

Ship investment under uncertainty: a real option approach.

Helen B. Bendall
Finance and Economics
University of Technology, Sydney
Lindfield NSW 2070, Australia

Alan F. Stent
Finance and Quantitative Analysis
University of Otago
P.O. 56, Dunedin, New Zealand

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Introduction

Discounted cash flow (DCF) methodology has long been advocated as the appropriate theoretical underpinning for maritime investments (Bendall, 1979; Evans, 1984; Gardner, Goss and Marlow, 1984). However, its limitations are also well-known and can relegate its practical use to a confirmatory role, or management may override its results in favour of an alternative preferred strategy (Teisberg, 1995; Bendall, 2002). Uncertainty, and the difficulty to value the flexibility that management has in adapting plans after a project is underway are significant limiting factors. This paper considers a maritime investment where there is uncertainty and alternative strategies, and uses *real option analysis* to value a flexible strategy that adapts to conditions as uncertainty is resolved.

The proposed investment is a new service based on a new technology: high speed container ships (Bendall and Stent, 1998). The service would be based in South East Asia with Singapore as a hub port and offer fast turn-around times to neighbouring ports, Klang and Penang. The level of service that is offered depends on the number of ships purchased for the project. Being a new technology they must be purchased new and, clearly, in discrete units. There is a one year time to build. The demand for the service is the main uncertainty. Besides day to day variations in demand and economic cycles which affect long run achievable load factors, there are the initial uncertainties associated with the new type of service and technology. While market research might reduce this latter uncertainty, it can only be resolved over time as customers gain experience and adapt or otherwise to the particular characteristics. It could be, for instance, that an eventual high level of demand would favour putting two ships on the service whereas an eventual low level of demand might favour just one. Competition is a consideration. Offering an inadequate level of service to either port leaves an opportunity for a competitor and adversely affects a strategy such as servicing just one port initially with a view to extending the service to the other port if conditions prove favourable. Freight rates also have a significant bearing upon the decision. The paper considers the case where demand for the service as well as freight rates are uncertain. It provides a methodology using Real Option Analysis for adapting the number of ships employed to actual market experience.

Real option analysis, ROA, is a methodology for valuing flexible strategies in an uncertain world (Trigeorgis, 1995). It builds on the traditional DCF technique. It owes its quantitative antecedents to the seminal works of Black and Scholes (1973), Merton (1973) and to the binomial approach of Cox, Ross and Rubinstein (1979) in pricing financial options. Like financial options, real options are the owner's right but not obligation to trade an underlying asset or income stream under predetermined conditions. The simplest financial options are the right to buy (call) or sell (put) the asset at a predetermined price (exercise price) for a predetermined period of time (life of the option). Real option parallels are management's ability to expand or contract a project in light of actual outcomes after it is underway. Like financial options, management's strategies can be far more complex than simple calls and puts, such as abandoning or delaying a project. The methodology, including risk-neutral valuation as a general principle, has evolved to price complex financial options and can be applied to

real options too. There is the matter of selecting an appropriate underlying asset which spans the project's uncertain states. Although projects are not usually traded *per se*, the process of capital budgeting models the market value of their cash flows (Kasanen and Trigeorgis, 1993). Copeland and Antikarov (2000) advance the concept of Marketable Asset Disclaimer to justify the use of the present value of inflexible strategies, estimated by the traditional discounted cash flow techniques, as the appropriate underlying asset. No stronger assumption is required than for the traditional analysis.

This is the method employed to value the strategies in the present paper. A simulation model is first built to model particular fixed services. These are the underlying assets. The model provides estimates of their present values which are used as market prices. It also provides estimates of their volatilities and correlations which are used to model the evolution of their prices in a second step when options are valued. The particular option used in the paper is that to exchange one risky income stream for another. The different income streams are the alternative services which are to be valued as a flexible strategy. The methodology is applied to a hub and spoke system. It is readily adaptable to other situations.

The traditional DCF analysis.

STRATEGIES

Three strategies for a hub and spoke system, using 100 TEU high speed container ships centred at Singapore and serving two ports, are first explored in a traditional DCF analysis. The data are taken from the earlier paper (Bendall and Stent, 2001) where they are explained in more detail. The three strategies are summarised in Table 1. In the first strategy there are two round voyages from Singapore to Klang each week, two round voyages from Singapore to Penang, and three voyages from Singapore to Klang, proceeding to Penang and then returning to Singapore. The total time to complete these seven voyages is 144 hrs. This strategy requires just one ship, with the remaining 24 hours to make up the 168 hours available in the week (seven days at 24 hours per day) being used for scheduled maintenance.

-----Table 1 about here-----

Table 1: Three strategies for a hub and spoke system

	Voyages per week			Total Time	Number of Ships	PV†
	Klang (14hrs)‡	Penang (22hrs)	Klang/Penang (24hrs)			
Strategy 1	2	2	3	144 hrs	1	61.06
Strategy 2	9	7	0	280 hrs	2	121.00
Strategy 3	10	0	0	140 hrs	1	67.56

† USD Million.

‡ Round voyage times including turn-around.

The other two strategies in Table 1 are explained similarly. Strategy 2 would deploy two ships while strategy 3 would deploy one, both strategies leaving a few spare hours per week after allowing 24 hours per ship per week for scheduled maintenance.

The Present Values (PVs) in Table 1 are derived from the following model for annual cash flows:

$$F_t = \sum_{p,q} (X_{ptq} + Y_{ptq})(w_p R_{ptq} - h) - nc \quad (1)$$

where, for a given strategy and year t :

F_t is the calculated free cash flow,

X_{ptq} is the number of TEUs carried *out* from the hub to spoke port p in quarter q ,

Y_{ptq} is the number of TEUs carried *in* from spoke port p to the hub in quarter q ,

R_{ptq} is the base freight rate per TEU carried to or from port p in quarter q ,

w_p is a loading factor for port p ,

h is the variable cost per TEU carried,

n is the number of ships deployed in the strategy, and

c is the operating cost of a ship per annum.

The number of TEUs carried (X_{ptq}, Y_{ptq}) and base freight rates (R_{ptq}) are modelled as stochastic variables with quantities measured in quarters to allow variation within years. The other variables, denoted by lower case letters, are modelled as fixed parameters. A 15-year time horizon is adopted as this is the life of a ship. Stochastic simulation is performed to estimate average annual cash flows for the strategy, $\bar{F}_1, \bar{F}_2, \dots, \bar{F}_{15}$, and the PV calculated:

$$PV = \sum_{t=1}^{15} \bar{F}_t / (1+k)^t \quad (2)$$

where k is the cost of capital.

STOCHASTIC VARIABLES

For a TEU to be carried there must be both a demand and available ship capacity. Thus simulated values for X_{ptq} and Y_{ptq} are calculated as the minimum of simulated quarterly demands and ship capacity. For each port p , the quarterly demand for carrying TEUs out from the hub to the port, and in to the hub from the port are denoted by A_{ptq} and B_{ptq} respectively. These demands, and the stochastic base freight rates R_{ptq} , are modelled as triangular probability distributions with three parameters each, namely the two extremes values (best and worst cases) and the most likely value. The parameters express management judgement and are listed in Table 2. The triangular distribution can take a wide range of shapes and is often used for a subjective distribution (Law and Kelton, 1991).

-----Table 2 about here-----

Table 2: Parameters for triangular distributions

	Worst case		Most likely		Best case	
Demand, TEUs per week†						
Port	From hub	To hub	From hub	To hub	From hub	To hub
Klang	400	500	700	750	1000	1100
Penang	200	300	600	650	900	1000
Base freight rates per TEU‡						
Klang	200 USD		240 USD		260 USD	
Penang	200 USD		260 USD		300 USD	
† TEUs per week are multiplied by 13 to get quarterly parameters.						
‡ The same rate is used for both from the hub and to the hub.						

Two sources of dependencies are modelled. The first recognises dependencies between stochastic variables, such as positively correlated demands between ports. If demand at one port is high due to positive economic conditions, then it is likely to be high at the second port too. Such between-variable correlation is modelled using a normal copula function.¹ The second source of dependency allows correlation over time, such as that attributable to the business cycle: periods of positive or negative economic conditions tend to persist. This type of dependency is modelled by generating first order autocorrelated standard normal variates as inputs to the method of copulas above. The parameters used to induce both type of dependencies are listed in Table 3. The actual values have just a small effect on the average cash flows estimated for Equation (2) and thus on the present values estimated for strategies. However, they do effect their estimated volatilities and correlations and hence option values. A sensitivity analysis is performed below.

-----Table 3 about here-----

Table 3: Correlation coefficients

Demand between and within (to and from hub) ports	0.30
Between freight rates	0.30
Between demands and freight rates	0.40
First order autocorrelation	0.70

At the time of writing economic conditions have been negative with freight rates at the lower end of those in Table 2. The copula function's normal variates for base freight rates in quarter 1, year 1 were therefore initialised to their 5th percentiles for each simulated run.²

Observations on the number of TEUs carried per quarter, X_{ptq} and Y_{ptq} , for Equation (1) were calculated from the simulated demands, A_{ptq} and B_{ptq} , as follows:

$$X_{ptq} = \min(A_{ptq}, 1300nl_p), Y_{ptq} = \min(B_{ptq}, 1300nl_p) \quad (3)$$

where the factor 1300 is the product of the 13 weeks in a quarter and the 100 TEU size of a ship, n is the number of ships deployed in the strategy, and l_p is a loading factor to recognise that even under the best conditions, with stochastic demands full loading

might not be achieved. The factors used in the analysis are $l_p = 96\%$ for voyages each way to single ports, Klang or Penang alone, and the lower value $l_p = 94\%$ for a joint voyage to Klang/Penang.

FIXED PARAMETERS

The values used for the remaining variables, in lower case letters in Equations (1) and (2), are summarised in Table 4. The loading factor is applied to base freight rates and reflects the regular, high-speed characteristics of the service. Penang, being a longer journey, supports the higher premium. Voyage costs were calculated as the sum of port costs of 1000 USD per port visited, and fuel costs based on an average cruising speed of 45 knots, an engine power rating of 58,000 kW and a fuel cost of 160 USD per tonne. All dollar values were modelled in constant real terms over time.

-----Table 4 about here-----

Table 4: Fixed parameters

w_p	Loading factor - to/from Klang	1.40
	- to/from Penang	1.60
h	Variable cost per TEU	50 USD
c	Annual operating cost per ship, made up of:	
	Crew, supplies, per week	8500 USD
	Voyage: per voyage to Klang	20,622 USD†
	per voyage to Penang	36,212 USD†
	per voyage to Klang/Penang	37,645 USD†
	Ship maintenance	400,000 USD
	Ship insurance	1.75% of ship cost
k	Cost of capital	10% PA

† Port costs plus fuel costs, explained further in the text.

THE SIMULATION

A simulation of 15,000 iterations was performed.³ To minimise random variation between strategies the same sample of demands and freight rates stochastic values was used to compute cash flows for each of the three strategies. Cash flows were calculated using Equation (1) for the first five years. To model the decrease in uncertainty expected as the new service becomes familiar, the years 3, 4 and 5 cash flows were averaged for each iteration and the average was used for its following ten years (years 6 though 15). Since PVs are calculated from expected cash flows, estimated by averaging over the iterations, this has little effect on the values estimated for them. However, it does limit the volatility of returns estimated for option values.

NET PRESENT VALUES (NPV)

A period of one year, the time to build a ship, elapses between the decision to commit to a particular strategy and the commencement of its cash flow. NPVs are therefore evaluated in Table 5 by discounting PVs back a further year at the cost of capital, and subtracting the cost of the ship expressed as the delivery price discounted at the risk-free rate, 5%.

-----Table 5 about here-----

Table 5: NPVs (millions USD)

	Service	PV(Table 1)	Ship Cost [†]	NPV [‡]
Strategy 1	Both ports, 1 ship	61.06	60	-1.63
Strategy 2	Both ports, 2 ships	121.00	120	-4.29
Strategy 3	Klang only, 1 ship	67.56	60	4.27

[†]60 million USD per ship due when it is delivered.

[‡] $PV/(1+k) - \text{Ship Cost}/(1+r)$

The third strategy, employing one ship and servicing Klang only, is the only strategy with a positive NPV. This is consistent with the earlier paper where all variables, including demands and freight rates, were modelled as deterministic quantities. Klang is the closer port to Singapore and can fit in more journeys per week. Providing demand is there to service it will be preferred.⁴

NPVs are averaged over a range of possible paths for demands and freight rates whereas in reality just one path will be followed. Should Penang demands or rates turn out to be in their upper ranges then servicing Penang can be highly profitable. This is demonstrated in Table 6 where NPV has been recalculated under more favourable conditions for Penang. In fact, the second strategy happens to be the best in more than 27% of simulated iterations. While interesting and suggestive of further strategies that explore servicing Penang, a static NPV analysis is not the appropriate tool. There is no theoretical justification to apply the same cost of capital that was deemed appropriate for the original levels of uncertainty, 10%, to individual paths or to compute conditional NPVs as in Table 6.

-----Table 6 about here-----

Table 6: NPV millions USD, recalculated for strategy 2 (two ships).

Penang	Original demand	High demand [†]
Original freight rates	-4.29	17.87
High freight rates [†]	14.06	39.71

[†] For "High" scenarios rates and demands were modelled as centered triangular distributions with limits taking the original *most likely* and *best case* values.

Real option analysis (ROA)

A FLEXIBLE STRATEGY

The flexible strategy valued here is to proceed to service both ports Klang and Penang with one ship (original strategy 1), and expand to two ships (original strategy 2) or change to servicing Klang only (original strategy 3) as uncertainty is resolved. A regular service is provided initially with five visits to each port per week. This is sufficient for a high-speed service, though capacity is limited as three of the visits would include both ports on the one round voyage – see Table 1. Experience would be gained with both markets and the ship operator well placed to preempt competition. It is possible that conditions turn out to favour one ship continuing to service both ports and the decision to continue with the first strategy remains. Another option would be to terminate the project completely if conditions warrant this. This is unlikely to add much value in the present circumstances and is not carried through here.

ROA is used to value the flexible strategy outlined. The original strategy 1 had a negative NPV of –\$1.63 million. The options add value which ROA calculates. If the combined value of the NPV and the options exceeds the NPV of the original strategy 3, then it is the better.

AN OPTION ON THE MAXIMUM OF THREE ASSETS

The ship operator proceeds with the original strategy 1 and has the option to change to strategies 2 or 3. The ship operator will exchange strategies to adopt that with the maximum market value. While analytical formulas have been developed for pricing European-style options on the maximum of several non-dividend paying assets (Johnson, 1987), these are not applicable here. The decision to change strategies is not restricted to a fixed point in time, and more significantly, the asset is depreciating. The life of the ship is finite and its value reduces as the project goes forward in time; dividends must be included in the model. Numerical procedures can cope with both these features. An efficient multinomial lattice model developed by Kamrad and Ritchken (1991) is used to calculate the option value.⁵

Adapting the ROA methodology in Copeland and Antikarov (2000) the present values of the three strategies, estimated by simulation above, are market prices. Their movements over time are modelled as multivariate geometric Brownian motion with drift being the risk-free rate of return, and with volatilities and correlations estimated from the first year of the simulation.

ESTIMATION OF VOLATILITIES AND CORRELATIONS

The present value of i th strategy at starting time, P_{i0} , is from Equation (2) above:

$$P_{i0} = \sum_{t=1}^{15} \bar{F}_{it} / (1+k)^t \quad (4)$$

where the subscript i , $i=1,2,3$, has added to the average cash flow in year t , \bar{F}_{it} , to denote the strategy. In particular, for each strategy and year t :

$$\bar{F}_{it} = \sum_{j=1}^N F_{ijt} / N \quad (5)$$

where j runs over the N iterations of the simulation and F_{ijt} is the cash flow calculated for strategy i , iteration j of year t . It will be recalled that while iterations are independent, the same simulated demands and freight rates for each iteration j were used to calculate the cash flows for each strategy. This dependency between strategies is required for the estimation of their covariances below. For strategy i and iteration j the present value of the cash flows at the start of year 1, denoted P_{ij1} , is estimated as:

$$P_{ij1} = \sum_{t=2}^{15} F_{ijt} / (1+k)^t \quad (6)$$

and the return for that strategy and iteration:

$$z_{ij} = \ln((F_{ij1} + P_{ij1}) / P_{i0}) \quad (7)$$

The $3 \times N$ sample of returns (z_{ij}) contains one column for each strategy and one row for each iteration. It is an estimate of the variance-covariance matrix of market returns, and thus volatilities and correlation coefficients. The latter estimates, from the present simulation, are shown in Table 7.

-----Table 7 about here-----

Table 7: Statistics estimated for market returns

	Strategy 1	Strategy 2	Strategy 3
Volatility	0.0976 PA	0.2320 PA	0.2185 PA
<i>Correlation coefficients</i>			
Strategy 1		0.8813	0.7023
Strategy 2			0.8438

OTHER CHARACTERISTICS OF THE OPTION

Other assumptions needed to value the option are summarised in Table 8.

-----Table 8 about here-----

Table 8: Other assumptions

Life time of option	3 years
Decision periods	Every six months
Dividends	Market values reduced by the ratio of average present values each six months.
Ship depreciation	Straight line.
Risk free rate of interest	5% effective PA

The option is valued over three years with the decision whether to exercise modelled at regular six-monthly intervals. Three years is a reasonable timeframe for making a final decision. There is a reason for keeping this period short, namely to limit the potential increases in market values that are modelled. Ship capacity is finite. Although it is feasible that demand can turn out to be very favourable, practical loading factors bound the number of TEUs that can be carried. Equation (3) imposes this constraint in the simulation model, but asset prices in the market are modelled as geometric Brownian motion with no such explicit constraint. The volatilities in Table 7 are assumed to remain constant over time. That the range of asset prices in the market model continue to be consistent with the simulation model, at least over a period of three years, is supported by some simple calculations. The upper ranges of asset prices over this period remain less than present values calculated with demands and freight rates varying in their upper ranges.⁶

Dividends are modelled by scaling down the market values of assets over time in the same ratios that the present values of cash flows diminish in Equation (4). The scaling is done in the multinomial lattice during the numerical calculation of the option value, at points after making exercise decisions.⁷ The modelling of dividends is an important feature of the option valuation because of the finite life of a ship. The longer a decision to change strategies is delayed, the less will be the time for its benefits to accrue. For instance, if expectations continue as modelled for Table 1 then delaying a change to strategy 3 is foregoing value.

The shipper requires one ship to proceed with the first strategy. This ship is retained should the decision be made to change to either of the other two strategies, so its cost is left from the calculation of the option value. However, the cost of a second ship must be deducted on a change to the second strategy. Since the earliest that such a change could occur is after six months plus another one year time to build, and the life of a ship is the same as that of the project, a second ship will have a terminal value. Straight line depreciation is assumed to derive the terminal value. The latter is discounted at the cost of capital, 10%, then deducted from the ship's price to model the cost of the second ship.

THE OPTION VALUE

The formula for valuing the option is at the end of the multinomial tree (time denoted by T) is

$$\max(0, M_{2T} / (1+k) - s_T / (1+r) - M_{1T}, M_{3T} - M_{1T}) \quad (8)$$

where M_{iT} is the generated market value of the original strategy i . Since the value of

the second strategy is subject to the one-year time to build, its value is discounted one year, and the discounted cost of the second ship, as calculated above, deducted. Folding back is performed in the usual manner using risk-neutral probabilities from Kamrad and Ritchken (1991) and the risk-free rate. At a decision step, every 6-months folding back, the option value is the maximum of usual fold-back value (wait) and the early exercise value calculated as in Equation 8.

THE RESULT

The calculated value of the option is \$13.43 million. The additional value it adds to the NPV of strategy 1 is shown in Table 9. Since the NPV of the flexible strategy exceeds that of the original strategy 3, \$4.27 million, it is preferable.

-----Table 9 about here-----

Table 9: NPV of flexible strategy (millions USD)

Service		(Per Table 5)	
		Inflexible	Flexible
Strategy 1	Initially serve both ports with 1 ship	61.06	61.06
Options	Increase service to 2 ships or change to Klang only		13.43
		61.06	74.49
Ship cost		60.00	60.00
NPV†		-1.63	10.58

† $PV/(1+k) - \text{Ship Cost}/(1+r)$

SENSITIVITY ANALYSIS

As noted previously, the option value is dependent on the correlation coefficients in Table 3. A sensitivity analysis was performed. The option value was recalculated after increasing all the coefficients by 0.1, and again after decreasing all the coefficients by 0.1. Table 10 contains the results.

-----Table 10 about here-----

Table 10: Sensitivity analysis (millions USD)

Correlations (Table 3):	Lower	Original	Greater
PV of original strategy 1	61.92	61.06	59.83
Value of options	11.06	13.43	18.61
NPV of flexible strategy	9.20	10.58	14.16

Summary and conclusions.

ROA can be applied to value flexible strategies in conditions of uncertainty.

It has been applied to value a new venture with new technology and new markets in shipping.

A flexible strategy of servicing two markets, Klang and Penang, with one ship, with the option to expand the service by employing two ships, or contracting to Klang alone as uncertainty is resolved, was valued as an option to exchange one risky income stream for the best of two others. This flexible strategy was found to be more valuable than either of three alternative, inflexible strategies.

ROA is a useful tool for decision makers

ENDNOTES

- ¹ Correlated standard normal variates are first generated; the resulting observations on their cumulative densities become the inputs for the Inverse Transformation Method which is then applied in the usual manner to generate observations on the individual marginal distributions. While the originating normal variates have the specified correlation coefficients, when measured for generated marginal distributions their values are perturbed slightly. A reference to copula functions is Dall'Aglio, Kotz, and Salinetti (1991).
- ² Other stochastic variables were initialised randomly. Since all stochastic variables are dependent the initial freight rates are random, too. In almost all runs their initial values were in the left side of their triangular distribution (Table 2). Their average initial value was near their 10th percentiles.
- ³ Thirty independent batches of 500 iterations each, which facilitates the later measurement of sampling error in the option value. The model was written and simulations performed using the Matlab program (The MathWorks, Inc, Natick, MA, USA).
- ⁴ The negative contribution from servicing Penang would manifest directly by including an additional strategy servicing Penang alone. This strategy has not been included as it is an extremely unlikely outcome. Also, going beyond three states (one ship Klang/Penang, two ships Klang/Penang, one ship Klang) essentially prohibits the numerical calculation of option values below with the available desktop computing facilities.
- ⁵ The model was coded in C++ and tested against known analytical results. It was also tested against an alternative numerical procedure developed by Boyle et.al. (1989), which was coded in C++ as well for the purpose. The method is iterative with both the storage required for intermediate calculations and the total number of calculations growing exponentially as the number of states and iterations is increased. This is the limiting factor referred to in Note 4 above. With three states, $n=75$ iterations were performed to obtain the option values in the study. Varying n slightly about this value had little effect on the values obtained. The method also needs a value for a parameter

λ . The value 1.25 was used. The option values obtained were insensitive to this parameter.

⁶ From Table 7, the “up” factors over three years for a binomial model, $e^{\sqrt{3}\sigma}$, are 1.19, 1.53 and 1.48 for the three strategies respectively. Recalculating present values as was done in Table 5 with “high” demands and freight rates for both Klang and Penang, scales them by the greater factors 1.21, 1.67 and 1.73 respectively. While the meaning of such values is imprecise they do support the validity of the market model. Capacity constraints are not a significant issue over a three year period.

⁷ Copeland and Antikarov (2000), Chapters 5 and 9. For the purpose of calculating 6-monthly ratios, annual cash flows were assumed to fall evenly over the year. Further, since the ratios did not differ much over the three strategies the same value, the maximum, was used for each. The ratios are, expressed as percentage decreases in value for the first six months, the second six months, etc, in sequence: 5.2%, 5.4%, 6.0%, 6.4%, 6.3%, 6.7%.

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