US Stock Prices and Macroeconomic Fundamentals

by

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SUMMARY

Using 54 years of US quarterly data and a VAR model underpinned by a theory of the relationship between stock prices and output, this paper considers the deviations of US stock prices from their fundamental value. To do this we derive the fundamental price-output ratio and the fundamental stock price under different assumptions regarding the time-variability of returns, and proceed to compare these to actual data. Despite differences between model results, all imply cyclical deviations of actual values from values warranted by the expected growth in output - these deviations being relatively large since 1996.

JEL Codes: C22, E0, E3, G1, G12

1.   INTRODUCTION

Both macroeconomists and finance specialists are giving increasing attention to the relationship between the stock market and the rest of the economy. There can be little doubt about the growing importance of the stock market from the point of view of the aggregate economy. In the US the ratio of stock-market capitalisation to GDP has approximately tripled in the last 25 years – from less than 30% in the mid-1970s to over 80% in the late 1990s. Not only has the stock market increased relative to the real economy, but it appears that the inter-relationship between them has strengthened. It has always been recognised that the stock market reflects to some extent the goings on in the rest of the economy but recently there has been widespread recognition that the influence is also in the opposite direction – dramatic events in the stock market are likely to have an impact upon the real economy.

There are various ways in which the stock market and the macroeconomy have been related in the literature. Consider first the effect of macroeconomic events on stock prices. One approach has been from an asset-pricing perspective in which the Arbitrage Pricing Theory (APT) was used as a framework to address the question of whether risk associated with particular macro variables is reflected in expected asset returns; the original work in this area is by Chen, *et al.,* (1986) who applied the model to the US.   A closely-related analysis is that of the consumption-CAPM which concentrates on a single macro influence, the growth of aggregate consumption; see, e.g., Breeden (1979) and Grossman and Shiller (1981).

The direction of influence underlying the asset-pricing literature is the traditional one – from the economy to the stock market. A similar focus is found in the literature which explores the response of aggregate stock prices to the (expected) inflation rate; early work carried out in this area is by Bodie (1976), Fama and

Schwert (1977), Jaffe and Mandelker (1976) and Nelson (1976) whereas more recent applications include those by Balduzzi (1995), Graham (1996) and Siklos and Kwok (1999). Similar studies assess the response of the stock market (often, but not always, at an aggregate level) to other macro variables such as those which capture monetary and fiscal policy shocks; e.g. Pearce and Roley (1985), Jain (1988), Aggarwal and Schirm (1992), and Singh (1993).

An alternative to this direction of influence from the economy to the stock market is to analyse the effects of stock prices on the macroeconomy or selected macroeconomic variables. A relationship of this nature that has received considerable attention is between stock prices and investment (in the sense of capital formation). Studies of this type start with Tobin's q-theory of investment (Tobin, 1969) and also include Fischer and Merton (1984), Morck, *et al.,* (1990), Blanchard, *et al.,* (1993) and Chirinko and Schaller (1996). The question in that literature is whether firms, in making investment decisions, should or do pay any heed to stock prices or whether stock prices are simply a veil over the real part of the economy which can be dispensed with when making decisions about real variables such as investment.

More recently, empirical models without any specific theoretical structure have been applied in a more pragmatic fashion to the two-way relationship between stock prices and macroeconomic variables. The vector auto-regressive (VAR) model has been particularly popular in this area given that it can be used as a framework for formal examination of inter-relationships within a given data set without the need to specify a theoretical framework *a priori*. Once estimated, the model can be used to simulate the effects of shocks in a way that is consistent with the data by the use of impulse response functions and forecast-error-variance decomposition.

A relatively early application of the VAR model to the analysis of the relationship between stock prices and the macroeconomy is by Lee (1992) and more recent ones can be found in Cheung and Ng (1998) and Gjerde and Saettem (1999). While the VAR analysis is useful for the simulation of the effects on the endogenous variables of shocks to equation error terms, the non-theoretical nature of such models makes the interpretation of these shocks difficult.

In this paper we extend the standard VAR analysis of the economy-stock-market relationship by imposing restrictions based on a simple but widely-used theory of the relationship between stock prices and output. The procedure can alternatively be seen as an extension to the macroeconomic level of the work on the relationship between stock prices and dividends initiated by Campbell *et al.* (see Campbell and Shiller, 1987, 1988, 1989 and Campbell and Ammer, 1993) and also applied by Lee *et al.* (see Lee, 1995, 1998, Chung and Lee, 1998 and Hess and Lee, 1999).

We begin, in section 2, by setting out the theoretical framework and deriving from it the restrictions to be imposed on our VAR model. We go on to use the restricted VAR to define the fundamental stock price as stock prices that are consistent with the model. We discuss the data used and preliminary statistics in section 3. In section 4 we report the estimation[1] results and we then use the estimated restricted model to compute a series for fundamental stock prices which are the stock prices warranted by the expected growth in output as generated by the restricted model. Conclusions are presented in section 5.

2. THE MODEL

We begin by assuming that in the framework of efficient financial markets and wealth holders who have rational expectations, the real value of the representative

firm, V_t, will be the expected value of its future real profits discounted at the real discount rate. Therefore, V represents the real value of the firm.

Note that in contrast to the dividend-discount model common in undergraduate finance text books, we use profits rather than dividends. This reflects the view that for the economy as a whole (with which we will be concerned), profits contain more information about fundamentals than dividends. For example, the work of Ackert and Smith (1993) suggests that dividends are managed over time and may not capture all current information on macroeconomic fundamentals.

We assume that the representative firm uses two factors to produce output, according to Cobb-Douglas production technology. One factor, called labour, is a variable one which the firm buys in the labour market at a real wage w and the other, which is called capital, is in fixed supply and owned by the firm. We assume that the firm operates in competitive product and labour markets so that both output prices and wages, and therefore the real wage, are taken as given. A profit-maximising firm will set the real wage equal to the marginal product of labour, which is proportional to average product so that the wage bill and therefore real profits are proportional to output:

$$real\ profits_t = Y_t - w_t L_t = Y_t - \textbf{\textit{g}}Y_t = (1\text{-}\textbf{\textit{g}}).Y_t$$

where Y represents real output, w the real wage, L employment and $\textbf{\textit{g}}$ is the exponent of employment in the production function.[2] Hence the real value of the firm can be written:

$$V_t = (1-\textbf{\textit{g}})E_t\sum_{i=1}^{\infty}\left(\frac{1}{\prod\limits_{j=1}^{i}(1+\textbf{\textit{r}}_{t+j}^{*})}\right)Y_{t+i} \qquad (1)$$

where r_t^* is the real (possibly time-varying) rate of return required by shareholders.

Following the representative agent method, we assume equation (1) to hold for the economy as a whole and in our empirical work we use an aggregate stock-price index rather than market capitalisation as the variable on the left-hand side. This involves the assumption of a constant relationship between the real stock-price index, P, and market capitalisation, V. Moreover, the application of (1) to the whole economy involves a further constancy assumption about the relationship between the value of all firms (to which Y applies) and those covered by the index. These two constancy assumptions are therefore maintained in the empirical work that follows and failure of the model could result from the failure of one or both of these assumptions rather than the failure of the rational-valuation model. However, given that we use quarterly data and that our sample period is relatively short, these ratios are unlikely to vary substantially in practice – most of the variation in V reflects fluctuations in P and index coverage change infrequently.

We therefore assume that the stock-price index is proportional to market capitalisation:

$$P_t = \mathbf{b}' V_t$$

Hence, defining $\mathbf{b} = \mathbf{b}'(1-\mathbf{g})$ and $Q_t = \mathbf{b}. Y_t$, equation (1) can be re-written as:

$$P_t = E_t \sum_{i=1}^{\infty} \left(\frac{1}{\prod_{j=1}^{i}(1+ r_{t+j}^*)} \right) Q_{t+i} \qquad (2)$$

This equation is the basis for the computation of our fundamental stock-price index.

In order to use this we need to forecast output and the discount rate for which we use a linearised version of the model obtained using a method similar to that of

Campbell and Shiller (1987).   First, we define the time stream of real realised discount rates, r_t, to satisfy:

$$P_t = \sum_{i=1}^{\infty} \left(\frac{1}{\prod_{j=1}^{i}(1 + r_{t+j})} \right) Q_{t+i} \qquad (3)$$

It follows that

$$(1 + r_{t+1}) = (P_{t+1} + Q_{t+1})/P_t \qquad (4)$$

Taking logs and using lower case letters to represent the logs of their upper-case counterparts, we can write:

$$r_{t+1} = ln(1 + exp(q_{t+1} - p_{t+1})) + p_{t+1} - p_t \qquad (5)$$

where r is defined as $ln(1+r)$ and $(q\text{-}p)$ is the economy-wide log "dividend-price ratio".   The first term in (5) can be linearised using a first-order Taylor's approximation and (5) can then be written as:

$$r_{t+1} = -(p_t - q_t) + m(p_{t+1} - q_{t+1}) + \Delta q_{t+1} + k \qquad (6)$$

where k and m are linearisation constants:

$$m = 1/(1 + exp\overline{(q - p)})$$
$$k = -ln\,m - (1-m)\overline{(q - p)}$$

where $\overline{(q-p)}$ is the sample mean of (q-p) about which the linearisation was taken. Clearly, $0 < \boldsymbol{m} < 1$ and in practice is close to 1.

Empirically it is common that both p and q are I(1) so that the variables are transformed to ensure stationarity. Denote by $\boldsymbol{p}$ the (log) stock-price-output ratio, p-q, and rewrite equation (6) as:

$$\boldsymbol{p}_t = k + \boldsymbol{m}\boldsymbol{p}_{t+1} + \boldsymbol{D}q_{t+1} - r_{t+1} \tag{7}$$

After repeated substitution for $\boldsymbol{p}_{t+1}, \boldsymbol{p}_{t+2},\ldots$ on the right-hand side of (7), we get:

$$\boldsymbol{p}_t = \frac{k(1-\boldsymbol{m}^i)}{(1-\boldsymbol{m})} + \sum_{j=0}^{i-1} \boldsymbol{m}^i \Delta q_{t+j+1} - \sum_{j=0}^{i-1} \boldsymbol{m}^i r_{t+j+1} + \boldsymbol{m}^i \boldsymbol{p}_{t+i} \tag{8}$$

Letting $i \circledR \yen$ and assuming that the limit of the last term is 0, results in the following alternative form of (8):

$$\boldsymbol{p}_t = \frac{k}{(1-\boldsymbol{m})} + \sum_{j=0}^{\infty} \boldsymbol{m}^i \Delta q_{t+j+1} - \sum_{j=0}^{\infty} \boldsymbol{m}^i r_{t+j+1} \tag{9}$$

Hence, if $q_t \sim$ I(1) then $\boldsymbol{D}q_t \sim$ I(0) and, assuming that $r_t \sim$ I(0) (recall that it is the real discount rate), then $\boldsymbol{p}_t$ will be I(0) and we have the model linearised and expressed in terms of stationary variables. Finally, taking conditional expectations of both sides:

$$\boldsymbol{p}_t = \frac{k}{(1-\boldsymbol{m})} + \sum_{j=0}^{\infty} \boldsymbol{m}^i E_t \Delta q_{t+j+1} - \sum_{j=0}^{\infty} \boldsymbol{m}^i E_t r_{t+j+1} \tag{10}$$

where we interpret $E_t r_{t+j+1}$ as shareholders' required return.

In order to use (10) to generate a series for p^*_t, the price-output ratio implied by the model and from it the implied or fundamental stock price, p^*, we need to obtain empirical counterparts to the terms on the right-hand side involving expectations. For the first of these, the expectation of output growth, we follow Campbell and Shiller (1987) and use a VAR model while for the second we make three alternative assumptions and assess the sensitivity of our fundamental stock-price series to these alternatives.

We begin with the simplest case and assume that the rate of return required by shareholders is constant and equal to r. In that case (10) becomes:

$$\boldsymbol{p}_t = \frac{k-r}{(1-\boldsymbol{m})} + \sum_{j=0}^{\infty} \boldsymbol{m}^j E_t \Delta q_{t+j+1} \tag{10$'$}$$

Next, we assume expected output growth to be generated by a VAR in the price-output ratio and output growth. Define the vector $z_t = (\boldsymbol{p}_t, \boldsymbol{D}q_t)'$ so that the VAR may be written as:

$$z_{t+1} = A z_t + \boldsymbol{e}_{t+1} \tag{11}$$

where A is a (2x2) matrix of coefficients and $\boldsymbol{e}$ is a vector of error terms. We assume a lag length of 1 for ease of exposition. If, in the empirical application, a longer lag length is required, the companion form of the system can be used. Using the VAR for forecasting allows us to replace expected output growth by:

$$E_t \boldsymbol{D}q_{t+j+1} = e_2 \boldsymbol{A}^{j+1} z_t$$

Where e_2 is the second unit vector. Hence the value of p_t generated by the combination of the present-value model and the forecasting assumptions (denoted p^*) is:

$$p_t^* = \frac{k-r}{1-m} + \mathbf{e}_2' \mathbf{A}(\mathbf{I} + m\mathbf{A} + m^2\mathbf{A}^2 + ...)\mathbf{z}_t = \frac{k-r}{1-m} + \mathbf{e}_2' \mathbf{A}(\mathbf{I} - m\mathbf{A})^{-1}\mathbf{z}_t \quad (12)$$

which is the equation we use to generate p^* and hence the fundamental stock price series once we have estimated the VAR coefficients and the constants m, k, and r. Given that we wish to generate a series for stock prices which are warranted by (predicted) output growth, we simply generate (the log of) fundamental stock prices as:

$$p_t^* = p_t^* + q_t \quad (13)$$

Equation (12) can also be used to derive tests of the model's ability to explain actual stock prices. This is simply a test of $p_t = p_t^*$ for all t. Since $p_t = e_1' \mathbf{z}_t$ where e_1 is the first unit vector, we can write (12), after transforming the variables to deviations from their means to remove the constant term, as:

$$e_1' \mathbf{z}_t = e_2' \mathbf{A}(\mathbf{I}\text{-}m\mathbf{A})^{-1}z_t \quad (14)$$

For this to hold for all t we require:

$$e_1' = e_2' \mathbf{A}(\mathbf{I}\text{-}m\mathbf{A})^{-1} \quad (15)$$

which constitutes a set of non-linear restrictions on the coefficients of the VAR. They can be tested using the "delta method" (see Campbell, Lo and MacKinlay, 1997, p.540) which is based on writing equation (14) as

$$\boldsymbol{p}_t^* = \boldsymbol{k}\,\boldsymbol{\xi}_t$$

where $\boldsymbol{k}' = (k_1, k_2) = \boldsymbol{e}_2'\boldsymbol{A}(\boldsymbol{I}\text{-}\boldsymbol{mA})^{-1}$ so that a test of $\boldsymbol{p}_t = \boldsymbol{p}_t^*$ is equivalent to a test of $\boldsymbol{k} = \boldsymbol{e}_1$ which can be tested using the Wald statistic:

$$\text{Wald} = (\boldsymbol{k}\text{-}\boldsymbol{e}_1)'(\partial\boldsymbol{k}/\partial\boldsymbol{A})'\boldsymbol{W}(\partial\boldsymbol{k}/\partial\boldsymbol{A})(\boldsymbol{k}\text{-}\boldsymbol{e}_1) \qquad (16)$$

where $\boldsymbol{W}$ is the variance-covariance matrix of the VAR coefficients and the matrices of the partial derivatives of $\boldsymbol{k}$ with respect to the elements of $\boldsymbol{A}$ are evaluated at the estimated value of $\boldsymbol{A}$ and can be computed numerically. Under the hypothesis that the model is true, the Wald statistic is asymptotically χ^2-distributed with 2 degrees of freedom.

An alternative form of the restrictions is obtained by post-multiplying both sides of (**15*****) by $(\boldsymbol{I}\text{-}\boldsymbol{mA})$ to obtain:

$$\boldsymbol{e}_1'(\boldsymbol{I}\text{-}\boldsymbol{mA}) = \boldsymbol{e}_2'\boldsymbol{A} \qquad (17)$$

which is linear in the elements of $\boldsymbol{A}$ and in the present case simply amounts to:

$$\boldsymbol{m}a_{11} + a_{21} = 1$$
$$\boldsymbol{m}a_{12} + a_{22} = 0$$

and can be tested with a standard Wald test given $\boldsymbol{m}$ Gregory and Veall (1985) have shown that the results of the Wald test may not be invariant to such non-linear transformations.

All the above has been derived on the basis of the assumption that the return required by wealth-holders is constant. We now relax this assumption in two alternative models. Both are based on the inter-temporal CAPM model of equilibrium returns due to Merton (1973, 1980), whereby the return required by shareholders is divided into a risk-free component and a risk premium.

In the first of the alternatives we assume that the risk-free rate is time-varying but the risk premium is constant while in the second this assumption is reversed: the risk-free rate is assumed constant while the risk-premium is allowed to vary. Accommodating a time-varying risk-free rate is analogous to allowing changes in the rate of interest that equates individuals inter-temporal demand and supply of funds – the discount rate - while the inclusion of a time-varying risk premium can be explained in terms of 'habit persistence'. Campbell and Cochrane (1999) explain the latter in terms of business cycle risk: as we enter a recession, current consumption will fall toward the level of the habit and individual risk aversion will start to increase affecting future returns, which in turn, will be expected to rise. Conversely, in expansionary times, consumption will rise above the habit, risk aversion will decline and, vis-à-vis the risk premium, expected future returns will fall.

We decompose the real required return into the real risk-free rate, f, and a real risk premium, q

$$E_t r_{t+j} = E_t f_{t+j} + E_t q_{t+j} \qquad (18)$$

In the case where the risk premium is constant we have:

$$E_t r_{t+j} = E_t f_{t+j} + q \qquad (19)$$

and the expression for p_t, equation (10), becomes:

13

$$p_t = \frac{k-q}{(1-m)} + \sum_{j=0}^{\infty} m^j E_t \Delta q_{t+j+1} - \sum_{j=0}^{\infty} m^j E_t f_{t+j+1} \qquad (20)$$

Clearly, once we have removed the constant from this equation by using variables in terms of deviations from their means, this is formally equivalent to the case where the required return is time-varying in an unrestricted way – in the empirical application they would be distinguished only by the data used for r and f.

In this case we use a three-variable VAR in p, Dq, and f and use it to forecast both the last two variables. Re-define z_t as $(p_t, Dq_t, f_t)'$ and use it to forecast both Dq and f:

$$E_t Dq_{t+j+1} = e_2'A^{j+1}z_t$$

and $$E_t f_{t+j+1} = e_3'A^{j+1}z_t$$

so that in this case we compute the fundamental value of the log price-output ratio from:

$$p_t^* = \frac{k-r}{1-m} + (e_2' - e_3')A(I - mA)^{-1}z_t \qquad (21)$$

We again use (13) to compute fundamental stock prices and, as in the previous case, the model's ability to explain actual stock prices can be tested as the hypothesis that $p_t = p_t^*$ for all t which requires that

$$e_1' = (e_2' - e_3')A(I - mA)^{-1} \qquad (22)$$

in non-linear form or

$$e_1'(I-mA) = (e_2' + e_3')A \tag{23}$$

in linear form.   The first can be tested using the delta method as before while the second can be tested using a standard Wald test.

The second alternative to the constant-required-return model is one with a constant risk-free rate but a time varying risk premium.   Here we follow the empirical work of Cuthbertson *et al.* (1997) and model the time varying risk premium as the product of the coefficient of relative risk aversion, a, and the variance of returns, s_t^2. The equation for the price-output ratio then becomes:

$$p_t = \frac{k-f}{(1-m)} + \sum_{j=0}^{\infty} m^j E_t \Delta q_{t+j+1} - a\sum_{j=0}^{\infty} m^j E_t s_{t+j+1}^2 \tag{24}$$

In this case we forecast both real output growth and the return variance using a three variable VAR in $z_t = (p_t,\ \Delta q_t,\ s_t^2)'$ The equation from which we compute the fundamental price-output ratio (and hence the fundamental stock price) is similar to equation (21):

$$p_t^* = \frac{k-f}{1-m} + (e_2' - ae_3')A(I-mA)^{-1}z_t \tag{25}$$

and the linear and non-linear forms of the restrictions that the model explains actual stock prices are similar to those in the previous case.   The non-linear form is:

$$e_1' = (e_2' - ae_3')A(I-mA)^{-1} \tag{26}$$

and the linear form is:

$$e_1'(I-mA) = (e_2' - ae_3')A \tag{27}$$

Restrictions (26) may be tested using the delta method and (27) using the Wald test.

3. DATA AND PRELIMINARY STATISTICS

The US data used in this study are sampled on a quarterly basis over the period March 1947 to June 2001. The financial data were collected from the database of Ibbotson Associates and consist of the S&P500 inflation-adjusted price and accumulation indices and the US 30-day TBILL inflation-adjusted rate of return. US output data were collected from the Federal Reserve Bank of St Louis database, FRED$^{®}$ as constant-dollar GDP.

In the empirical work below, real output data is scaled so that the (log) real stock-price-output ratio, p_t, has the same dimension as the (log) real stock-price-dividend ratio. The scale factor is calculated as: $((1+R) \cdot P_{t-1} - P_t)/Y_{t-1}$ where, R is the real required rate of return of wealth-holders, P_t, is the value of the S&P500 index at time t, and Y_{t-1}, is lagged real output. The value of R is calculated as the sample average quarterly change in the real accumulation stock price index. The (log) of the product of the scale variable and real output gives a time series of real output of the same dimension as those of dividends and is denoted q_t.

The variables included in the VAR models specified above in section 2 were constructed as follows. The S&P500 price index was used to calculate the (log) price-(scaled) output ratio (p), while output growth over the sample period was measured as the change in (log) (scaled) output (Δq_t). The risk-free rate was measured as the continuously compounded quarterly return from the 30-day TBILL rate (f_t). The variance of the quarterly return based on the S&P500 accumulation index was used as a proxy for risk in the time-varying risk model (s^2_t). A time series for the variance

was constructed by taking the square of the deviation of the return from its sample mean. All variables in the VAR are in terms of deviations from their sample mean value thus avoiding the necessity of including a constant term in the VAR equations. The restrictions imposed on the time-varying risk model also require a measurement of the CRRA, a. This we calculate as being *(AVE+(XRVAR/2))/XRVAR)*, where *AVE* is the mean of the returns of the S&P500 accumulation index in excess of the risk-free rate and *XRVAR* is the variance of these excess returns (see, Campbell and Shiller (1989)).

INSERT TABLE 1

Table 1 provides summary statistics on the variables of interest. The mean of the (log) price-output ratio, p_t, implies a quarterly yield of 3.9% per annum,[3] average US real output growth over the period is 3.6% per annum, and the average real return from the TBILL rate is 0.24% per annum. The mean variance of inflation-adjusted market returns averaged 2.4% per annum over the period.

Table 1 also reports the results of various tests. The J-B statistic is relevant to a test of normality and, consistent with previous studies, it convincingly rejects the null of normality for all series under consideration. Autocorrelation statistics and tests for unit roots are also reported. With the exception of the variance series, s^2_t, all the series exhibit some significant autocorrelation. For f_t and Dq_t, the autocorrelation coefficients fade away quite quickly and, with the exception of p_t, the P-P unit roots tests easily reject the null of a non-stationary series.[4] The autocorrelation statistics are particularly high for the price-output ratio, p_t, and appear to be persistent – features which, when considered alongside the reported P-P unit root test, support the hypothesis of a non-stationary series. The non-rejection of the null of a unit root in

the price-output series indicates that the two variables underpinning this ratio are not cointegrated. Figure 1 below plots these two variables.

INSERT FIGURE 1

From Figure 1 we can see that the two series deviate quite considerably over the period, this being particularly noticeable in the middle of the sample. However, stock prices and output also display a tendency to come together again. The correlation between the two series is 0.756 (not reported) but there are large and infrequent swings in the price-output relationship. It is these swings, and the fact that these are large relative to the data sample, that account for the failure to reject the null of non-stationarity for this variable. Similar time series characteristics have been reported using dividend-price ratios. Campbell (1988) and Cuthbertson (1996), for example, use US and UK data extending back to 1871 and 1918 respectively and find that over this longer period similar ratios tend toward stationarity. As the visual evidence and results from very long-term unit root tests tend to support the stationary of the series, we proceed under the assumption that over the long term stock prices and (scaled) output are cointegrated.

4. EMPIRICAL RESULTS

4.1. The Constant Return Present Value Model

INSERT TABLE 2

Table 2 reports the OLS estimated VAR coefficients, residual diagnostics and Wald tests for the constant return specification of the present value model. For both equations in the VAR model, the Q statistic suggests residual autocorrelation is not a problem and that the 1-period lag imposed on the system captures all relevant information on the variables in the VAR. With a t-statistic of 75.69, the own lag on

the price-output ratio would appear to drive this variable and with a coefficient estimate of 0.984, it is statistically insignificantly different from unity. Such persistence is not surprising given the autocorrelations, unit root test and plots of the series reported above. While the reported R^2 is considerably less for the GDP growth equation than that for the price-output ratio equation, for the former both coefficient estimates are significant at conventional levels, with the price-output ratio appearing to Granger cause output growth.

As far as the Wald tests are concerned, we are unable to reject the null that the two series (actual and fundamental price-output ratio) are equal in the linear case but can convincingly reject the non-linear restrictions. Plots of actual and fundamental price-output series (Figure 2) and the actual S&P500 price index and its derived fundamental price (Figure 3) are shown below.

INSERT FIGURES 2 AND 3

Inspection of Figures 2 and 3 shows that the model prediction is far less volatile than the actual data. The two series are however, highly correlated with a correlation coefficient of 0.998 for the actual and fundamental price-output series, and 0.843 for the actual and fundamental price series.

Figure 3 is particularly interesting – it shows the fundamental stock price series generated by the constant-return model. According to this model there have been periods of around 20 years of considerable under/overvaluation of stock prices. Overvaluation is prevalent from the 1950s through to the early 1970s, undervaluation from the mid-late 1970s through to the early 1990s, and followed by overvaluation since the mid-late1990s.

The analysis of fundamental stock prices presented above however, is based on the assumption that the return required by wealth holders is constant. In practice the return may well vary over time and, as discussed earlier, time variation can come from two sources: time-variation in the risk-free rate and time-varying risk premia.

INSERT FIGURE 4

Consider the case of a time-varying risk-free rate first. We can see from Figure 4 that the risk-free rate – as proxied by the real 30-day TBILL rate – follows a not dissimilar pattern to the gap between actual prices and their constant return prediction, particularly in the early and late parts of the sample period, suggesting that the difference between actual and fundamental stock prices presented above may be largely a reflection of the constant required return assumption. The risk-free rate is relatively low up to the late 1970s, rising dramatically in the very early 1980s and falling again in the late 1980s. It is clear that the risk-free rate was not constant over the period and that its behaviour may help explain the long-run difference between the constant return model prediction and actual data. This leads us to a discussion of the evidence from the Time-Varying Risk-Free Rate Present Value Model.

4.2. The Time-Varying Risk-Free Present Value Model

INSERT TABLE 3

The OLS estimates and tests are presented in Table 3 for the time-varying risk-free rate model. Again the price-output ratio is dominated by its own predictive power, while output growth remains influenced by its own past values and those of the price-output ratio. However, the TBILL rate has no predictive power for either the

price-output ratio or output growth, and is itself influenced solely by its own lagged value. There is also considerable autocorrrelation in the residuals of the TBILL equation. The Wald test continues to reject the non-linear form of the restrictions that the actual and predicted series are equal and not the linear restrictions. Figures 5 and 6 plot the actual and fundamental series from this model.

INSERT FIGURES 5 AND 6

The correlations between the actual and fundamental price–output ratio is 0.999 while for actual and for prices it is 0.849. These are very close to those reported for the constant required return model, and again, indicate a close association between movements in the two series. Comparing Figures 5 and 6 with those of Figures 2 and 3 as well as the statistics discussed above, leads us to conclude that a time-varying discount rate has no appreciable role to play in explaining the apparent gap between actual and fundamental stock prices, notwithstanding the suggestive nature of Figure 4. We, therefore, turn to the third formulation of the model which allows the required rate of return to vary by incorporating a variable risk premium while keeping the risk-free rate constant.

4.3. The Time-Varying Risk Present Value Model

INSERT TABLE 4

Table 4 reports the estimates and tests for the time-varying risk model discussed in section 2 of this paper. Clearly, the lagged time-varying risk proxy – the market variance variable – has predictive power for the price-output ratio and, both market variance and the price-output ratio has predictive power for output growth at 5% and 10% respectively. The coefficient estimates suggest that if the previous quarter's price-output ratio increases, then current output growth will also increase.

Similarly, an increase in market variance (market risk) will lead to a decrease in the output growth rate next period. Causation also runs in the opposite direction with lagged output growth having a dominant, and significantly negative, relationship with next periods risk. Overall, this model implies dual, negative causality between output growth and market risk, one-way, positive causality leading from the price-output ratio to output growth, and one-way, positive causality leading from market risk to the price-output ratio. According to the Q statistic, the lag-length is adequately specified to capture autocorrelation up to the fourth order. The CRRA, computed as described in section 3 above and used in equations 26 and 27, is 3.851, and lies well within the empirically acceptable bounds of between 1 and 10. While the R^2s for the price-output ratio equation and the output growth equation are the same and larger respectively, than those reported for the same equations in Table 3, the R^2 for the variance equation is lower than that reported for f_t in Table 3. This feature however, is due to the low significance of the own lag on the variance variable in Table 4 compared to significance of the own lag on the risk-free variable of Table 3. The possible superiority of the time-varying risk model however is reflected in the Wald statistics where, in both the linear and non-linear case we now cannot reject the null hypothesis that the actual and predicted series are statistically equal (for the non-linear case this is at the 1% significance level).

Figures 7 and 8 display plots of the actual and fundamental price-output ratio series and the actual and fundamental price series.

INSERT FIGURES 7 AND 8

Figures 7 and 8 display a closer association between the actual and predicted series than those derived from the constant and time-varying risk-free rate models. The correlation between the price-output ratio series is 0.996, and is 0.903 for the two

price series: the time-varying risk model produces fundamental stock prices which are considerably closer to those implied by the previous two models considered. While, the results still show periods of persistent under- and over-valuation lasting around 20 years, over the sample period they are insignificantly different from zero. Notably however, the most recent period has seen the largest overvaluation of the sample. Investors appear to have become particularly bullish in late 1996 and by the end of the sample period (June 2001) the market was overvalued with respect to the benchmark model at around 26.9%.[5], which, as the graph depicts, is high relative to previous cycles. This begs the question of whether this gap will continue until 2016?

5. CONCLUSION

Using a VAR model underpinned by a theoretical framework describing the relationship between US stock prices and the macroeconomy, this paper analyses the extent to which US stock prices deviate from economy-wide fundamentals. Focusing on real output and using a present value approach, we derive the fundamental price-output ratio (the economy-wide price-dividend ratio) and the fundamental stock price, and proceed to compare these to actual data.

We consider three cases. We begin by assuming that the return required by wealth-holders is constant and then relax this assumption by first, allowing the risk-free rate to vary over time, and second, the risk premium to be time-varying, with the time-varying risk model producing a series for fundamental prices which is closest to actual. Despite the differences between model results, all imply that since 1996, the stock market has been relatively overvalued compared to its value warranted by the expected growth in output.

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[1] All the estimation for this paper is carried out using RATS Version 5. All programs and derived data are available from the authors upon request.

[2] Alternatives to the above structure are possible without losing the central property of the model that the wage bill is proportional to the value of output. Thus, as an alternative to perfect competition, if we have monopolistic competition of the type that is popular in the macro labour literature (see, e.g., Layard, Nickell and Jackman, 1991) and a homogeneous single-factor production function, the proportionality result continues to hold although the constant of proportionality involves the elasticity of demand as well as the production parameter. On the labour-market front it is possible to introduce efficiency wages and the proportionality relationship continues to hold as long as the production function is homogeneous in labour in efficiency terms. Similar results can be obtained from a union-bargaining model. The convenient property that factor payments are proportional to output is not restricted to the Cobb-Douglas production function. It holds for all homothetic function such as the CES and the generalised CES production functions.

[3] Taking $e^{-4.640} = 0.97\%$ per quarter $= 3.9\%$ per year, which is sensible

[4] Phillips-Perron unit root tests were conducted over varying lag lengths and with and without a time trend for a range of Phillip-Perron tests.

[5] The lack of data observations post 1996 prevents us from providing a credible analysis of this most recent period of history in isolation.

Table 1. – Summary Statistics on the VAR Variables*

Variable	Mean	Variance	J-B	AR(1)	AR(2)	AR(3)	AR(4)	P-P
p_t	4.640	0.153	13.778	0.980	0.958	0.939	0.919	-1.396
	(0.391)		(0.001)	(0.068)	(0.116)	(0.148)	(0.173)	
Δq_t	0.009	0.0001	12.428	0.336	0.184	-0.001	-0.120	-10.396
	(0.010)		(0.002)	(0.068)	(0.076)	(0.077)	(0.077)	
f_t	0.0006	4.345-e06	228.176	0.501	0.400	0.485	0.417	-8.902
	(0.002)		(0.000)	(0.068)	(0.083)	(0.091)	(0.103)	
σ^2_t	0.006	0.0001	1587.167	0.012	0.110	0.090	0.023	-14.599
	(0.012)		(0.000)	(0.068)	(0.068)	(0.069)	(0.070)	

* p is the price-output ratio; Δq_t, is output growth; f_t, is the risk-free rate of return; σ^2_t, is the variance of the market return. The statistics above are computed on series with means. J-B is the Jarque-Bera test for normality. The AC(.) are autocorrelation coefficients at lags 1-4. Figures in parenthesis below mean values and autocorrelation coefficients are standard errors and under the J-B are marginal significance levels. P-P is the Phillips-Perron t-test for a unit root in the series – it incorporates 4 lags and an intercept. The critical values for the P-P test are –2.58 (10%), -2.89 (5%), and –3.51 (1%).
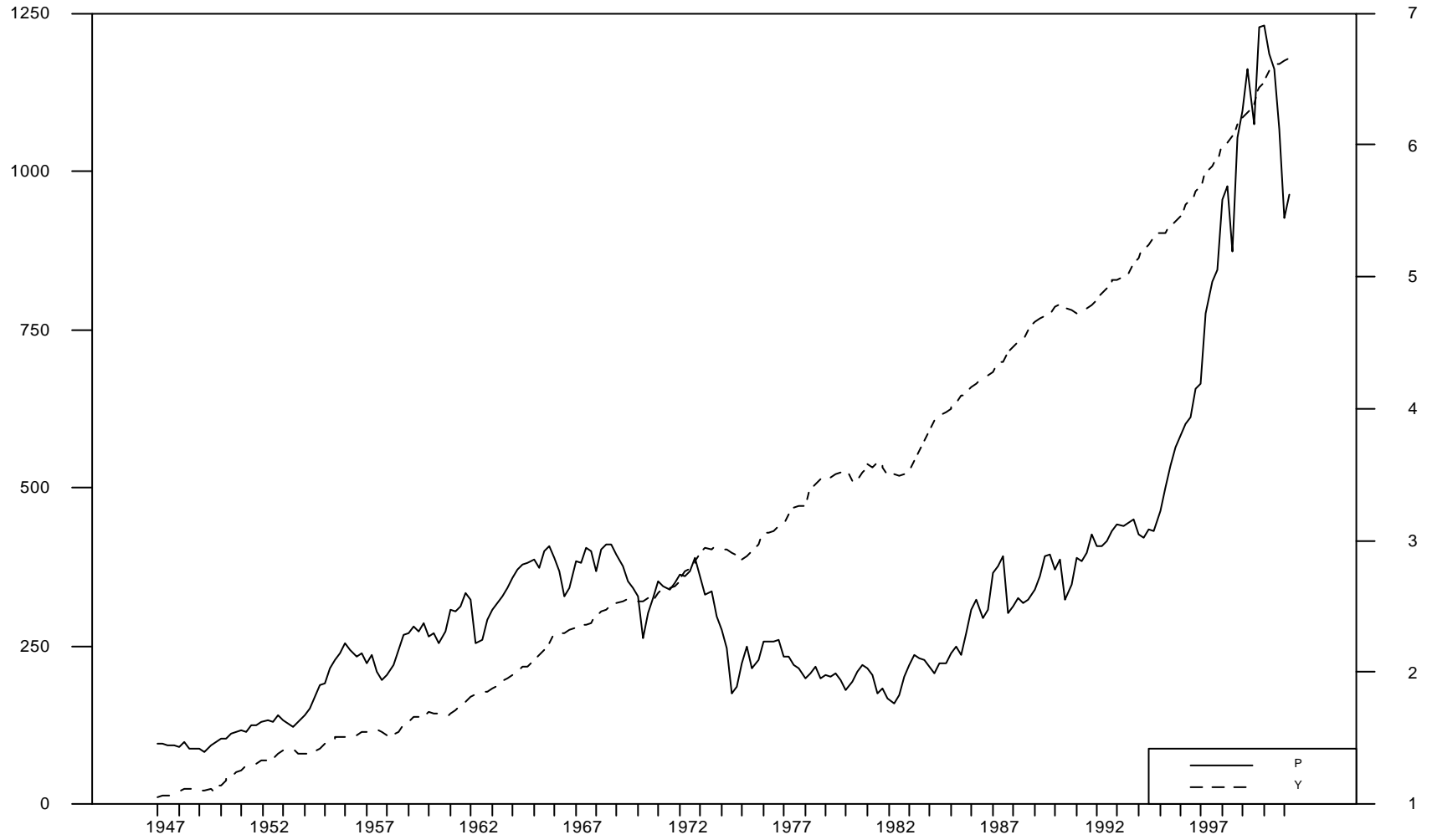
STOCK PRICE AND SCALED OUTPUT

Table 2. - VAR Statistics and Tests for the OLS estimation of the Constant Return Present Value Model *

$$z_{t+1} = A z_t + \boldsymbol{\varepsilon}_{t+1}$$

z_{t+1}	a_{i1}	a_{i2}	R^2	Q(4)
$\boldsymbol{p}_t$	0.984	-0.423	0.961	2.983
	(0.013)	(0.517)		(0.560)
$\boldsymbol{\Delta q}_t$	0.003	0.317	0.106	4.369
	(0.002)	(0.065)		(0.358)

Wald Tests

Linear Wald Restrictions	3.019
$\boldsymbol{m} a_{11} + a_{21} = 1$	(0.221)
$\boldsymbol{m} a_{12} + a_{22} = 0$	
Non-Linear Wald Restrictions	55.148
$\boldsymbol{p} = \boldsymbol{p}^*$	(0.000)

*$\boldsymbol{p}$ is the price-output ratio and $\boldsymbol{\Delta q}_t$, is output growth. The model was estimated using OLS and the figures below the estimated coefficients are the standard errors. a_{in} are the elements of the coefficient matrix A. The Q(4) statistic is the Ljung-Box test statistic for joint significance of the first four autocorrelation coefficients. Figures in parentheses below the Q(4) statistic are marginal significance levels. The Wald test statistics correspond to tests of restrictions in equations (17) and (15) respectively and $\boldsymbol{m}$ is a linearisation constant which takes a value of 0.99093. Under the hypothesis that the model is true, the Wald statistics are asymptotically χ^2-distributed with 2 degrees of freedom; marginal significance levels appear in parentheses below the reported Wald statistics.

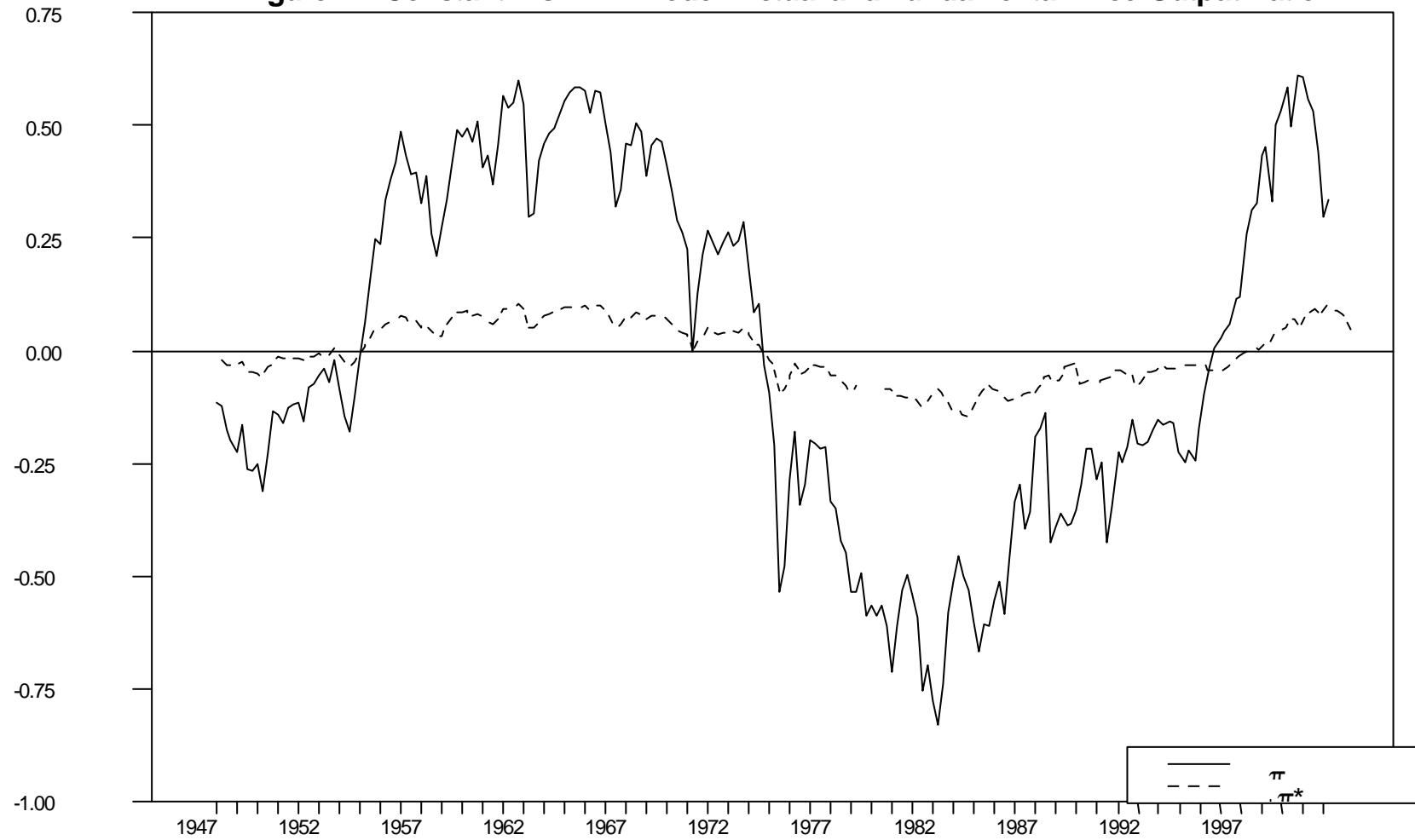Figure 2. - Constant Risk P.V. Model: Actual and Fundamental Price-Output Ratio

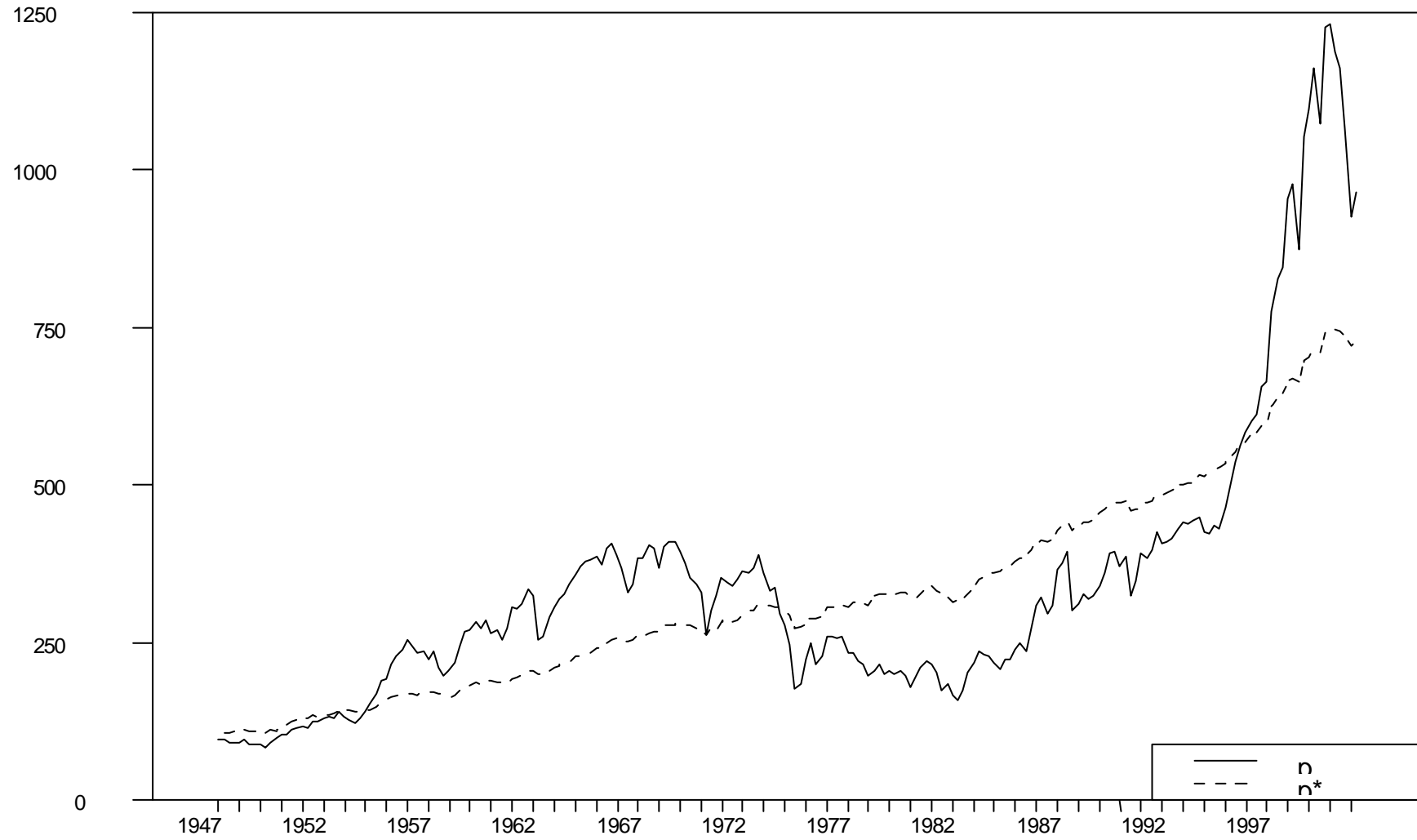**Figure 3. - Constant Risk P.V. Model: Actual and Fundamental Price**

Figure 4. – Constant P.V. Model: Actual Price less Fundamental Price and the TBILL Rate
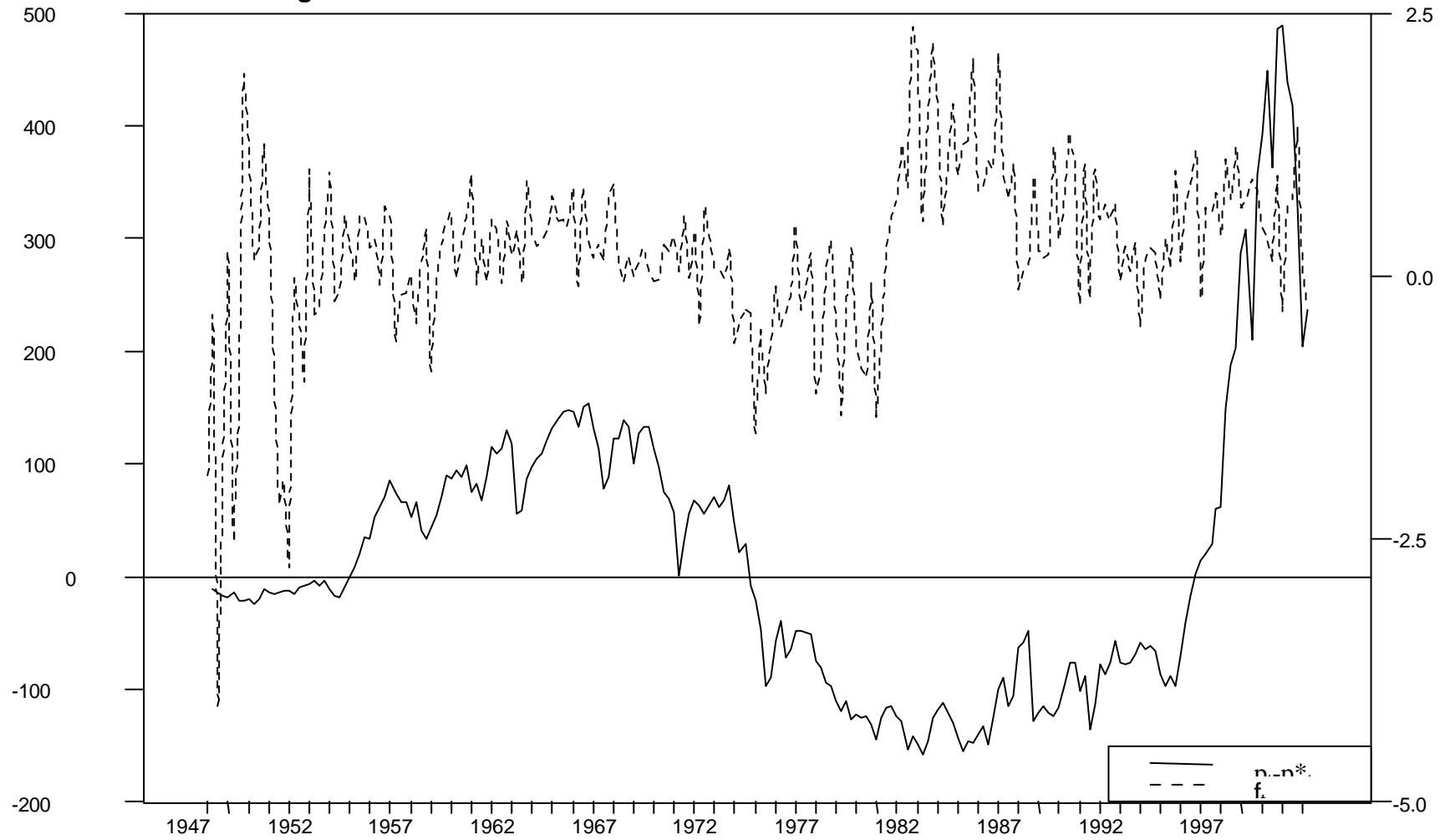
Table 3. - VAR Statistics and Tests for the OLS Estimation of the Time-Varying Risk-Free Present Value Model *

$$z_{t+1} = Az_t + e_{t+1}$$

z_{t+1}	a_{i1}	a_{i2}	a_{i3}	R^2	Q(4)
p_t	0.984	-0.400	3.344	0.961	3.059
	(0.013)	(0.515)	(2.535)		(0.548)
Dq_t	0.003	0.316	-0.131	0.080	4.193
	(0.002)	(0.065)	(0.317)		(0.381)
f_t	-0.0002	-0.015	0.497	0.219	32.198
	(0.0003)	(0.012)	(0.059)		(0.000)

Wald Tests

Linear Wald Restrictions	4.095
$m_{11}+a_{21}-a_{31}=1$	(0.251)
$m_{12}+a_{22}-a_{32}=0$	
$m_{13}+a_{23}-a_{33}=0$	
Non-Linear Wald Restrictions	68.852
$p = p^*$	(0.000)

*p is the price-output ratio and Dq_t, is output growth. The model was estimated using OLS and the figures below the estimated coefficients are the standard errors. a_{in} are the elements of the coefficient matrix A. The Q(4) statistic is the Ljung-Box test statistic for joint significance of the first four autocorrelation coefficients. Figures in parentheses below the Q(4) statistic are marginal significance levels. The Wald test statistics correspond to tests of restrictions in equations (17) and (15) respectively and m is a linearisation constant which takes a value of 0.99093. Under the hypothesis that the model is true, the Wald statistics are asymptotically c^2-distributed with 3 degrees of freedom; marginal significance levels appear in parentheses below the reported Wald statistics.

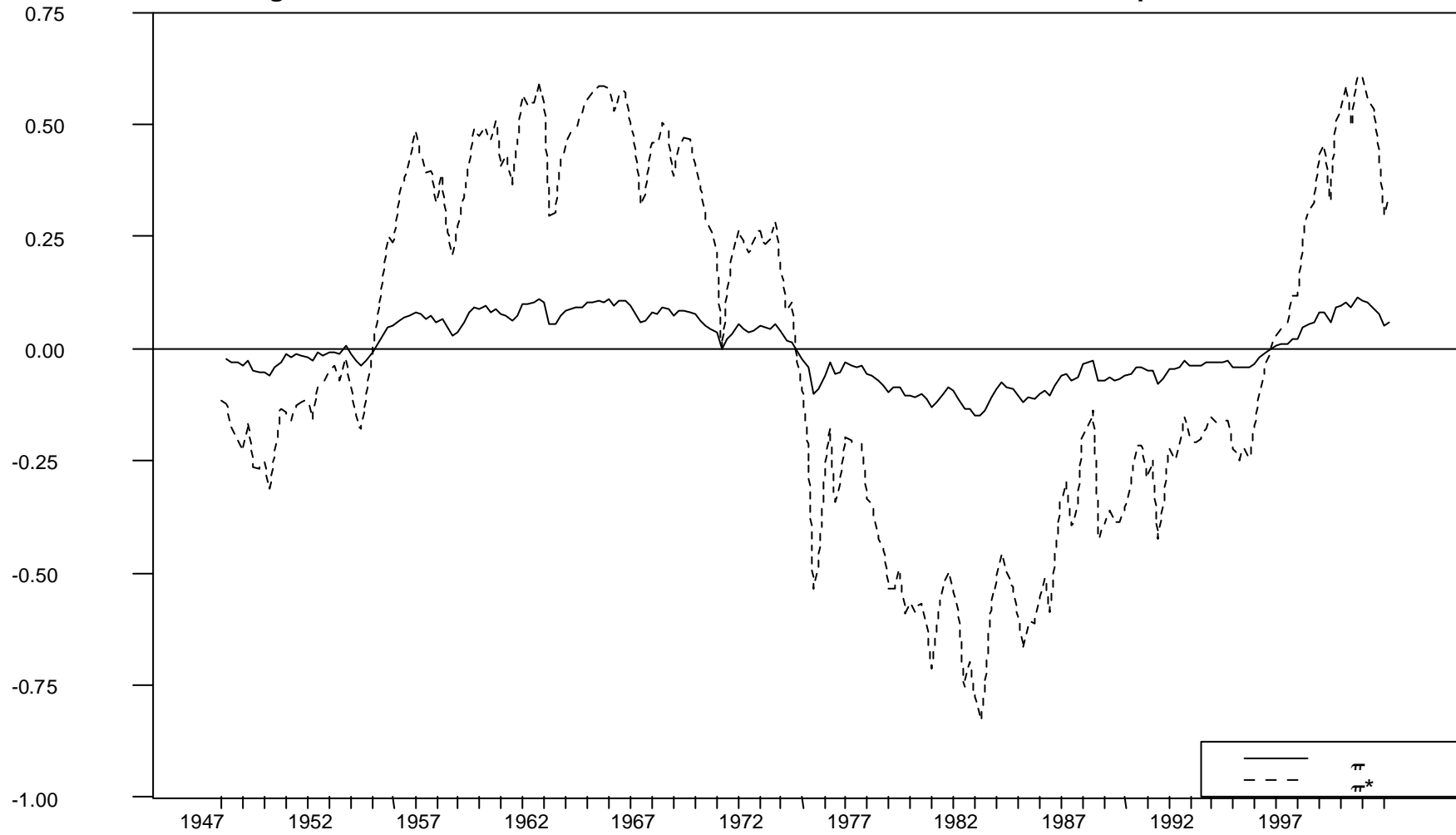**Figure 5. - TV Safe Rate P.V. Model: Actual and Fundamental Price-Output Ratio**

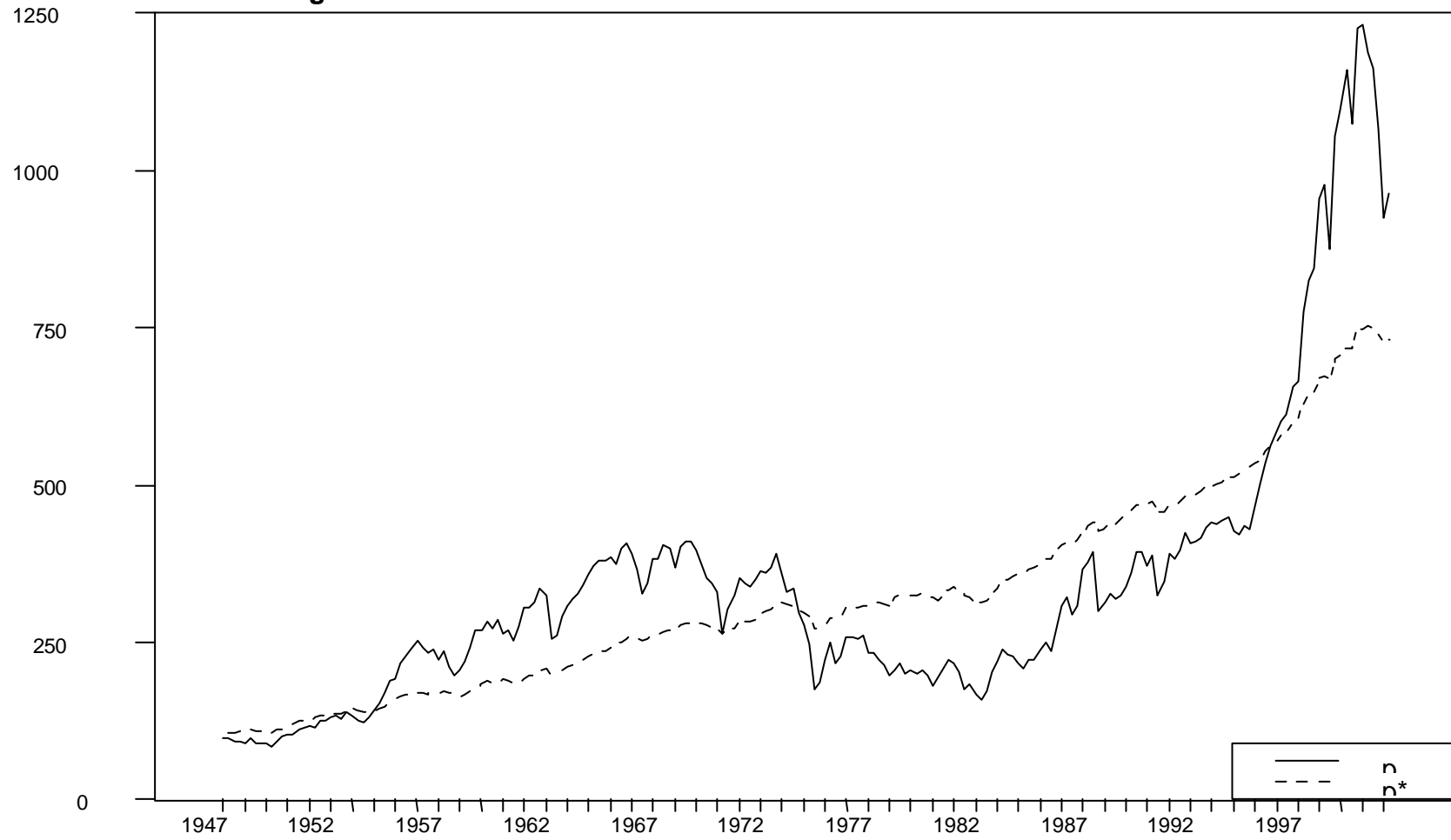**Figure 6. - TV Safe Rate P.V. Model: Actual and Fundamental Price**

Table 4. - VAR Statistics and Tests for the OLS Estimation of the Time-Varying Risk Present Value Model *

$$z_{t+1} = Az_t + \boldsymbol{e}_{t+1}$$

z_{t+1}	a_{i1}	a_{i2}	a_{i3}	R^2	Q(4)	CRRA $\boldsymbol{a}$
$\boldsymbol{p}_t$	0.986	-0.343	0.849	0.961	4.802	3.851
	(0.134)	(0.514)	(0.427)		(0.308)	
$\boldsymbol{D}_{q t}$	0.003	0.308	-0.088	0.091	4.734	
	(0.0017)	(0.064)	(0.053)		(0.316)	
$\boldsymbol{s}^2_t$	-0.0003	-0.163	-0.002	-0.034	2.215	
	(0.002)	(0.082)	(0.069)		(0.696)	

Wald Tests

Linear Wald Restrictions	3.295
$\boldsymbol{m}_{11}+a_{21}-\boldsymbol{a}a_{31}=1$	(0.348)
$\boldsymbol{m}_{12}+a_{22}-\boldsymbol{a}a_{32}=0$	
$\boldsymbol{m}_{13}+a_{23}-\boldsymbol{a}a_{33}=0$	
Non-Linear Wald Restrictions	8.897
$\boldsymbol{p}=\boldsymbol{p}^*$	(0.031)

*$\boldsymbol{p}$ is the price-output ratio and $\boldsymbol{D}_{q t}$, is output growth. The model was estimated using OLS and the figures below the estimated coefficients are the standard errors. a_{in} are the elements of the coefficient matrix $\boldsymbol{A}$. The Q(4) statistic is the Ljung-Box test statistic for joint significance of the first four autocorrelation coefficients. Figures in parentheses below the Q(4) statistic are marginal significance levels. The Wald test statistics correspond to tests of restrictions in equations (17) and (15) respectively and $\boldsymbol{m}$ is a linearisation constant which takes a value of 0.99093. Under the hypothesis that the model is true, the Wald statistics are asymptotically $\boldsymbol{c}^2$-distributed with 3 degrees of freedom; marginal significance levels appear in parentheses below the reported Wald statistics. The CRRA, $\boldsymbol{a}$ is the coefficient of relative risk aversion.

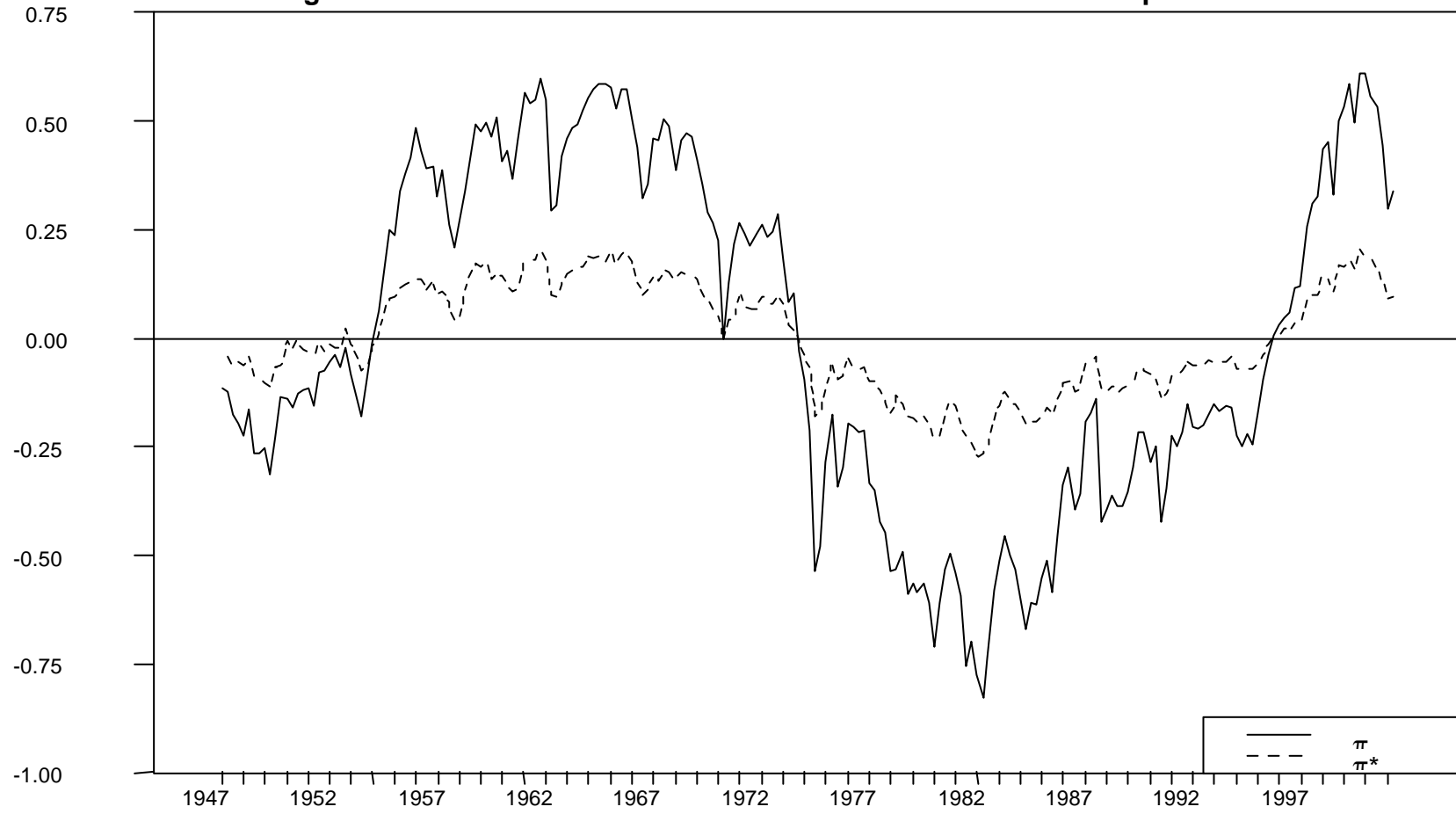**Figure 7.- TV Risk P.V. Model: Actual and Fundamental Price-Output Ratio**

Figure 8.-TV Risk P.V. Model: Actual and Fundamental Price