility Forecast Testing Methodology



## Descriptive Statistics

The descriptive statistics of each data series are shown in Table 1. The mean and standard deviation of all the series were similar. Also, the range of each series was similar with the RV having the largest range. The values of note were the relatively high values of skewness and kurtosis, especially for the HV21 series. There

is a possibility that this would influence the regression results for this data set. Overall, however, the data was qualitatively similar to previous research and was adequate to warrant further analysis.

The subsequent methodology used to test forecast power was that set out by Canina and Figlewski (1993) and is consistent with Szakmary et al (2003) in their tests of futures markets. It comprised the standard tests for the suitability of a time series for regression analysis and three individual regressions. These statistical procedures are discussed in detail shortly, but first, the research hypotheses are presented.

## **Research Hypotheses**

IV is regarded as an unbiased expectation of the RV under the assumption that the market is informationally efficient (weak form market efficiency) and the option pricing model is specified correctly. If the Asay pricing model for options with futures style margining is accurate for the short-term near-the-money options used in this study, then rejection of the IV as a superior predictor compared to HV would imply that the options market does not efficiently stabilise prices. This would mean that the Australian short-term interest rate market is not informationally efficient and that arbitrage opportunities could exist.

Consistent with the existing literature, the following three hypotheses are examined.<sup>10</sup>

- H<sub>1</sub>: IV is an unbiased estimate of the future RV.
- H<sub>2</sub>: IV has more explanatory power than the HV in forecasting RV.
- H<sub>3</sub>: IV efficiently incorporates all information regarding future volatility; HV contains no information beyond what is already included in IV.

These hypotheses are in order of power. Failure to reject H<sub>1</sub> would mean failure to reject H<sub>2</sub> and H<sub>3</sub>. However, failure to reject H<sub>3</sub> would not mean failure to reject H<sub>1</sub> or H<sub>2</sub>. The statistical procedures used to test these hypotheses are described in the next section.

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<sup>10</sup> These hypotheses are taken from Szakmary et al. (2003).

## Statistical Procedures

### Stationarity tests

In studying time-series data, examining stationarity is the most important prerequisite to further analysis. A non-stationary time series has an integrated process in which a given shock to the series does not die out. Sims et al. (1990) demonstrated that an econometric model with an integrated time series will generally be misspecified, meaning that the regression results will be spurious. To test stationarity, the augmented Dickey–Fuller (ADF) method<sup>11</sup> is used with the ADF and Phillip-Perron (PP) tests completed. Further details of the stationarity tests are given in the appendix. It is important to note that if any of the data series failed to meet the stationarity requirements then the regression tests discussed in the next section were not be completed.

### Regression Tests

The procedure to test the forecasting ability of IV versus HV in Canina and Figlewski (1993) and Szakmary et al. (2003) was employed by estimating the following regressions:

Unbiased Predictor Regressions

$$RV_t = \alpha_0 + \beta_1 IV_t + \varepsilon_t \quad (1)$$

$$RV_t = \alpha_1 + \beta_2 HV_t + \varepsilon_t \quad (2)$$

Encompassing Regression

$$RV_t = \alpha_0 + \beta_1 IV_t + \beta_2 HV_t + \varepsilon_t \quad (3)$$

These regressions were also run for the non-overlapping series of RV, IV and HV respectively, but to avoid repetition the equations are not explicitly stated here.

Consistent with hypothesis H<sub>1</sub>, IV is an unbiased predictor of the RV, it was expected that  $\alpha_0 = 0$  and  $\beta_1 = 1$  in regression (1).

$$H_1: \quad \alpha_0 = 0, \beta_1 = 1$$

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<sup>11</sup> Dickey and Fuller (1979).

H<sub>1A</sub>: IV is a biased predictor of RV

Consistent with hypothesis H<sub>2</sub>, IV includes more information (i.e., current market information) than HV, then IV should have greater explanatory power than HV, and it would be expected that regression (1) has a higher adjusted R<sup>2</sup> than regression (2).

H<sub>2</sub>:  $adjR^2(1) > adjR^2(2)$

H<sub>2A</sub>: HV contains more information about future volatility than IV

Finally, consistent with hypothesis H<sub>3</sub>, when IV and HV appear in the same regression, it was expected  $\beta_2 = 0$  since HV should not add explanatory power beyond that already contained in IV. The adjusted R<sup>2</sup> (*adj R<sup>2</sup>*) should also be the same for regressions (1) and (3) if no extra explanatory power is gained from HV. Adjusted R<sup>2</sup> was used to measure extra explanatory power as it is well-known that adding any independent variable increases R<sup>2</sup>.

H<sub>3</sub>:  $\beta_2 = 0, adjR^2(3) = adjR^2(1)$

H<sub>3A</sub>: HV contains information about future volatility not found in IV

The results of the statistical procedures and hypothesis test are discussed in the following section.

## **Empirical Results**

### **Results of Stationarity Tests**

The unit root test results are reported in Table 2. All of the test statistics are significant at the 1% level, indicating the null hypothesis of a unit root is rejected in all cases. Therefore, IV, HV and RV are all stationary time series. This is a necessary result, meaning that the subsequent regression results are not spurious and that meaningful conclusions can be formed from them.

### **Forecasting Regression Results**

The results of the tests for the relative predictive power of IV and HV appear in Table 3. The first three regressions correspond to the overlapping data set of 10,608 observations. The next three correspond to the 21 trading-day non-overlapping series, the following three to the 41 trading-day non-overlapping series and the last three regressions relate to the 62 trading-day non-overlapping series. For each regression, the coefficients and adjusted  $R^2$ s are shown, with relevant  $t$ -stats reported in brackets below the estimate. The results of the relevant hypothesis tests are shown in Table 4.

### **Unbiased Predictor Regressions**

Regressions (1), (4), (7) and (10) represent the tests of whether or not IV is an unbiased predictor of RV. The coefficients for IV range from 0.438 to 0.583, and all of the intercepts are significantly greater than zero. If IV is an unbiased predictor of RV, as hypothesis one states, then the intercept should not be significantly different from zero and the coefficient for IV should not be significantly different than one. This is clearly not the case in any of the series, with the hypothesis rejected in all of the regressions. This means that IV is not an unbiased predictor of RV in any of the series tested. This result is consistent with Jorion (1995), Szakmary et al. (2003) and other previous studies, where the coefficient on IV is found to be between 0.300 and 0.650. The slope coefficients for IV are significantly greater than zero in all of the regressions except for (7), the 21 trading-day non-overlapping series. Even in this case, it is still positive and nearly significant. The adjusted  $R^2$ s from the regressions range from 0.0828 for the 21 trading-day series to 0.2341 for the 62 trading-day series, with the other two close to the 0.220. This indicates that IV is useful in

predicting RV, but it is not very accurate. However, the results are comparable with previous research into interest rate futures markets, where  $R^2$  values ranges from 0.106 to 0.500.<sup>12</sup> Therefore, on a relative basis, the results of IV's ability to predict RV in the 90-Day Australian Bank Bills Futures market are in line with world markets.

Regressions (2), (5), (8) and (11) represent the tests of the predictive power of HV. The slope coefficients for HV range from 0.23127 to 0.38145, with only one significantly greater than zero. This was for regression (2), the full overlapping series. This means that for the non-overlapping data, HV does not have any significant predictive power. This point is also illustrated by the result that the intercept term is positive and highly significant in all four regressions. The adjusted  $R^2$ 's for HV range from 0.0199 to 0.0827. This is very low relative to previous research and makes it clear that HV is not very useful in predicting RV in this market. Hypothesis two (IV has more explanatory power than HV) required that the adjusted  $R^2$ 's from regressions (1), (4), (7), and (10) be higher than those from regressions (2), (5), (8), and (11) respectively. This is true in every instance, with the difference ranging from 0.0620 to 0.1969. Clearly, hypothesis two, that IV has more explanatory power than HV in forecasting RV in the 90-Day Australian Bank Bills Futures market, can be accepted.

### **Encompassing Regressions**

Regressions (3), (6), (9) and (12) represent the encompassing regression tests. They are used to directly compare the predictive power of IV versus HV. If IV contains all of the information that HV does and any more relevant information, then it would be expected that the coefficient on HV would fall to zero in the encompassing regressions and the adjusted  $R^2$ 's would be no greater than those for the regression with IV as the only independent variable. This is effectively what happens. The coefficients on HV range from 0.03159 to 0.08024, with the only significant result obtained in the overlapping series. Even then, the value is 0.03159. Compared to the  $t$ -stats of 45.79 and 24.27 for IV and the intercept term respectively, it is clear that HV, while statistically significant, is not economically significant. The adjusted  $R^2$ 's from the encompassing regressions are comparable to the regressions where IV is

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<sup>12</sup> Szakmary et al. (2003).

the only independent variable. This indicates that adding HV to the model provides no extra predictive power above that gained from IV. Therefore, hypothesis three (the IV efficiently incorporates all information regarding future volatility and that HV contains no information beyond that contained in IV) can be accepted. The coefficients for IV are significant in three out of the four regressions, with a range from 0.43383 to 0.56832. Again, the only series where the coefficient of IV is not significant is the 21-day non-overlapping series. All of the coefficients for IV are significantly closer to one than those for HV and the  $t$ -stats ranges from three to over 20 times larger for IV.

Ideally, if IV was an unbiased forecaster of RV and contained all relevant forecasting information it would be expected that the intercept term and coefficient for HV would not be significantly different from zero and the coefficient for IV would not be significantly different from one. While this is not the case, indicating that IV is a biased estimator of RV, it is clear that it is the best predictor.

## Summary and Conclusions

This paper tests the ability of two volatility measures to forecast future volatility using data from the 90-Day Australian Bank Bill Futures market from September 1992 to December 2000. The two measures are the volatility implied in option prices and the historical standard deviation of futures returns.

This paper is an improvement on and an extension of previous work. The improvement results from the unique characteristics of the market studied. The options market is one of only a growing few in the world that employs futures style margining on the option contracts. The result is that the option pricing model is simpler and the possible bias introduced by early exercise of the options is avoided. This means that the market should be more efficient and the results do not dispute this.

This research extends previous work by analyzing a market from a geographically unique location, Australia, broadening the net of available information on volatility forecasting. The results will also be of interest to direct market participants, as it tests whether the market they operate in is efficient or not.

The method of regression analysis used was introduced by Canina and Figlewski (1993) and provides a simple test of the predictive power of various volatility measures. The study finds that, for the 90-Day Australian Bank Bill Futures market, implied volatility is a biased forecaster of realized volatility. This is consistent with previous literature and an expected result. All that can be drawn from this result is that accurately predicting volatility is very difficult.

The study then compared implied volatility against historical volatility to gauge the level of efficiency in the market. The simple tests of the predictive power of historical volatility indicate that it is far less useful than implied volatility measures. However, the most interesting results come from the encompassing regression test, where the realized volatility is regressed against both the implied and historical measures. What is found is that with the presence of implied volatility in the

regression, the coefficient for historical volatility loses all significance. Furthermore, the adjusted  $R^2$ 's from the encompassing regressions are less than or equal to those from the regressions with implied volatility as the only independent variable. This indicates that there is absolutely no extra forecasting information in the historical volatility that is not already found in the implied volatility. From this it can be concluded that the 90-Day Australian Bank Bill Futures market is efficient in the manner at which historical volatility information is processed.

The hypothesis of market efficiency also has a joint hypothesis that the pricing model was correct. Therefore, following from the conclusion that the market is efficient, it can be concluded that the Asay option pricing model for options with futures style margining accurately prices at-the-money options in the 90-Day Australian Bank Bill Futures market.

This study also has a point of difference in that both overlapping and non-overlapping data sets are used. The results from each are similar, indicating that neither method appears superior and that comparison between papers that employ these different sampling techniques is appropriate.

Overall, the nature of the data and the market ensure that the results obtained and conclusions drawn from this research are a significant contribution to the field of volatility forecasting

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## Tables of Results

**TABLE 1**  
Descriptive Statistics

<b>Panel A: Overlapping Data</b>						
	RV	IV	HV			
<i>N</i>	10608	10608	10608			
Mean	0.173683	0.182278	0.201012			
Std. Dev.	0.050284	0.040791	0.034940			
Min	0.009858	0.032612	0.041887			
Max	0.495051	0.351655	0.394674			
Skewness	-0.097878	-0.461940	0.134627			
Kurtosis	0.984226	0.400898	2.906848			
<b>Panel B: Non-Overlapping Data</b>						
	IV21	HV21	IV41	HV41	IV62	HV62
<i>N</i>	33	33	33	33	32	32
Mean	0.123574	0.170756	0.156267	0.190533	0.166054	0.141527
Std. Dev.	0.040541	0.048699	0.055295	0.061199	0.044504	0.050338
Min	0.033951	0.067886	0.051428	0.102927	0.070259	0.062922
Max	0.227886	0.275745	0.328489	0.344936	0.248963	0.278259
Skewness	0.359191	2.079366	0.936946	0.995257	0.009245	1.172837
Kurtosis	0.507092	6.290270	2.043612	2.454742	-0.668968	1.932901

A positive value of Skewness indicates the distribution is skewed to the right and a negative value indicates a left skew.

The Kurtosis statistic represents excess kurtosis. A value above zero indicates a leptokurtic distribution (high peak and fat tails) and a negative value indicates a platykurtic distribution (smaller peak and thin tails).

The descriptive statistics for the non-overlapping series of RV are not given because they are the same as those for the non-overlapping series of HV. This is because the non-overlapping series of HV represents that lag of the non-overlapping series of RV.

$RV_t$  represents the overlapping series of realized volatility,  $IV_t$  represents the overlapping series of implied volatility, and  $HV_t$  represents the overlapping series of historical volatility.  $RV_{21_t}$ ,  $RV_{41_t}$ , and  $RV_{62_t}$  represent the non-overlapping series of realized volatility for 21, 41 and 62 trading-days respectively.  $IV_{21_t}$ ,  $IV_{41_t}$ , and  $IV_{62_t}$  represent the non-overlapping series of implied volatility for 21, 41 and 62 trading-days respectively.  $HV_{21_t}$ ,  $HV_{41_t}$ , and  $HV_{62_t}$  represent the non-overlapping series of historical volatility for 21, 41 and 62 trading-days respectively.

**TABLE 2**  
Unit Root Tests

Test	Data Series		
	RV	IV	HV
ADF test statistic	-7.84**	-10.70**	-10.72**
PP test statistic	-8.03**	-10.81**	-10.73**

For each series the following regression is estimated:

$$\Delta X_t = \gamma_0 + \gamma_1 X_{t-1} + \sum_{i=1}^N \phi_i \Delta X_{t-i} + e_t$$

Where  $X_t$  is the series being tested for unit roots,  $X_{t-i} = X_t - X_{t-1}$ , and  $N$  is the lag length. For each series, the lag length is selected by examining the autocorrelation function (ACF) and the partial autocorrelation function (PACF) and is chosen to eliminate any significant autocorrelation of the residuals. The null hypothesis is that a unit root exists and, thus, a significant test statistic indicates a stationary series. The test statistic is the  $t$ -statistic on the coefficient  $\gamma_1$ ; however, it does not have a standard  $t$ -1 distribution. The critical values are given in Fuller (1976).

The Phillip and Perron (PP) test generalizes the ADF test by allowing for serially correlated residuals. The critical values are the same as for the ADF test.

\* and \*\* denote significance at the 5% and 1% level respectively.

**TABLE 3**  
Forecasting Regression Results

	Model	Coefficient Estimates			$adjR^2$
		$\alpha_0$	$\beta_1$	$\beta_2$	
(1)	$RV_t = \alpha_0 + \beta_1 IV_t + \varepsilon_t$	0.06749** (34.25)	0.58259** (55.23)		0.2233
(2)	$RV_t = \alpha_0 + \beta_2 HV_t + \varepsilon_t$	0.09701** (35.29)		0.38145** (28.31)	0.0702
(3)	$RV_t = \alpha_0 + \beta_1 IV_t + \beta_2 HV_t + \varepsilon_t$	0.06374** (24.37)	0.56832** (45.79)	0.03159* (2.18)	0.2236
(4)	$RV_{21_t} = \alpha_0 + \beta_1 IV_{21_t} + \varepsilon_t$	0.03401 (0.93)	0.55248 (1.97)		0.0828
(5)	$RV_{21_t} = \alpha_0 + \beta_2 HV_{21_t} + \varepsilon_t$	0.07902** (3.57)		0.23127 (1.29)	0.0208
(6)	$RV_{21_t} = \alpha_0 + \beta_1 IV_{21_t} + \beta_2 HV_{21_t} + \varepsilon_t$	0.02774 (0.74)	0.54951 (1.66)	0.08024 (0.41)	0.0753
(7)	$RV_{41_t} = \alpha_0 + \beta_1 IV_{41_t} + \varepsilon_t$	0.05567* (2.38)	0.43831** (3.10)		0.2168
(8)	$RV_{41_t} = \alpha_0 + \beta_2 HV_{41_t} + \varepsilon_t$	0.09389** (3.75)		0.23532 (1.27)	0.0199
(9)	$RV_{41_t} = \alpha_0 + \beta_1 IV_{41_t} + \beta_2 HV_{41_t} + \varepsilon_t$	0.04843 (1.71)	0.43383* (2.69)	0.06618 (0.37)	0.1935
(10)	$RV_{62_t} = \alpha_0 + \beta_1 IV_{62_t} + \varepsilon_t$	0.04313 (1.40)	0.58059** (3.24)		0.2341
(11)	$RV_{62_t} = \alpha_0 + \beta_2 HV_{62_t} + \varepsilon_t$	0.09068** (3.37)		0.3525 (1.93)	0.0827
(12)	$RV_{62_t} = \alpha_0 + \beta_1 IV_{62_t} + \beta_2 HV_{62_t} + \varepsilon_t$	0.03003 (0.89)	0.55136* (2.64)	0.13450 (0.74)	0.2393

$RV_t$  represents the overlapping series of realized volatility,  $IV_t$  represents the overlapping series of implied volatility, and  $HV_t$  represents the overlapping series of historical volatility.  $RV_{21_t}$ ,  $RV_{41_t}$ , and  $RV_{62_t}$  represent the non-overlapping series of realized volatility for 21, 41 and 62 trading-days respectively.  $IV_{21_t}$ ,  $IV_{41_t}$ , and  $IV_{62_t}$  represent the non-overlapping series of implied volatility for 21, 41 and 62 trading-days respectively.  $HV_{21_t}$ ,  $HV_{41_t}$ , and  $HV_{62_t}$  represent the non-overlapping series of historical volatility for 21, 41 and 62 trading-days respectively.

Regressions (1), (4), (7), and (10) test the predictive power of IV. Regressions (2), (5), (8), and (11) test the predictive power of HV. Regressions (3), (6), (9), and (12) are encompassing regression tests to directly compare the predictive power of IV versus HV.

$t$ -stats are reported in brackets below the relevant parameter estimate and follow the normal  $t$  distribution with  $N-k$  degrees of freedom.

\* and \*\* denote significance at the 5% and 1% level respectively.

**TABLE 4**  
Hypothesis Test Results

	Model	Null Hypothesis		
		$\alpha_0 = 0,$ $\beta_i = 1$	$\beta_i = 1$	$\alpha_0 = 0,$ $\beta_1 = 1,$ $\beta_2 = 0$
(1)	$RV_t = \alpha_0 + \beta_1 IV_t + \varepsilon_t$	982.40**	1565.79**	
(2)	$RV_t = \alpha_0 + \beta_2 HV_t + \varepsilon_t$	2738.56**	2107.28**	
(3)	$RV_t = \alpha_0 + \beta_1 IV_t + \beta_2 HV_t + \varepsilon_t$			656.75**
(4)	$RV21_t = \alpha_0 + \beta_1 IV21_t + \varepsilon_t$	3.09*	2.55	
(5)	$RV21_t = \alpha_0 + \beta_2 HV21_t + \varepsilon_t$	9.17**	18.33**	
(6)	$RV21_t = \alpha_0 + \beta_1 IV21_t + \beta_2 HV21_t + \varepsilon_t$			1.53
(7)	$RV41_t = \alpha_0 + \beta_1 IV41_t + \varepsilon_t$	16.55**	15.74**	
(8)	$RV41_t = \alpha_0 + \beta_2 HV41_t + \varepsilon_t$	8.53**	16.98**	
(9)	$RV41_t = \alpha_0 + \beta_1 IV41_t + \beta_2 HV41_t + \varepsilon_t$			8.99**
(10)	$RV62_t = \alpha_0 + \beta_1 IV62_t + \varepsilon_t$	8.43**	5.47*	
(11)	$RV62_t = \alpha_0 + \beta_2 HV62_t + \varepsilon_t$	6.69**	13.33**	
(12)	$RV62_t = \alpha_0 + \beta_1 IV62_t + \beta_2 HV62_t + \varepsilon_t$			4.63**

The test statistics reported are  $F$ -values. Critical values follow the usual  $F$ -distribution.

The null hypothesis:  $\alpha_0 = 0, \beta_i = 1$ , represents the test for an unbiased forecaster of RV. The null hypothesis:  $\alpha_0 = 0, \beta_1 = 1, \beta_2 = 0$ , tests whether IV contains all the predictive information of HV.

$RV_t$  represents the overlapping series of realized volatility,  $IV_t$  represents the overlapping series of implied volatility, and  $HV_t$  represents the overlapping series of historical volatility.  $RV21_t, RV41_t,$  and  $RV62_t$  represent the non-overlapping series of realized volatility for 21, 41 and 62 trading-days respectively.  $IV21_t, IV41_t,$  and  $IV62_t$  represent the non-overlapping series of implied volatility for 21, 41 and 62 trading-days respectively.  $HV21_t, HV41_t,$  and  $HV62_t$  represent the non-overlapping series of historical volatility for 21, 41 and 62 trading-days respectively.

Regressions (1), (4), (7), and (10) test the predictive power of IV. Regressions (2), (5), (8), and (11) test the predictive power of HV. Regressions (3), (6), (9), and (12) are encompassing regression tests to directly compare the predictive power of IV versus HV.

\* and \*\* denote significance at the 5% and 1% level respectively, indicating rejection of the null hypothesis.

## **Appendix**

### **Market Overview**

A 90-Day Australian Bank Bill is a discounted fixed-income security that returns a par value in exactly 90 days. It is bought and sold at a price of less than par value with the return representing the short-term interest rate. It can be thought of as the short-term risk-free rate for the Australia economy. It therefore plays an important role in determining the cost of capital for companies and is thus a commonly traded asset. The most common form that this short-term rate is traded in, however, is 90-Day Australian Bank Bill Futures contracts.

A 90-Day Australian Bank Bill Futures contract is an agreement to buy a 90-Day Australian Bank Bill at a specified price on a specified future date. It has a par value of A\$1 million (this was A\$500k for part of the sample) and the term to maturity is exactly 90-days and represents the market's expectations of future short-term interest rates or the short-term forward rate. These time periods can range from one day to five years into the future. Its value is marked to market each day, so entering into a futures contract is costless and only requires posting a margin. If the losses are significant enough to reduce the margin account below a specified level then a margin call is made, meaning further funds must be deposited into the account. Likewise if gains are made on the contract, the trader may withdraw funds from the account down to a certain level. This concept of marking to market is very important in futures markets.

Completing the short-term interest rate market in Australia are option contracts on the futures contract. An Option on a 90-Day Australian Bank Bill Futures contract is the right to buy (call) or sell (put) a 90-Day Australian Bank Bill Futures contract for a specific price up to a specific date. Time to maturity of option contracts on 90-Day Australian Bank Bill Futures can range from one day to two years. Both the futures contract and the options on the futures contract are traded on the Sydney Futures Exchange (SFE).

## Pricing Conventions

One difference between the Australian market and other futures markets where volatility forecasting has been tested is the pricing conventions used for interest rate products on the SFE. Instead of pricing bank bill futures on the basis of their capital price,<sup>13</sup> instruments are priced on the basis of their yield to maturity. The settlement price is calculated as 100 minus the yield to maturity expressed in per cent per annum.<sup>14</sup> The tick size of the futures contract is not constant but varies accordingly to changes in the yield to maturity. Roughly speaking, a 0.01% change in the interest rate will lead to approximately a \$25 change in the current value of the futures contract. Formally, the bank bill formula can be written as:

$$P = \frac{\text{Face Value} \times 365}{365 + \left( \frac{\text{yield} \times \text{Days to Maturity}}{100} \right)}$$
$$= \frac{1,000,000 \times 365}{365 + \left( \frac{\text{yield} \times 90}{100} \right)}$$

Similarly, premiums for options are quoted in terms of annual percentage yield with the value of the premium calculated by comparing its contract value at the exercise price and its value at the same exercise price less the option premium percentage.

## Example of Methodology

### Overlapping Observations

On 1 June 1995 the Sep-95 futures contract had 99 days to maturity. On the same day, the nearest-the-money call and put options on the Sep-95 futures contract a time to maturity of 92 days. Using a Newton-Raphson method, the IV is solved from the straddle option (one call and one put) to give the market's expectation of volatility in the Sep-95 futures price for the 92 days till maturity. Similarly, the historical standard deviation of daily futures returns for the 92 days prior to 1 June 1995 is used

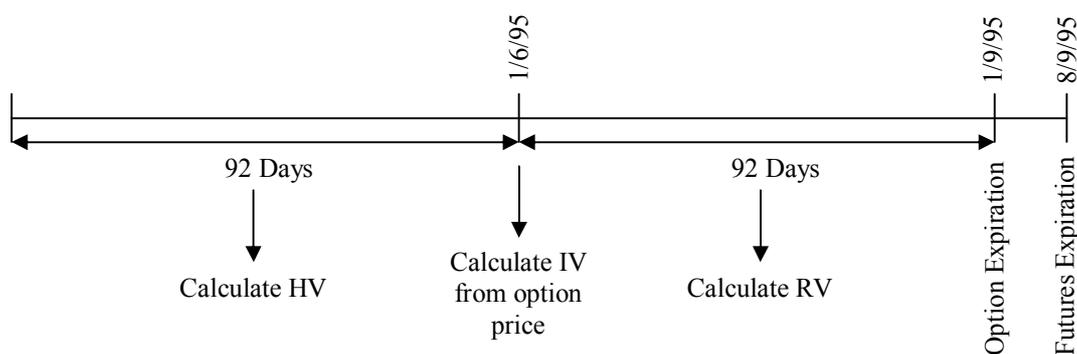
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<sup>13</sup> \$1,000,000 discounted at the current yield.

<sup>14</sup> e.g. quoted as 95.00 (yielding 5% p.a. for 90 days) instead of \$987,921. See "SFE – A Guide to the Pricing Conventions of SFE Interest Rate Products" for more information.

as the HV forecast for the same 92 days. The RV is then calculated as the daily standard deviation of futures returns for these subsequent 92 days from 1 June 1995 until the option expires. Altogether, these three values represent one set of observations used for the regression analysis. This methodology is shown in diagrammatic form below:

**DIAGRAM 2**  
Example of Volatility Forecast Testing Methodology



### Non-Overlapping Observations

Alternatively, when non-overlapping observations are used, the RV series is simply lagged to provide the HV series. For example, on 3 August 1995, the Sep-95 futures option contract has 21 trading-days to maturity. The RV and IV observations are obtained the same way as above, but the HV series is different. The HV21 observation is the RV from the Jun-95 futures option contract when it has 21 trading-days to maturity. Thus, the HV21 series is simply a lag of the RV series made up of only observations with 21 trading-days to maturity. The methodology is similar for the 41 and 62 trading-day series.

### Stationarity Tests

The ADF test is performed by running ordinary linear regressions of the single-mean zero-trend form. This is because it is reasonable to assume that the volatility varies randomly about a mean that is greater than zero; therefore a zero-mean is acceptable. There is also no reason to believe that there is a systematic trend

in volatility and, thus, a trend component is not included in the model. The stationarity test model and hypotheses are:

$$\Delta X_t = \gamma_0 + \gamma_1 X_{t-1} + \sum_{i=1}^N \phi_i \Delta X_{t-i} + e_t$$

With:  $H_0: \gamma_1 = 0$   
 $H_A: \gamma_1 < 0$

Where:  $X_t$  is the series being tested for unit roots;  
 $\Delta X_t = X_t - X_{t-1}$ ;  
 $\Delta X_{t-i}$  is  $\Delta X_t$  with the  $i$ th lag, and;  
 $N$  is the lag length.

The null hypothesis is that  $X_t$  contains a unit root and is therefore non-stationary. Rejection of the null hypothesis indicates that the time series is stationary and is therefore suitable for regression analysis.

The Phillip-Perron (PP) test<sup>15</sup> is also used to test for the presence of a unit root in the time series. It modifies the ADF test to allow for the presence of serially correlated error terms. The critical values are the same as for the ADF test.

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<sup>15</sup> Phillip and Perron (1987).

## Microsoft Visual Basic Macro for Calculating Implied Volatilities

```
Public Sub MatchFuture()  
Application.Calculation = xlManual  
Application.ScreenUpdating = False  
  
ATMCounter = 11150  
  
Worksheets("Options").Select  
  
Do  
    ATMCounter = ATMCounter + 1  
  
If IsEmpty(Cells(ATMCounter, 1)) Then Exit Do  
TargetDate = Cells(ATMCounter, 1)  
ExpiryTarget = Cells(ATMCounter, 2)  
  
Worksheets("Futures").Select  
  
ZipCount = ATMCounter - 2000  
    Do  
        ZipCount = ZipCount + 1  
        If Cells(ZipCount, 1) = TargetDate And _  
            Cells(ZipCount, 2) = ExpiryTarget Then  
            Futures = Cells(ZipCount, 5)  
            Exit Do  
        End If  
    Loop  
        'Futures Stops  
  
Worksheets("Options").Select  
    Cells(ATMCounter, 20) = Futures  
  
Loop  
    'Options Stops  
  
Application.Calculation = xlAutomatic  
Application.ScreenUpdating = True  
End Sub  
Function AsayStradVol(CallPrice As Double, PutPrice As Double, strike  
As Double, expiry As Double, _  
    asset As Double, error As Double)  
  
mktprice = CallPrice + PutPrice  
lstrike = 100 - strike  
lasset = 100 - asset  
  
Volatility = 0.3  
  
dv = 1 + error  
  
While Abs(dv) > error  
    d1 = Log(lasset / lstrike) + (0.5 * Volatility * Volatility *  
expiry)  
    d1 = d1 / (Volatility * Sqr(expiry))  
    d2 = d1 - Volatility * Sqr(expiry)  
  
    Straddle = (lstrike * (cdf(-d2) - cdf(d2))) - (lasset * (cdf(-d1)  
- cdf(d1)))
```

```

'MsgBox ("Check Call" & Putprice)

    priceerror = Straddle - mktprice
    vega = 2 * (lasset * Sqr(expiry / 3.1415926 / 2) * Exp(-0.5 * d1
* d1))
    dv = priceerror / vega
    Volatility = Volatility - dv

Wend

AsayStradVol = Volatility
End Function

Function cdf(x)
d = 1 / (1 + 0.33267 * Abs(x))
a = 0.4361836
b = -0.1201676
c = 0.937298
cdf = 1 - 1 / Sqr(2 * 3.1415926) * Exp(-0.5 * x * x) *
(a * d + b * d * d + c * d * d * d)
If x < 0 Then cdf = 1 - cdf
End Function

```

## SAS code

```
PROC IMPORT OUT= Allfuts
  DATAFILE= "G:\Futures.xls"
  DBMS=EXCEL2000 REPLACE;
  RANGE="Futs$";
  GETNAMES=YES;
RUN;

Proc Sort;
by      expiry date;

Data    futures;
set     AllFuts;
  if    date = . then delete;
  Expire1=lag1(expiry);
  LAG1SETTLE=lag(SETTLE);
  FUTRET=log((100-SETTLE)/(100-LAG1SETTLE));
  If    Expire1 ne expiry then Futret=.;
  FUTRETSQD=FUTRET*FUTRET;

proc means;
Title  'Futures Info';

PROC IMPORT OUT = Options
  DATAFILE= "G:\Ops4ISd.xls"
  DBMS=EXCEL2000 REPLACE;
  RANGE="Options$";
  GETNAMES=YES;
RUN;

data    Options;
set     Options;
If expiry > 200012 then delete;

proc means;
Title 'Options Info';

proc sql;
  create table  CleanData2 as

  select ID, Options.Date, Options.Expiry, OpEx, FutEx, Tot_Vol, ISD,
Options.TTE,
          FutRet, FutRetSqd

  from    futures, options

  where   futures.Date <= options.OpEx          and
          futures.Expiry = options.Expiry      and
          futures.Date >= options.Date ;

proc sort;
  by      id date;

data    four;
set     CleanData2;
  by     id;
  if    first.id then do;
    SumYldRet      =      0;
    SumRetSq       =      0;
    Count          =      0;
  end;

  SumYldRet + FutRet;
  SumRetSq + FutRetSqd;
  Count + 1;
  if last.id then output four;

Data    Five;
Set     Four;
  If    Count < 10 then delete;

  MeanRet      =      SumYldRet/Count;
  MeanSqRet    =      SumRetSq/Count;
```

```

RealizedVol = Sqrt(MeanSqRet - (MeanRet*MeanRet))*sqrt(250);
VolDiff = ISD - RealizedVol;
Run;

data Nathan_ISD;
Set Five;

proc sort data = Nathan_ISD;
by expiry date;

proc reg data = Nathan_ISD;
title 'Realized Vol on ISD';
model RealizedVol = ISD/stb ;
test intercept = 0, ISD = 1;
test ISD = 1;

proc model data = Nathan_ISD;
parms a b;
RealizedVol = a + b*ISD;
Title 'Regression 1:Realized Vol on ISD Lag 1';
Fit RealizedVol / KERNEL=( bart, 2, 0);
run;

proc sql;
create table CleanData3 as

select Nathan_ISD.ID, Nathan_ISD.Date, Nathan_ISD.Expiry,
Nathan_ISD.OpEx, Nathan_ISD.FutEx, Nathan_ISD.Tot_Vol,
Nathan_ISD.TTE, futures.FutRet, futures.FutRetSqd,
Nathan_ISD.Count,
Nathan_ISD.RealizedVol, Nathan_ISD.ISD

from futures, Nathan_ISD

where futures.Expiry = Nathan_ISD.Expiry and
futures.Date <= Nathan_ISD.Date ;

proc sort;
by id descending date;

data six;
set CleanData3;
by id;
if first.id then do;
SumYldRetH = 0;
SumRetSqH = 0;
Counter = 0;

end;

SumYldRetH + FutRet;
SumRetSqH + FutRetSqd;
Counter + 1;
if Counter = Count then output six;

Data seven;
Set six;
If Count < 10 then delete;

MeanRetH = SumYldRetH/Counter;
MeanSqRetH = SumRetSqH/Counter;
HistVol = Sqrt(MeanSqRetH - (MeanRetH*MeanRetH))*sqrt(250);
VolDiffH = HistVol - RealizedVol;

proc means data= seven;
Run;

data Nathan_HVmatchTTM;
Set seven;

proc sort;
by expiry date;

proc reg data = Nathan_HVmatchTTM;
title 'Regression 2:Realized Vol on Hist Vol';
model RealizedVol = HistVol/stb ;

```

```

test    intercept = 0, HistVol = 1;
test    HistVol = 1;

proc model data = Nathan_HVmatchTTM;
parms   a b;
        RealizedVol = a + b*HistVol;
Title   'Realized Vol on Hist Vol Second Lag 1';
Fit     RealizedVol / KERNEL=( bart, 2, 0);
run;

proc reg data = Nathan_HVmatchTTM;
title   'Realized Vol on ISD 2';
model   RealizedVol = ISD/stb ;
test    intercept = 0, ISD = 1;
test    ISD = 1;

proc reg data = Nathan_HVmatchTTM;
title   'Regression 3:Encompassing Regression';
model   RealizedVol = ISD HistVol/stb ;
test    HistVol = 1;
test    ISD = 1;
test    HistVol = ISD;
test    HistVol = ISD - 1;
test    intercept = 0, Isd = 1, HistVol=0;
run;

proc model data = Nathan_HVmatchTTM;
parms   a b c;
        RealizedVol = a + b*ISD + c*HistVol;
Title   'Encompassing Regression Third Lag 1';
Fit     RealizedVol / KERNEL=( bart, 2, 0);
run;

proc univariate normal;
Title   ' Is ISD Normal?';
var ISD;

proc univariate normal;
Title   ' Is Historical Normal?';
var HistVol;

proc univariate normal;
Title   ' Is Realized Normal?';
var RealizedVol;

proc arima;
title   'Is Realized Stationary?';
identify var = RealizedVol stationarity = (adf=(0,1,2,3,4,5,6,7,8,9,10));
identify var = RealizedVol stationarity = (pp=(0,1,2,3,4,5,6,7,8,9,10));

proc arima;
title   'Is Historical Stationary?';
identify var = HistVol stationarity = (adf=(0,1,2,3,4,5,6,7,8,9,10));
identify var = HistVol stationarity = (pp=(0,1,2,3,4,5,6,7,8,9,10));

proc arima;
title   'Is ISD Stationary?';
identify var = ISD stationarity = (adf=(0,1,2,3,4,5,6,7,8,9,10));
identify var = ISD stationarity = (pp=(0,1,2,3,4,5,6,7,8,9,10));

RUN;

data    eight;
set     Nathan_HVmatchttm;
        lagID = lag(ID);
        if Count ne 21 then delete;
        if lagID = ID then delete;

proc means data= eight;
        title HV21 data;
Run;

data    Nathan_HV_21nonoverlapping;
Set     eight;
        lagHV21 = lag(RealizedVol);

```

```

proc sort data=Nathan_HV_21nonoverlapping;
by expiry date;

proc reg data = Nathan_HV_21nonoverlapping;
title 'Regression 4:Realized Vol on Hist Vol 21 Trading Day Non-Overlapping';
model RealizedVol = lagHV21/stb ;
test intercept = 0, lagHV21 = 1;
test lagHV21 = 1;

proc reg data = Nathan_HV_21nonoverlapping;
title 'Regression 4:Realized Vol on ISD 21 Trading Day Non-Overlapping';
model RealizedVol = ISD/stb ;
test intercept = 0, ISD = 1;
test ISD = 1;

proc reg data = Nathan_HV_21nonoverlapping;
title 'Regression 4:Encompassing Regression 21 Trading Day Non-Overlapping';
model RealizedVol = ISD lagHV21/stb ;
test intercept = 0, ISD = 1, lagHV21=0;
test ISD = 1;
test lagHV21 = 1;

proc model data = Nathan_HV_21nonoverlapping;
parms a b;
RealizedVol = a + b*lagHV21;
Title 'Realized Vol on Hist Vol 21 Trading Day Non-Overlapping Second Lag 1';
Fit RealizedVol / KERNEL=( bart, 250, 0);

proc model data = Nathan_HV_21nonoverlapping;
parms a b;
RealizedVol = a + b*ISD;
Title 'Realized Vol on ISD 21 Trading Day Non-Overlapping Second Lag 1';
Fit RealizedVol / KERNEL=( bart, 250, 0);

Run;

data nine;
set Nathan_HVmatchttm;
lagID = lag(ID);
if Count ne 41 then delete;
if lagID = ID then delete;

proc means data= nine;
title HV41 data;
Run;

data Nathan_HV_41nonoverlapping;
Set nine;
lagHV41 = lag(RealizedVol);

proc sort data=Nathan_HV_41nonoverlapping;
by expiry date;

proc reg data = Nathan_HV_41nonoverlapping;
title 'Regression 5:Realized Vol on Hist Vol 41 Trading Day Non-Overlapping';
model RealizedVol = lagHV41/stb ;
test intercept = 0, lagHV41 = 1;
test lagHV41 = 1;

proc reg data = Nathan_HV_41nonoverlapping;
title 'Regression 5:Realized Vol on ISD 41 Trading Day Non-Overlapping';
model RealizedVol = ISD/stb ;
test intercept = 0, ISD = 1;
test ISD = 1;

proc reg data = Nathan_HV_41nonoverlapping;
title 'Regression 5:Encompassing Regression 41 Trading Day Non-Overlapping';
model RealizedVol = ISD lagHV41/stb ;
test intercept = 0, ISD = 1, lagHV41=0;
test ISD = 1;
test lagHV41 = 1;

proc model data = Nathan_HV_41nonoverlapping;
parms a b;
RealizedVol = a + b*lagHV41;

```

```

Title   'Realized Vol on Hist Vol 41 Trading Day Non-Overlapping Second Lag 1';
Fit     RealizedVol / KERNEL=( bart, 250, 0);

proc model data = Nathan_HV_41nonoverlapping;
parms  a b;
       RealizedVol = a + b*ISD;
Title   'Realized Vol on ISD 41 Trading Day Non-Overlapping Second Lag 1';
Fit     RealizedVol / KERNEL=( bart, 250, 0);

Run;

data    ten;
set     Nathan_HVmatchttm;
       lagID = lag(ID);
       if Count ne 62 then delete;
       if lagID = ID then delete;

proc means data= ten;
       title HV62 data;
Run;

data    Nathan_HV_62nonoverlapping;
Set     ten;
       lagHV62 = lag(RealizedVol);

proc sort data=Nathan_HV_62nonoverlapping;
by expiry date;

proc reg data = Nathan_HV_62nonoverlapping;
title   'Regression 6:Realized Vol on Hist Vol 62 Trading Day Non-Overlapping';
model   RealizedVol = lagHV62/stb ;
test    intercept = 0, lagHV62 = 1;
test    lagHV62 = 1;

proc reg data = Nathan_HV_62nonoverlapping;
title   'Regression 6:Realized Vol on ISD 62 Trading Day Non-Overlapping';
model   RealizedVol = ISD/stb ;
test    intercept = 0, ISD = 1;
test    ISD = 1;

proc reg data = Nathan_HV_62nonoverlapping;
title   'Regression 6:Encompassing Regression 62 Trading Day Non-Overlapping';
model   RealizedVol = ISD lagHV62/stb ;
test    intercept = 0, ISD = 1, lagHV62=0;
test    ISD = 1;
test    lagHV62 = 1;

proc model data = Nathan_HV_62nonoverlapping;
parms  a b;
       RealizedVol = a + b*lagHV62;
Title   'Realized Vol on Hist Vol 62 Trading Day Non-Overlapping Second Lag 1';
Fit     RealizedVol / KERNEL=( bart, 250, 0);

proc model data = Nathan_HV_62nonoverlapping;
parms  a b;
       RealizedVol = a + b*ISD;
Title   'Realized Vol on ISD 62 Trading Day Non-Overlapping Second Lag 1';
Fit     RealizedVol / KERNEL=( bart, 250, 0);

proc univariate normal data=Nathan_HV_21nonoverlapping;
Title   ' Is HV21 Normal?';
var lagHV21;

proc univariate normal data=Nathan_HV_41nonoverlapping;
Title   ' Is HV41 Normal?';
var lagHV41;

proc univariate normal data=Nathan_HV_62nonoverlapping;
Title   ' Is HV62 Normal?';
var lagHV62;

RUN;

```