An Explanation of Unbiased Expectations and Efficient Market Hypothesis Using Markov Switching Framework

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Abstract

This paper uses Indian stock futures data to explore unbiased expectations and efficient market hypothesis. Having experienced voluminous transactions within a short time span after its establishment, the Indian stock futures market provides an unparalleled case for exploring these issues involving expectation and efficiency. Besides analyzing market efficiency between cash and futures prices using cointegration and error correction frameworks, the efficiency hypothesis is also investigated after explicitly modeling the underlying state of the market (expansion or contraction) through the first-order Markov switching set-up. The results based on Markov switching analysis show that relatively longer time horizon is more effective in eliminating arbitrage opportunities than the short run.

Keywords: Efficient market hypothesis, Futures market, Cointegration, Error correction, Markov switching.

JEL Codes: G13, G14.

Introduction

The investigation of the validity of the efficient market hypothesis has been and still is one of the favorite topics of the financial literature. Fama (1970) is the first study proposing the *market efficiency* theory which argues that prices should always reflect all available information in an efficient market. In this situation, the only price changes that can occur are the ones resulting from any new information received in the market. In an efficient market setup, one just should not expect to be able to generate positive excess profits. According to the futures market literature, the model that futures prices are unbiased estimators of future spot prices seems to be the appropriate framework to test efficiency.

The hypothesis that futures prices are unbiased estimators of the maturity date spot prices involves joint investigation of two aspects: (i) market efficiency and (ii) no risk premium payment to the speculators. This issue has been explored in the published literature under various names, viz., simple efficiency (Hansen and Hodrick (1980)) and speculative market efficiency (Hokkio and Rush (1989)). The rejection of the unbiased expectations hypothesis implies that the market is inefficient and/or that speculators are receiving a risk premium (see Ivanovic and Howley (2004) for further reference on this aspect).

In this paper, the market efficiency hypothesis is analyzed for Indian stock index futures market for two time horizons, one month ahead and two months ahead.[1] India's

[1] In India, trading takes place on S&P CNX Nifty Futures (which is the stock index futures of National Stock Exchange, Mumbai, India) in three month trading cycle – the near month (one), the next month (two) and the far month (three).

National Stock Exchange (NSE) commenced trading in these derivatives only from mid - June 2000. From the very beginning, NSE established itself as the market leader in the segment of financial futures and options in India with more than 99% market share.[2] Within five years of its initial opening in mid 2000, NSE attracted large transactions in volume of futures and is now ranked 7th [3] in the world in terms of transaction volumes in futures exchanges. Apart from huge amount of volumes related transactions, the average number of traded contracts per month also increased by a staggering 5548% within these five years.[4] This massive but consistent jump in the trading volumes and contracts therefore necessitates studying the *efficiency* of such a fast growing market. In doing so, contribution to the existing literature has been made in two ways: (i) unexplored performance (in terms of efficiency) of index futures market in India is examined, and (ii) for the first time, the first-order Markov switching analysis is used to establish the extent or degree of the market efficiency hypothesis over one month and two months time frames.[5] The existing literature employs cointegration tests and error correction models to determine market efficiency. Cointegration tests establish a potential long-run, equilibrium relationship, but these tests do not address the underlying state of the market which may generate extra information that can influence arbitrage opportunities. Sometimes, error terms in a cointegrating framework can look into this extra information issue through short-term shocks. However, the cointegration and error correction framework do not explicitly model the characteristics of the short-term shocks affecting

[2] Another exchange is Bombay Stock Exchange (BSE).
[3] http://www.futuresindustry.org/fimagazi-1929.asp?a=1100
[4] During financial year 2001-02, when the stock index market was at initial stage in India, the average monthly traded contracts were 85,000, which have gone up to 4.8 million contracts per month during financial year 2005-06.
[5] Looking at only one month and two months time horizons may restrain comparability effectiveness. Nevertheless, the way the issue is modeled in a regime-switching setup shows some important insights into the market efficiency hypothesis even with these small time frames.

this long-term relationship. As a result, the degree or the extent of the market efficiency hypothesis cannot be fully comprehended from an error correction setup. Therefore, an attempt is made in this study to generate extra information about the degree of market efficiency hypothesis through explicitly modeling the underlying state of the market using a Markov switching framework. From the results, the following conclusion can be made: the market is efficient in both one month and two month horizon, but the extent or degree of market efficiency from the Markov switching framework is supporting two month spread better than the one month spread.

In the present study, market efficiency is explored using a consistent framework in the following way. First, the time series properties of individual spot and futures price series in both time horizons are investigated. Second, a vector error correction model (VECM) is built after establishing the cointegrating relationship between the spot and futures prices. Thereafter, the speed of adjustment from this VECM is compared to determine any deviation from short term shocks. Finally, a first-order Markov switching methodology is employed based on the cointegrating relationship established earlier to figure out how effective the market efficiency hypothesis is over these two time horizons. Overall results show that for the Indian stock index futures market, the two month spread is more effective than the one month spread, as the speed of adjustments and Markov switching probabilities show greater support towards maintaining market efficiency hypothesis in the relatively longer[6] (two months) horizon. In this way, we can determine the significance of Indian stock index futures market as a price discovery centre where

[6] Strictly speaking, it should not be termed as longer, as the difference is only for one month. But given the nature of data and recognizing the fact that this market is still developing, terming two months horizon as the longer time horizon makes sense. An extra month's information in a nascent market (like the Indian market for this study) may well lead to arbitraging opportunities within that time frame, leading to possible inefficiency.

information about future supply and demand conditions are disseminated and interpreted in an efficient manner.

The existing literature on forward pricing function mostly employ Johansen's (1988, 1991) cointegration technique and rely on vector error correction framework to conclude about the efficiency in the futures market transactions. However, the studies differ regarding establishing efficiency over different time horizons. Some researchers argue that market efficiency hypothesis holds both in the short and long run. For instance, Haigh (2000) tests the unbiasedness and efficiency hypothesis in the Baltic International Freight Futures Exchange (BIFEX) market and finds that the futures contract appears to become more efficient over time in predicting the spot rate. Kavussanos and Nomikos (1999) analyze the shipping freight futures market and report that futures prices one and two month ahead from maturity provides unbiased forecasts of the realized spot prices. Pizzi, Economopoulos and O'Neill (1998) check for cointegration in both the spot index and three-month futures as well as in the spot index and six-month futures market and conclude that markets are cointegrated in both time horizons, indicating market efficiency in both short-run as well as in the long-run .

On the other hand, some studies in the literature report conflicting evidence regarding market efficiency hypothesis over different time horizons. For example, looking at stock index futures market in Australia, Ivanovic and Howley (2004) indicate that futures prices with one, two and three months to maturity are unbiased predictors of the spot price and hence provide an efficient hedging mechanism for Australian equity index market participants in the short-run. On the other hand, they find that six, nine and twelve month futures prices are biased predictors of spot prices, indicating that speculative opportunities may exist in futures contracts for these time spreads. Kavussanos and Nomikos (1999) also indicate that futures prices three months from maturity are biased estimates of the realized spot prices. In a recent country-specific futures market study for Greece, Kenourgios (2005) has rejected the joint hypothesis of market efficiency and unbiasedness for one month, two month and three month time period, indicating market inefficiency in both short and long run horizons. Looking at the copper futures market, Kenourgis and Samitas (2004) has concluded that the market is not efficient in the sense that the three and fifteen month futures prices do not provide unbiased estimates of the future spot prices in both the long-run and short-run. In another commodity specific study of Australian wool market, Higgs, Rambaldi and Davidson (1999) report that the futures market is efficient for up to only a six month spread, but no further into the future.

The above discussion clearly shows why market efficiency is extensively debated (over the time horizon) in the existing literature. But, surprisingly there is no effort to study the market efficiency hypothesis beyond the cointegration and error correction framework. This may be true as the existing studies rely on comparing the speed of adjustments over time from the error correction framework and conclude about the relative effectiveness of the market efficiency hypothesis over time. However, the cointegration and error correction framework do not explicitly model the characteristics of the short-term shocks which may contain additional information affecting the futures market. Consequently, reliance only on error correction setup to determine the market efficiency hypothesis can at best be termed as a necessary but not sufficient condition. Therefore, the possibility of a sufficient condition (for the effectiveness of market efficiency hypothesis) is explored through the Markov switching framework, which looks at the dynamics of the cointegration process over time based on underlying state of the market. From the results, it can be established that even if the market is efficient in both one and two month horizons (from the cointegration tests and error correction setup), the degree of market efficiency (using additional evidence from the Markov switching framework) is supporting two month spread better than the one month spread.

The rest of the paper is organized in the following way. In the next section the methodology is discussed in detail. Section three looks at the nature of the data after providing a brief overview of Indian stock index futures market. Section four has all the result related discussions. The fifth section concludes with possibility of further research in this area. All the results are reported in the first appendix after the reference section. Some illustrative figures are reported in the second appendix. The third appendix provides additional details on methodology.

1. Methodology

To test the long-run efficiency of the forward pricing function in Indian stock index futures market, first, the parameters β_1 and β_2 need to be estimated from the following equation:

$$S_{t+k} = \beta_1 + \beta_2 F_t + \varepsilon_{t+k} \quad (1)$$

where, S_{t+k} is the actual spot price at time $t+k$, F_t is the futures price at time t of time period $t+k$, ε_{t+k} is the error term with the usual assumptions of zero mean and constant variance, i.e., $E(\varepsilon_t) = 0$ and $E(\varepsilon_t^2) = \sigma^2$, β_1 is the intercept term and β_2 is the slope coefficient. To establish the long-run efficiency, it needs to be determined if the joint restrictions $\beta_1 = 0$ and $\beta_2 = 1$ holds true or not. If the null hypothesis is accepted against the alternative that these restrictions are not true, then the market supports efficiency hypothesis.

The above equation is easy to estimate using ordinary least squares (OLS) method, provided the spot and futures prices series are stationary. To test for stationarity in the spot and futures series, five different test statistics have been used: Augmented Dickey-Fuller test statistic (ADF), Phillips-Perron test statistic (PP), Kwiatkowski-Phillips-Schmidt-Shin test statistic (KPSS), Ng-Perron test statistic (NP), and Elliott-Rothenberg-Stock DF-GLS test statistic (DF-GLS). All the unit root tests are done in presence of constants and linear trends, as it has been found that both the spot and futures price series in one month and two month spread contain linear trends and support affine transformations in their logarithmic forms. For the ADF and DF-GLS tests the lag lengths are chosen by the Schwartz (BIC) criterion; for the PP, KPSS and NP tests the bandwidths are set to the default values corresponding to the Bartlett kernel functions. The order of integration is also tested, and it has been found that for both horizons, the spot and futures are integrated of order 1, i.e., $I(1)$ in log-levels, which supports the possibility of long-run cointegrating relationships between the spot and futures in both one and two month horizons. As both price series are of the same order, Johansen multivariate cointegration framework (as in Johansen, 1988, 1991; Johansen and Juselius, 1990) is applied and a vector error correction model (VECM) is built to check for possible long-run equilibrium relationship between the spot and futures price series. The market efficiency hypothesis can also be analyzed from this VECM, as error correction framework shows the speed of adjustment process if there is any short-run deviations (due to some unexpected shocks) from the long-run equilibrium cointegrating relationship. Market efficiency presumes that actual spot prices at a later time (ex post) can be accurately predicted by futures prices set beforehand (ex ante). Any deviation arising out of misinformation in the market can therefore be quickly corrected, and the speed of adjustment will reflect this process reasonably well.

Once patterns (i.e., whether there are trends, unit roots, structural breaks as well as nonlinearity evidence)[7] of all the four individual time series are established, an unrestricted vector autoregressive (VAR) model is built involving one month and two months spot and futures prices. This is needed to determine the optimal lag-length of the vector error correction model, and that can only be done through setting up an unrestricted VAR and statistically testing for optimal lag-lengths.

In generic terms, say $\mathbf{z}_t = (y_t, x_t)$ be a (2×1) vector. Then we can write the VAR model as:

$$\mathbf{\Phi}(B)\mathbf{z}_t = \mathbf{C}\mathbf{d}_t + \mathbf{S}\vartheta_t \qquad (2)$$

where the matrix autoregressive polynomial $\mathbf{\Phi}(B)$ is defined as $\mathbf{\Phi}(B) = \mathbf{I} - \sum_{i=1}^{p} \mathbf{\Phi}_i B^i$ which obeys the stability condition $\det\{\mathbf{\Phi}(1)\} \neq 0$. $\mathbf{d}_t$ is a vector of dummy variables and constant term with associated coefficient matrix $\mathbf{C}$, while ϑ_t is white noise with identity covariance matrix and $\mathbf{S}$ is the lower triangular decomposition for the covariance matrix $\mathbf{\Sigma} = \mathbf{SS}'$. Conditional least squares were used in estimating the model's parameters with the autoregressive order chosen via a combination of five different criteria: a sequential modified likelihood ratio test statistic, final prediction error criterion, Akaike information

[7] See Appendix 3 for the above test details.

criterion, Schwarz information criterion, and Hannan-Quinn criterion. All of these test results are tabulated in table 3 in appendix 1. Based on all of the five criteria, three lags are determined as the optimal lag length for the unrestricted VAR as well as the VECM. Thereafter, this unrestricted VAR model is also used in performing Granger causality tests. If the matrix polynomial $\mathbf{\Phi}(B)$ is portioned as:

$$\mathbf{\Phi}(B) = \begin{bmatrix} \phi_{yy}(B) & \phi_{yx}(B) \\ \phi_{xy}(B) & \phi_{xx}(B) \end{bmatrix}$$

then the null hypothesis of non-causality from x_t to y_t is equivalent to $\mathrm{H}_{01} : \phi_{yx}(B) = 0,$ while the corresponding null hypothesis of non-causality from y_t to x_t is equivalent to $\mathrm{H}_{02} : \phi_{xy}(B) = 0$. If both H_{01} and H_{02} are rejected then the feedback between x_t and y_t exist.

In the present context, the Granger causality test conveys and supports the market efficiency hypothesis in the following way. If there is a long-run equilibrium relationship between the spot and futures prices (in both one month and two months spreads), then Granger causality can also capture this fact. For example, say spot price is Granger causing future price, which implies that spot price can be used to predict future prices in exactly the same way we would expect from equation 1 under the null hypothesis of parameter constraints holds true. On the other hand, say we find evidence that the futures price is not Granger causing the spot price. This conveys the fact that futures are not unbiased predictors of spot prices, which should be the case under market efficiency hypothesis. Therefore, performing Granger causality tests assume special significance, as that can pinpoint the potential nature of the long run equilibrium relationship between spot and futures prices in both time horizons. Results for the Granger causality tests are reported in table 4 of appendix 1 and the findings support market efficiency hypothesis unambiguously.

Thereafter, the joint distribution of spot and futures prices is modeled as a vector error correction framework:

$$\Delta\mathbf{V}_t = \boldsymbol{\mu} + \sum_{i=1}^{m-1} \boldsymbol{\Psi}_i \Delta\mathbf{V}_{t-i} + \boldsymbol{\Pi}\mathbf{V}_{t-m} + \boldsymbol{\varepsilon}_t \tag{3}$$

where, $\mathbf{V}_t$ is the (2×1) vector, $[S_{t+k}, F_t]'$; $\boldsymbol{\mu}$ is a (2×1) vector of deterministic components which includes intercept terms, linear trends, as well as dummies for structural breaks; $\boldsymbol{\Psi}_i$ and $\boldsymbol{\Pi}$ are the parameter matrices to be estimated which contain information on both short run and long run adjustments on changes in $\mathbf{V}_t$; $\boldsymbol{\varepsilon}_t$ is a (2×1) vector of white-noise residuals and m is the lag-length (three in this case). Maximum likelihood techniques (following Johansen, 1988, 1991 as well as Johansen and Juselius, 1990) are used to estimate the above parameter matrices. $\boldsymbol{\Pi}$ determines the number of stationary, linear combinations of the spot and futures prices. The numbers of cointegrating vectors are equal to the rank of $\boldsymbol{\Pi}$. Denoting rank by r, the decision goes this way. When $r=1$, S_{t+k} and F_t are cointegrated and there exists at least two (2×1) matrices, say, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, such that, $\boldsymbol{\Pi} = \boldsymbol{\alpha\beta}'$, where, $\boldsymbol{\beta}'$ represents the vector of cointegrating coefficients and $\boldsymbol{\alpha}$ is the vector of error correction coefficients. These error correction coefficients measure the speed of adjustments from the short run disequilibrium to the long run equilibrium state. The Johansen procedure to determine rank, r, entails estimation of two statistics, $\lambda_{\max}$ (denoted by ‘Lambda max’ in the tables) and λ_{trace} (denoted by 'Trace test' in the tables). If both 'Lambda max' and 'Trace test' statistics suggest that the rank is not significantly different from unity then a cointegrating relationship between the spot and futures price series exist. Tables 5 and 6 in appendix 1 report the Johansen cointegration test results and VECM results. Diagnostic tests from the VECM residuals are also conducted, the results of which are presented in table 7 of appendix 1.

After exploring potential long run equilibrium relationship between spot and futures prices for both one and two months time horizons, the efficiency of this relationship between different time horizons is checked. The issue here is to analyze the extent of the cointegrating relationship at a certain time frame based on the underlying structure or state of the market and compare any potential change in that relationship when the time frame is changed. Alternatively, it is to be investigated if there is any deviation from the cointegrating relationship at time period $t+1$, provided the prices are already cointegrated at time t. If the market efficiency hypothesis is true, then it is expected that there will be either insignificant or at best negligible deviations from long term cointegrating relationship over time. This is modeled as a Markov switching process of the first order. For the present analysis, it is assumed that spot and futures prices are experiencing two states at all times, namely, "they are cointegrated" or "they are not cointegrated". These price series are not cointegrated at time period t is determined by the changes in their mean basis level (i.e., it is taking the value 1 if it is away from the mean basis level) in time period t. Therefore, cointegration evidence in period $t+1$ is determined by a value of 0 (i.e., not very much different from the mean basis level) in period $t+1$. Since, both cannot occur together at the same period, these events are mutually exclusive from a statistical viewpoint. Evidence of cointegration in period t is denoted by C_t and no cointegration in period t by N_t. Taking a first-order Markov process, initially the past periods' influence on current periods' cointegrating process is examined in the following way.

Let ϕ_{ij} be the conditional probability that the market is in state $i = C, N$ in period t and in state $j = C, N$ in period $t+1$. The (2×2) transition probability matrix between the prices' state in period t and $t+1$ is therefore:

$$\begin{bmatrix} \Pr(N_{t+1} \mid N_t) & \Pr(C_{t+1} \mid N_t) \\ \Pr(N_{t+1} \mid C_t) & \Pr(C_{t+1} \mid C_t) \end{bmatrix} = \begin{bmatrix} \phi_{NN} & \phi_{CN} \\ \phi_{NC} & \phi_{CC} \end{bmatrix} \tag{4}$$

where $\Pr$ denotes probability and each row of the above transition matrix sums to 1. Let, n_{ij} be the number of occurrences of state i in period t and state j in period $t+1$, and let n_i be the number of occurrences of state i in period $t+1$, so that for the (2×2) Markov matrix, $n_{CC} + n_{CN} = n_C$ and $n_{NC} + n_{NN} = n_N$. The log-likelihood function, $\ln(L)$ for the (2×2) Markov process can be written as follows:

$$\ln(L) = n_{CC}\ln(\phi_{CC}) + n_{CN}\ln(1-\phi_{CC}) + n_{NC}\ln(1-\phi_{NN}) + n_{NN}\ln(\phi_{NN}) \tag{5}$$

and the corresponding maximum likelihood estimators of the probabilities are $\hat{\phi}_{CC} = \frac{n_{CC}}{n_C}$ and $\hat{\phi}_{NN} = \frac{n_{NN}}{n_N}$, that is the corresponding sample proportions on the number of transitions. These estimated ϕ_{ij} are reported in table 8 of appendix 1.

2. Data and descriptive statistics

Data for this study are collected from India's National Stock Exchange (NSE) web site.[8] Monthly closing price observations for both S&P CNX NIFTY spot and S&P CNX NIFTY futures market from July 2000 to December 2005 constitute the data sample in this study. There are 66 observations for the one month sample and 64 observations for the two month sample. However, the same numbers of observations, i.e., 64 observations for both of the time frames in the present analysis are used. The small data points in this study can be explained in this way. Despite the existence of a well developed stock market for over one hundred years, trading on financial derivative contracts in India started only from June 2000. The first traded contract was stock index futures. Later on, individual stock futures and individual stock options have also been introduced on the National Stock Exchange (NSE, Mumbai). Stock Index Futures in India are traded in monthly series; the expiration date for each series being the last Thursday of the month. At any point in time, three monthly series (current month expiry, two month and three month expiry) are traded side by side. These contracts are cash settled and can be exercised either at expiration or before expiration. NSE and BSE (Bombay Stock Exchange) are the main stock markets for the underlying derivatives and they have the same trading time for both their cash and derivative segments (Monday–Friday; 8:55 a.m.–3:30 p.m.). Settlement price for the derivatives is the closing price of the underlying index (Nifty) in the cash- (equity-) market segment on the expiration day. Therefore,

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from the monthly time horizon, we could only match these small numbers of observations from both the spot and futures indexes.[9]

Descriptive statistics from spot and futures prices for both one month and two months time horizons are reported in table 1 of appendix 1. Individual spot and futures series as well as basis series are plotted in figures 1 to 4 in appendix 2. In the last two columns of table 1 in the first appendix, the decisions regarding structural breaks (denoted by Br) and linearity tests (denoted by L) in these data series are also reported. For the spot prices, the range (difference between maximum and minimum values in the data series) is equal for both time spreads, which shows uniformity in the spot prices series. The spot and futures price series are also well behaved in the sense that they do not show any conditional volatility evidence from the sampled kurtosis values. Jarque-Bera test statistic values show that all price series are non-normal. For the structural break analysis, Perron's (1997) structural break test (based on an additive outlier model) is used. This test shows evidences of structural break in the sampled price series. In the ongoing modeling process to investigate market efficiency hypothesis, dummies for these structural break evidences are used.[10] Based on extensions of ARIMA processes for univariate time series (as described in Appendix 3 later), the nonlinearity test (F-test, based on Tsay (1986)) are done to determine any nonlinear evidence in the price series. The results show no nonlinear evidence in the data sample of spot and futures prices for both one month and two months time horizons.[11]

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[9] We have ignored the three month expiration contract as these do not attract sufficient volumes compared to other two months case in the Indian stock futures market. For additional information on Indian stock market, see Vipul (2005).

[10] Dummies take values 1 whenever there are structural break evidences.

[11] It can well be argued that for such a small data sample, the evidence of nonlinearity can hardly be possible. We statistically tested that hypothesis even with this small data sample to be doubly sure.

3. Results and discussions

All the descriptive test statistics as well as unit roots tests for univariate time series, unrestricted VAR model and lag-order selection criteria results, Granger causality results, Johansen cointegration test results, VECM results, diagnostic tests from the residuals of the VECM and first-order Markov switching model results are reported in tables 1 to 8 in the first appendix after the reference section. Spot and futures prices figures, basis prices figures and cointegrating relationship figures are all reported in appendix 2. All evidence from tables in appendix 1 show two things: (i) market efficiency hypothesis holds true for both the one month and two months horizons but (ii) the two month spread proves to be more efficient in the sense that the speed of adjustment from a short-run deviation from the futures prices is faster than the one month horizon's case. This second finding is later supported by the Markov switching process. Based on (i) and (ii), the results show that the Indian futures market is more efficient over relatively longer time (two months) horizon, rather than the shorter, one month horizon.

Stationarity checks involving individual spot and futures price series over both horizons are presented in table 2 of appendix 1. Based on 95% level of significance, all the price series are integrated of order one in log-level. Additional checks for higher order integration levels only support the earlier result, i.e., these prices are integrated of only order one and not of any higher order. Therefore, cointegrating relation can be explored and VECM framework can be built afterwards with these price series. Before building VECM, the log-likelihood values from an unrestricted VAR model are used to select the number of optimal lags based on five different criteria: Likelihood Ratio (LR) test statistic, final prediction error (FPE) criterion, Akaike information criterion (AIC), Schwarz information criterion (SIC), and Hannan-Quinn (HQ) criterion. The results are presented with a maximum of ten lags as the starting point to carry out this test. However, the unrestricted VAR with fifteen and twenty lags is also tested, and the lag order selection criteria always pointed towards three lags as the optimal one. The diagnostic tests involving residuals from these three different sets of VAR models (with ten, fifteen and twenty maximum lags as starting points) are also analyzed and the results show that three lags remain consistent in outperforming all the other diagnostic results involving more than ten lags. Needless to say, this also supports the parsimonious principle. Based on these criteria, three lags are chosen as the optimal number of lags for both one month and two months time horizons. Once the optimal lag orders are determined, Granger causality tests are performed for both time horizons, the results of which are reported in table 4 of appendix 1. Causality results are very consistent in terms of pointing out that spot prices do Granger cause the futures prices in both one month and two months time horizons with lags one, three, four and ten. This supports (ex ante) the market efficiency hypothesis, i.e., in the long-run, the spot prices can be used to represent the futures prices, as there is unidirectional evidence that spot prices are affecting futures prices, and not the other way around. This result supports market efficiency hypothesis we want to prove from equation 1 discussed earlier in the methodology section.

Table 5 in the first appendix reports Johansen cointegration test results. Figures 5 and 6 in appendix 2 plot the actual cointegrating relationships. It is interesting to note that from the tabulated result, in one month spread, the ‘Lambda max’ test statistic shows no evidence of cointegrating relationship at 95% level of significance, but the ‘Trace test’ shows one cointegrating relation between spot and futures within the same time frame.

Two months spread results clearly show one cointegrating relationship exists between spot and futures price in the long run. This finding is at odds with the existing literature, which generally shows that within a shorter time frame (say one month or two months), there seems to be much more cointegrating evidence than a longer time frame (starting from six months or nine months onwards). On the contrary, this study shows that one month spread does not unequivocally support any cointegrating relationship, but two months spread unambiguously shows one cointegrating relation based on both maximum eigenvalue (Lambda max) test as well as trace test. Note that figures 5 and 6 in the second appendix also support the above evidence. A quick comparison between figures 5 and 6 show that the cointegrating relationship appears to be much more evident for the two months spread. The above findings are also supported in the later part of the analysis involving speed of adjustments from VECM and Markov switching results.

VECM results are presented in table 6 in the first appendix. In reporting these results, the cointegrating vectors are normalized with respect to the spot price parameters in both one and two months time horizons, and therefore, standard errors and t-statistics for the estimated spot price parameters in the long run (denoted by $\hat{\beta}_s$) are not listed in the table. From table 6, both the estimated futures price cointegrating vector parameters are close to one ($\hat{\beta}_f = -1.001$ for one month spread and $\hat{\beta}_f = -1.003$ for the two months spread), and this is what one would expect under the market efficiency hypothesis depicted in equation 1 with parameter restrictions $\beta_1 = 0$ and $\beta_2 = 1$. In the present context, the $\hat{\beta}_f$'s can be interpreted as β_2's. The speed of adjustment coefficients ($\hat{\alpha}_s$ and $\hat{\alpha}_f$) show one interesting fact: the speed of adjustment for futures prices, $\hat{\alpha}_f$, are much faster and statistically significant than the speed of adjustment for spot prices, $\hat{\alpha}_s$. Also, for two months spread, the adjustment speed for futures is faster than the one month spread. These show that the adjustment of futures price depends on the long-run disequilibrium between spot and futures prices, whereas, the adjustment of the spot price is independent of the long-run disequilibrium. Additionally, the longer the time horizon, the more is the dependency on future prices to adjust from a disequilibrium situation.

This is an interesting finding, as one would expect, for shorter time horizons (say, one month here), the long-run equilibrium relation would be restored faster had there been any deviation momentarily. Regarding the market efficiency hypothesis, this result seems consistent, as we previously found that spot prices are Granger causing futures prices, and now from the speed of adjustments results, spot prices are not affected by the past period's deviations from the long-run equilibrium. Both of the above results point to a very tight, long-run equilibrium relationship between spot and futures prices, where futures prices play the role of adjusters, in case there are deviations in the short run. It is much more evident for longer, two month time horizons. Diagnostic tests of residuals from the VECM (see table 7 in appendix 1 for reference) show that the results are consistent, as there are no serial correlations among the residuals, and they are not generally volatile, with a small evidence of conditional heteroskedasticity for the futures residuals from one month spread. On the whole, the results suggest that the VECM with three lags captures the long run equilibrium relationship between spot and futures quite well.

Table 8 in appendix 1 shows first-order Markov switching results. The issue here is to look at the probabilities depicting the change in cointegrating relationship between one month and two month time horizons. For this reason, comparing the numerical values in last two columns of table 8 will be sufficient, as they are showing the probabilities of no cointegration in period $t+1$ if there was cointegration in period t (denoted by ϕ_{NC}) and cointegration in period $t+1$ if there was cointegration in period t (denoted by ϕ_{CC}). Here again, the two months spread outperforms the one month spread in the sense that it shows higher probability of being cointegrated in $t+1$, provided it is already cointegrated in period t, and the one month spread shows higher probability of being not cointegrated in period $t+1$ if it is already cointegrated in period t. This finding also supports the earlier evidence that two months spread proves to be more effective than the one month spread (in terms of relatively long run evidence of cointegration and speed of adjustment from short run disequilibrium to the long run equilibrium path). Consequently, based on the analysis it can be said that for Indian stock index futures market, the two months spread seems to be more consistently supporting a long run equilibrium relationship. Therefore, the market efficiency hypothesis is more credible in the two months horizon, rather than the one month horizon.

4. Conclusion

In this paper, the unbiasedness and market efficiency hypothesis regarding the forward pricing function for Indian stock index futures market is tested. The analysis is done in a consistent time series framework. Using this framework, the patterns in both the stock index spot and stock index futures prices are minutely checked, an unrestricted VAR framework is built and statistically optimal lags are determined which are consistent with that framework. The findings from above help us to build a VECM based on the long run equilibrium cointegrating relationship established through Johansen's multivariate cointegration procedure. Thereafter, a new dimension is added in looking at the market efficiency hypothesis through a first-order Markov switching process. It is interesting to note that the Markov process is also built on the evidence of cointegration at the first instance, which is the central pillar for the VECM framework. The existing literature on forward pricing function is largely confined in building and drawing efficiency conclusion from a VECM framework only. Based on the VECM as well as the Markov switching process, two main results can be summarized from the present study: (i) the market efficiency hypothesis holds good for both the one and two months horizon and therefore, the arbitrage opportunities appear to be minimal and (ii) the forward pricing functional form is much more effective in the two months time frame, as the speed of adjustments and the switching probabilities from one cointegrating stage to another stage are much faster and higher than the one month horizon's case. The second finding is quite new, as the current literature largely supports the market efficiency hypothesis in shorter run than in the long run. For the present analysis, based on Indian stock index futures and spot index prices data, it can be concluded that the Indian stock index futures market appears to be more efficient in the two months spread, which reduces the arbitrage opportunities much more than the short run, one month time horizon. Future analysis will concentrate on further bolstering (or otherwise) this finding with a larger, high-frequency data set from Indian as well as other developing and developed stock futures markets in other countries.

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Appendix 1

Table 1. Descriptive Statistics for Spot and Futures Price Series

Series	$\bar{x}$	s	R	m	S	K	N	Br	L
				For one month spread					
Spot	1468.677	490.134	1945.250	1303.100	0.933	2.935	9.595	Yes	Yes
Futures	1462.234	470.262	1811.600	1317.350	0.837	2.680	7.981	Yes	Yes
				For two months spread					
Spot	1473.009	496.099	1945.250	1304.600	0.894	2.831	8.609	Yes	Yes
Futures	1442.308	447.546	1719.650	1295.500	0.768	2.453	7.104	Yes	Yes

Notes: The descriptive statistics are, in column order: sample mean, standard deviation, range, median, sample skewness, sample kurtosis, normality test statistic (Jarque-Bera) value, decision for structural break test and decision for linearity test.

Table 2. Unit Root Test Results for Spot and Futures Price Series

Series	ADF	PP	KPSS	NP	DF-GLS
			For one month spread		
Spot	-1.675**	-1.676**	0.231**	-2.265**	-1.036**
Critical value	*-3.480*	*-3.480*	*0.146*	*-17.300*	*-3.142*
Futures	-2.110**	-2.118**	0.231**	-2.445**	-1.142**
Critical value	*-3.480*	*-3.480*	*0.146*	*-17.300*	*-3.142*
			For two months spread		
Spot	-2.025**	-2.029**	0.218**	-2.496**	-1.143**
Critical value	*-3.482*	*-3.482*	*0.146*	*-17.300*	*-3.148*
Futures	-2.021**	-2.030**	0.219**	-3.232**	-1.352**
Critical value	*-3.482*	*-3.482*	*0.146*	*-17.300*	*-3.148*

Notes: ADF stands for Augmented Dickey-Fuller test statistic, PP stands for Phillips-Perron test statistic, KPSS stands for Kwiatkowski-Phillips-Schmidt-Shin test statistic, NP stands for Ng-Perron test statistic, and DF-GLS stands for Elliott-Rothenberg-Stock DF-GLS test statistics for testing unit roots and stationarity in presence of constants and linear trends. *Critical value* and ** represent 95% level of significance.

Table 3. VAR Lag Order Selection Criteria

Lag	LogL	LR	FPE	AIC	SIC	HQ
			For one month spread			
0	73.449	Na	0.001	-1.908	-1.185	-1.629
1	225.537	238.994	0.002	-7.197	-6.330**	-6.861**
2	227.043	2.259	0.002	-7.108	-6.096	-6.716
3	234.934	11.271**	0.003**	-7.247**	-6.090	-6.798
4	236.526	2.161	0.003	-7.161	-5.859	-6.657
5	240.637	5.285	0.003	-7.165	-5.718	-6.604
6	244.581	4.789	0.003	-7.163	-5.572	-6.546
7	246.481	2.171	0.003	-7.088	-5.352	-6.415
8	250.346	4.141	0.003	-7.083	-5.203	-6.355
9	252.200	1.854	0.004	-7.007	-4.982	-6.221
10	254.328	1.976	0.004	-6.940	-4.770	-6.098
			For two months spread			
0	51.841	Na	0.001	-1.179	-0.442	-0.895
1	182.581	203.374	0.001	-5.873	-4.989	-5.532
2	220.973	56.876	0.003	-7.147	-6.115	-6.749
3	232.228	15.841**	0.002**	-7.416**	-6.237**	-6.961**
4	234.041	2.416	0.002	-7.335	-6.009	-6.824
5	239.555	6.944	0.002	-7.390	-5.917	-6.822
6	241.478	2.278	0.003	-7.314	-5.693	-6.688
7	245.312	4.260	0.003	-7.308	-5.539	-6.626
8	247.740	2.518	0.003	-7.249	-5.334	-6.511
9	252.867	4.937	0.003	-7.291	-5.228	-6.496
10	257.424	4.050	0.003	-7.311	-5.102	-6.459

Notes: ** indicates lag order selected by the criterion. LogL stands for log-likelihood values, LR stands for sequential modified LR test statistic (each test at 5% level), FPE stands for Final prediction error criterion, AIC stands for Akaike information criterion, SIC stands for Schwarz information criterion, and HQ represents Hannan-Quinn criterion.

Table 4. Granger Causality Test Results

H_0	Lag 1	Lag 3	Lag 4	Lag 10
		For one month spread		
Spot $\neq$ Futures	481.992***	155.676***	113.918***	38.3325***
p-value	0.000	0.000	0.000	0.000
Futures $\neq$ Spot	0.136	1.484	0.835	1.095
p-value	0.715	0.229	0.509	0.393
		For two months spread		
Spot $\neq$ Futures	72.736***	176.827***	133.375***	49.016***
p-value	0.000	0.000	0.000	0.000
Futures $\neq$ Spot	0.001	1.349	0.646	1.054
p-value	0.992	0.268	0.632	0.423

Notes: *** denotes 99% level of significance. $\neq$ denotes 'does not Granger cause'. All the test statistics values are F-test statistics.

Table 5. Johansen Cointegration Test Results

Test Statistic	H_0	$\lambda_{calculated}$	$\lambda_{critical}$	MHM	Reject H_0
		For one month spread			
Lambda max	$r = 0$	13.399	*14.264*	0.068	No
	$r = 1$	2.726	*3.842*	0.098	No
Trace test	$r = 0$	16.125**	*15.494*	0.040	Yes
	$r \leq 1$	2.726	*3.842*	0.098	No
		For two months spread			
Lambda max	$r = 0$	35.921**	*14.264*	0.000	Yes
	$r = 1$	0.620	*3.842*	0.431	No
Trace test	$r = 0$	36.540**	*15.494*	0.000	Yes
	$r \leq 1$	0.620	*3.842*	0.431	No

Notes: Lambda Max and Trace Tests are Johansen cointegration test statistics. MHM denotes MacKinnon-Haug-Michelis (1999) p-values. $\lambda_{critical}$ shows 95% level of significance.

Table 6. Vector Error Correction Model Results

Values	$\hat{\alpha}_s$	$\hat{\alpha}_f$	$\hat{\beta}_s$	$\hat{\beta}_f$
		For one month spread		
Est. parameters	0.144	0.818***	1.000	-1.001***
Standard errors	*0.775*	*0.257*	*na*	*0.014*
t-statistic	*0.187*	*3.182*	*na*	*-69.917*
		For two months spread		
Est. parameters	0.343	1.196***	1.000	-1.003***
Standard errors	*0.590*	*0.202*	*na*	*0.010*
t-statistic	*0.581*	*5.931*	*na*	*-101.695*

Notes: *** denotes 99% level of significance. α 's are the speed of adjustment coefficients and β 's are estimated vector parameters (termed as long-run coefficients). Subscripts s indicates spot price coefficients and f denotes future price coefficients. 'Est. parameters' indicate estimated parameter coefficients.

Table 7. Diagnostic Tests of Residuals from the Vector Error Correction Model

Residuals	$Q(10)$	$LM(1)$	$LM(4)$	N	S	K	$ARCH(4)$
			For one month spread				
ε_s	6.233	0.001	0.296	0.015	-0.007	2.925	1.543
p-value	0.795	0.972	0.879	0.992	na	na	0.203
ε_f	3.507	0.130	0.153	41.066	-0.947	6.508	0.777
p-value	0.967	0.719	0.960	0.000***	na	na	0.545
			For two months spread				
ε_s	8.368	0.019	0.845	0.016	0.028	2.943	0.697
p-value	0.593	0.888	0.502	0.992	na	na	0.598
ε_f	11.605	0.105	1.146	4.110	-0.524	3.736	0.353
p-value	0.312	0.747	0.344	0.128	na	na	0.840

Notes: *** signifies 99% level of significance. The diagnostic tests are in column order: Ljung-Box Q-statistics on the first 10 lags of the sample autocorrelation function; $LM(1)$ and $LM(4)$ are the Lagrange Multiplier tests for serial correlation of order 1and 4, respectively; N stands for Jarque-Bera test for normality; S stands for skewness and K stands kurtosis; $ARCH(4)$ is the Engle (1982) test for ARCH effects. Except for the skewness and kurtosis, all other values represent estimated test statistics.

Table 8. Markov Switching Model Results

Series	ϕ_{NN}	ϕ_{CN}	ϕ_{NC}	ϕ_{CC}
For one month spread				
Spot and Futures	0.517	0.483	0.417	0.583
For two months spread				
Spot and Futures	0.714	0.286	0.257	0.743

Notes: Numerical values denote probabilities. ϕ_{NN} denotes not cointegrated in period "$t+1$" if there was no cointegration in period "t"; ϕ_{CN} denotes cointegrated in period "$t+1$" if there was no cointegration in period "t"; ϕ_{NC} denotes no cointegration in period "$t+1$" if there was cointegration in period "t" and ϕ_{CC} denotes cointegration in period "$t+1$" if there was cointegration in period "t".

Appendix 2

Figure 1. Spot and Futures Prices in One Month Time Horizon

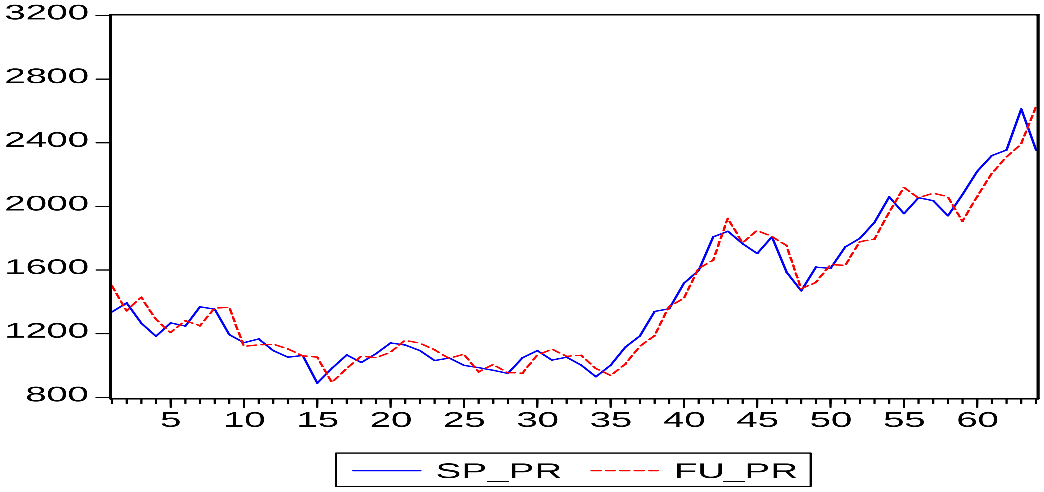


Figure 2. Spot and Futures Prices in Two Months Time Horizon

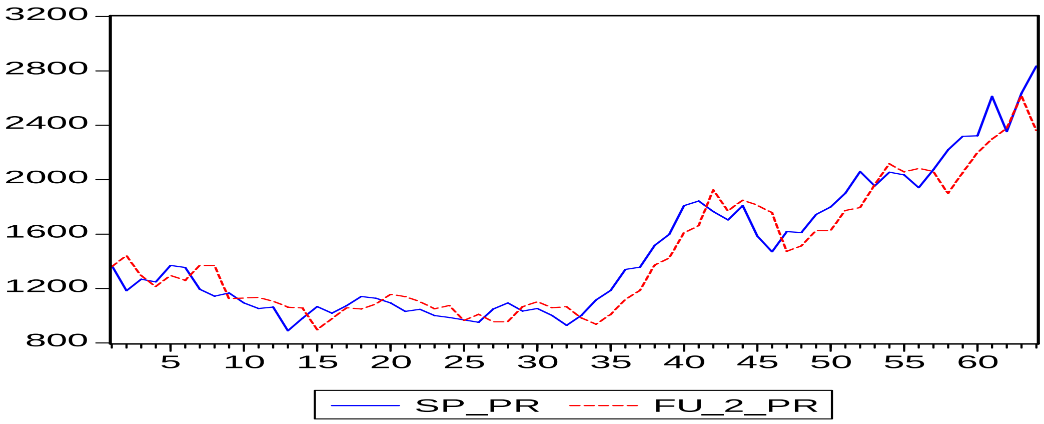


Figure 3. Basis Price for One Month Time Horizon

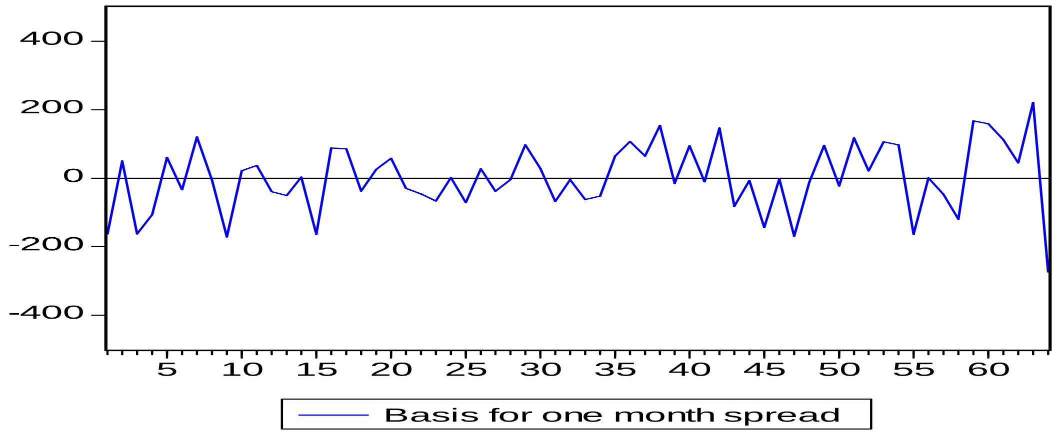


Figure 4. Basis Price for Two Months Time Horizon

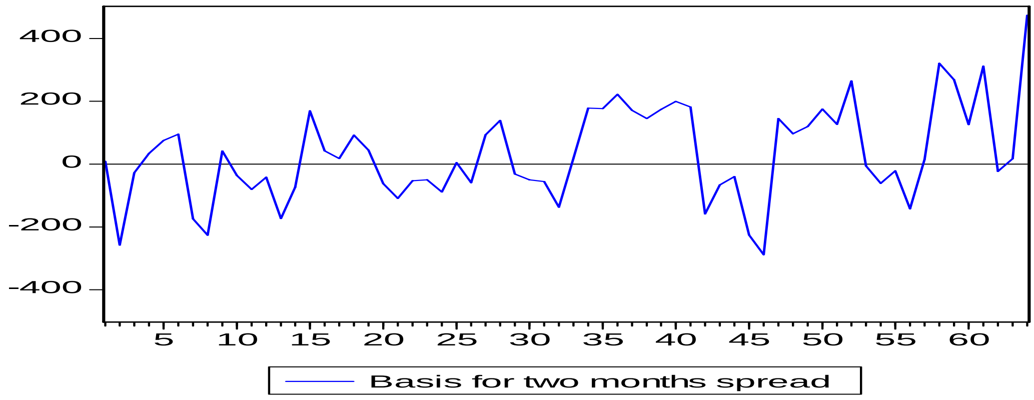


Figure 5. Cointegrating Relation for One Month Time Horizon

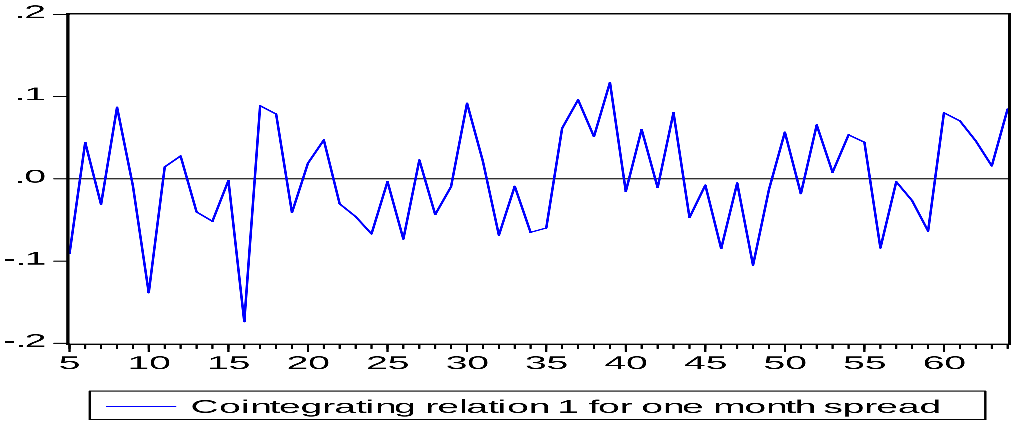
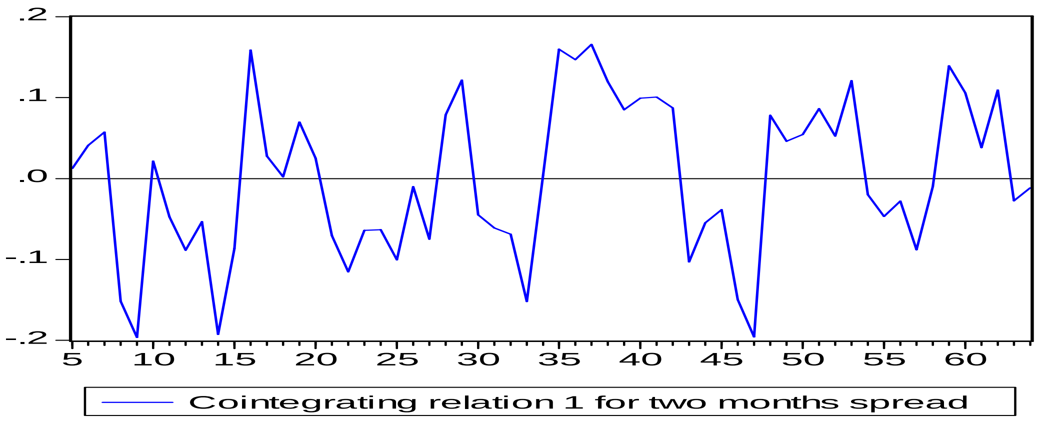


Figure 6. Cointegrating Relation for Two Months Time Horizon



Appendix 3

Before modeling the VECM framework (after establishing the number of cointegrating relationships between the spot and futures prices in both time horizons), two important aspects of these price series are investigated: (a) evidence of structural breaks and (b) evidence of nonlinearity. Both are done both after analysing the individual series in autoregressive integrated moving average (ARIMA) frameworks. In presence of constants and trending components in individual price series, one need to make sure that evidence of long-run relationship (and cointegration) is only driven by stationarity and not by any other nonlinear terms. Structural breaks are important to analyze before any full model setup (like VECM here), as the residuals from estimated models will not be well-behaved if there are structural breaks in the time series itself and these are not being taken care of through dummies in the beginning of the model building process. Perron's (1997) test is used for structural breaks and evidence of structural breaks in all the four price series is found. Dummies are used to take care of these structural breaks in testing for nonlinearity setup as well as the VECM setup later. Decisions for structural breaks are listed (and denoted by Br) in column nine of table 1 in appendix 1. Thereafter, the issue of nonlinearity is looked at. In what follows the setup of ARIMA models (in generic terms) is described and shown how nonlinearity vs. linearity can be tested by extending that framework. In reporting of results (see appendix 1 for reference), the decision about the linearity evidence (denoted by L) in the last column of table 1is reported and not the ARIMA model estimates.[12]

[12] Structural break test results, ARIMA model estimation results and nonlinear test statistic values are available upon request from the corresponding author.

For a generic time series w_t (it can be either spot or futures price series), we use B^j to denote the backshift operator $B^j w_t = w_{t-j}$. The univariate ARIMA models take the form:

$$\phi(B) y_t = \mathbf{d}_t'\boldsymbol{\beta} + \sigma\xi_t \tag{A3.1}$$

where y_t is $\log w_t$, and $\mathbf{d}_t$ is a $(k \times 1)$ vector of dummy variables (which takes care of structural breaks in the series, determined through Perron's (1997) test) and a constant term with associated coefficient vector $\boldsymbol{\beta}$. The autoregressive polynomial $\phi(B)$ is defined as $\phi(B) = 1 - \sum_{i=1}^{p} \phi_i B^i$ and is usually restricted, with some of the ϕ_i's being set to zero. It is also assumed that all the roots of $\phi(B) = 0$ are outside the unit circle. The error series ξ_t was white noise with unit variance for some of our models, $\xi_t = \varepsilon_t$. Occasionally, however, it takes the form of a restricted moving average $\xi_t = \theta(B)\varepsilon_t = \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}$, with some of the θ_j's set to zero. The orders (p, q) of the ARIMA models were chosen using standard techniques that is the correlogram, over fitting tests and diagnostic tests on the estimated residuals. Conditional least squares were used for estimation, with back casting employed only when the roots of the moving average polynomial $\theta(B)$ were outside the unit circle.

The model of equation (A3.1) is appropriate for a linear time series. Therefore, nonlinearity is tested using the autoregressive part of the model and the omnibus F-type test of Tsay (1986), see also Fuller (1995) and Tsay (2001). Let $\boldsymbol{\varphi} = (\phi_1, ..., \phi_p)'$ be the

$(p \times 1)$ vector of autoregressive coefficients, $\mathbf{y}_{t-1} = (y_{t-1}, ..., y_{t-p})'$ be the $(p \times 1)$ vector of lagged values and define the $p(p+1)/2 \times 1$ vector $\mathbf{h}_t$ that consists of all the squares and cross-products of the elements in $\mathbf{y}_{t-1}$. For example, if $p = 2$, then $\mathbf{h}_t = (y_{t-1}^2, y_{t-2}^2, y_{t-1}y_{t-2})'$. The test for linearity is the usual F-test for the null hypothesis $\mathrm{H}_0 : \boldsymbol{\delta} = 0$ in the augmented model:

$$y_t = \mathbf{d}_t'\boldsymbol{\beta} + \mathbf{y}_{t-1}'\boldsymbol{\varphi} + \mathbf{h}_t'\boldsymbol{\delta} + \sigma\xi_t \qquad \text{(A3.2)}$$

where $\boldsymbol{\delta}$ is the associated coefficient vector. The results (in terms of yes or no decisions) from this test, performed over the entire sample for all the four series, are given in the last column of table 1 in appendix 1. The null hypothesis of linearity is accepted for all the spot and futures series in both horizons.