ISSN 1178-2293 (Online)

University of Otago
Economics Discussion Papers
No. 1112

November 2011

Learning and Collusion in New Markets with Uncertain Entry Costs*

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*For helpful comments we are grateful to seminar participants at the Ecole Polytechnique, Massey University, Erasmus University Rotterdam, Queen Mary University of London, University of Auckland, University of Macerata and participants in the 29th Australasian Economic Theory Workshop (AETW), the 2nd Workshop 'Industrial Organization: Theory, Empirics and Experiments', the 11th Society for the Advancement of Economic Theory (SAET) Conference, the 40th Australian Conference of Economists, the 4th Transatlantic Theory Workshop at the Paris School of Economics, and the 1st Microeconomics Workshop at Victoria University of Wellington. We want to thank particularly Rossella Argenziano, Federico Boffa, Nisvan Erkal, Emeric Henry, Leo Fulvio Minervini, Hodaka Morita, Jose Rodrigues-Neto, and Francisco Ruiz Aliseda for their help and support. This work was supported by the Royal Society of New Zealand (Marsden Grant MAU0804 and UOO0821 to Fabrizi and Lippert, respectively). Correspondence: francis.bloch@polytechnique.edu, s.fabrizi@massey.ac.nz, and steffen.lippert@otago.ac.nz.

Abstract

This paper analyses an entry timing game with uncertain entry costs. Two firms receive costless signals about the cost of a new project and decide when to invest. We characterize the equilibrium of the investment timing game with private and public signals. We show that competition leads the two firms to invest too early and analyse collusion schemes whereby one firm prevents the other firm from entering the market. We show that, in the efficient collusion scheme, the active firm must transfer a large part of the surplus to the inactive firm in order to limit pre-emption.

JEL Classification Codes: C63, C71, C72, D81, D82, D83, F21, O32

Keywords: Learning; Pre-emption; Innovation; New Markets; Project Selection; Entry Costs; Collusion; Private Information; Market Uncertainty

1 Introduction

This paper studies investment decisions by two firms which compete to enter new markets or develop new products. Firms are uncertain about the cost of the investment. Before investing, they gradually acquire information about the entry cost on the market. After investment, they have the opportunity to collude by compensating one of the two firms for not entering the market.

The analysis of this paper can be applied to two different settings. When firms contemplate entering a new geographical market, they conduct market research to learn about the demand, and investigate various production and distribution alternatives. The decision to enter or not a foreign market is only taken after months of investigation. Furthermore, it is not uncommon that two rival firm join forces by creating a joint subsidiary in order to access foreign markets. When firms are engaged in a race to obtain an innovation, they often starts building small prototypes, or running small scale experiments before investing in a large scale research project. Once a firm has started developing a new product, the race will often be concluded with a merger, with one of the two firms acquiring the research output of the other before the new product reaches the market.

In this paper, our objective is to better understand the interplay between learning and preemption in entry timing games, and to study collusive mechanisms between two firms engaged in the development of new products or the access to new markets. Do firms invest too early or too late? How does the fact that signals are public or private affect the entry timing decisions? When do simple compensating payment allow firms to achieve the collusive outcome? Which share of the surplus should accrue to the two firms in the collusive transfer scheme? When is the optimal time to implement cooperation?

In order to answer these questions, we construct a model where firms initially ignore the fixed cost of entry into a new market. They gradually acquire signals about their entry cost through research and experimentation. (We consider here the case of *private values*, where the entry costs are independently distributed across firms.) Upon observing their signals, and forging beliefs about the signals received by their competitors, firms decide when to enter. If both firms enter, they collect duopoly profits on the market; if only one firm enters, it will receive monopoly profits. We suppose that firms make positive profits as duopolists only when their entry cost is low, and make positive expected profits as monopolists when they ignore their costs. We compute the cooperative outcome, where the two firms choose entry timing in order to maximize joint profits, and the outcome of the non-cooperative game of entry timing played by the two firms both when the signals are public and private. We analyze how the two firms can implement a collusive transfer scheme by which the firm which has entered the market compensates the other firm for not entering.

Our first result is that competition leads firms to invest excessively early, and that excess momentum is higher when signals are private than public. We consider situations of *project selection*, where it is optimal for the two firms to wait until they learn that one of the two projects is profitable before entering the market. For some range of parameters, competition leads the two firms to invest immediately in order to preempt entry by the rival firm. Furthermore, there exists a parameter region for which firms choose to wait when signals are public, but preempt at a finite time when signals are private. With private signals, firms ignore whether their competitor has abandoned the race or not. As time passes, firms become more and more convinced that the other firm has dropped from the race (no news is good news). Hence, at some finite date, firms become sufficiently confident that the other firm will not enter the market, and choose to enter before they learn their entry cost. This equilibrium is reminiscent of the preemption equilibrium in the innovation race studied by Fudenberg and Tirole (1985) and Grossman and Shapiro (1987). However, in our model, preemption occurs due to the endogenous dynamics of beliefs, whereas in their model, preemption results from the exogenous dynamics of the innovation cost.

Our second set of results deals with compensating payment schemes implemented to achieve collusion. Competing firms face three sources of inefficiency: (i) market competition, (ii) duplication of entry costs and (iii) excess momentum in market entry. In order to reach the collusive outcome, they may implement compensating payments paid by one firm to the other so that it stays out of the market. We show that collusion is possible only when a firm enters the market sufficiently early. After a finite date, collusion becomes impossible as the active firm becomes convinced that its rival has dropped from the market and is unwilling to compensate it at a level which would prevent entry. We also show that in order to achieve efficient entry timing decisions, the monopoly surplus should be shared between the active and inactive firms in an equitable fashion. The share of the active firm should be large enough to give it an incentive to invest immediately after it learns its cost. The share of the inactive firm should be large enough so that firms have no incentive to enter early in order to preempt their rival.

Our analysis thus sheds light on situations of project selection, where two independent firms run parallel research programs and a third party can enforce a cooperative scheme to prevent inefficiencies. The third party can for instance be a venture capitalist or a granting agency running competing research project, the editor of an academic journal or organizer of a scientific conference who discovers that two teams of scientists are working on the same problem. Our analysis suggests that selection should neither occur too early (before the profitabilities of the projects are known), nor too late (when the firms have become very optimistic about their prospects given that the other firm has not entered). It also shows that the share of the surplus transferred to the firm which is not selected should neither be too large (in which case the selected firm may have an incentive to delay the research project) nor too small (the higher the payoff transferred to the firm which is not selected, the smaller the gap between the payoffs of the leading and trailing firms, which reduces inefficiencies due to excess momentum.)

Our analysis is rooted in the literature on patent races in continuous time pioneered by Reinganum (1982) and Harris and Vickers (1985). The first extensions of patent races allowing for symmetric uncertainty are due to Spatt and Sterbenz (1985), Harris and Vickers (1987) and Choi (1991). Models of learning in continuous time with public information have been studied by Keller and Rady (1999) and Keller, Cripps and Rady (2005) in the more complex environment of bandit problems. Rosenberg, Solan and Vieille (2007) and Murto and Valimaki (2010) analyze general stopping games with common values where players' payoffs does not depend on the actions of other players. By contrast, we consider the simpler setting of private values but consider strategic interaction between the players after entry.)

The model of preemption we consider is formally identical to Fudenberg and Tirole (1985)'s models of technology adoption with preemption. Innovation timing games which can result either in preemption or in waiting games have been studied by Katz and Shapiro (1987). Hoppe and Lehmann-Grube (2005) propose a general method for analyzing innovation timing games. Fudenberg and Tirole (1985)'s model has been extended by Weeds (2002) and Mason and Weeds (2010) to allow for stochastic values of the technology. However, none of these models allows for private information. The closest papers to ours are the recent papers by Hopenhayn and Squintani (2010) on preemption games with private information and Moscarini and Squintani (2010) on patent races with private information. Moscarini and Squintani (2010) analyze a common values problem, where agents learn about the common arrival rate of the innovation, whereas we analyze a private values problem in which agents learn their individual market entry cost. Accordingly, our model displays very different results. Even though Hopenhayn and Squintani (2011)'s model is more general than ours in many aspects, it only covers situations where agents receive positive information over time. In our model, research teams may either receive positive or negative signals about the profitability of the research project, which impacts the results and so that the insights of Hopenhayn and Squintani (2011) do not directly apply. Cooperation among research teams with private information has been studied in a mechanism design context by Gandal and Scotchmer (1993). Goldfain and Kovac (2005) analyze the optimal design of contracts by a venture capitalist running two parallel projects. Gordon (2011) and Akcigit and Liu (2011) study patent races with private signals, focussing on the incentives to disclose information to competitors.

The rest of the paper is organized as follows. We introduce the model and describe the collusive benchmark in Section 2. Section 3 contains our core analysis of entry timing games with public and private signals. Section 4 discusses compensating payments and market monopolization. We analyze consumer surplus and extend the model to allow for stochastic profits in Section 5. Conclusions and directions for future research are given in Section 6. All proofs and derivations are collected in the Appendix.

2 The Model

2.1 Firms, new markets and entry costs

We consider two firms which may invest in order to enter a new market, launch a new product or exploit a new process. The monopoly and duopoly profits obtained after investment are given by π_m and π_d with $\pi_m > 2\pi_d$. The entry cost to the new market is uncertain, and can either take a high or a low value, $\theta_i \in \{\overline{\theta}, \underline{\theta}\}$. We consider a model of private values, where costs are independently distributed across the two firms. For simplicity, we suppose that the two values of the cost are equiprobable and denote the expected value of the entry cost by $\widetilde{\theta} = \frac{\underline{\theta}+\overline{\theta}}{2}$.

During the experimentation phase, each firm receives a perfectly informative signal about its entry cost according to a Poisson process with intensity μ. Hence, the probability that a firm receives a signal during the interval $[0, t]$ is $1 - e^{-\mu t}$. With probability $\frac{1}{2}$ the firm learns that it is of high type, and with probability $\frac{1}{2}$, it learns that it is of low type. We assume that the Poisson processes generating signals to the two firms are independent. Given our assumption of private values, independence furthermore means that the signals received by the two firms are independent. We let r denote the common discount rate of the two firms.

We assume that high cost firms never have an incentive to invest, even if they receive monopoly profit. Low cost firms always have an incentive to invest even if they receive duopoly profit. When entry cost remains unknown, firms have an incentive to invest as monopolists but not as duopolists. Formally,

Assumption 1

$$\underline{\theta} \leq \pi_d \leq \widetilde{\theta} \leq \pi_m \leq \overline{\theta}. \tag{1}$$

2.2 Entry timing and strategies

At any date $t = 0, \Delta, 2\Delta, ...$, both firms choose whether to enter the market. (We will analyze situations where the time grid becomes infinitely fine, and Δ converges to zero.) If firm i enters the market, it pays the fixed cost θ_i and starts collecting monopoly (or duopoly) profits immediately. Investments to enter the market are immediately observed by the other firm.

Given Assumption 1, it is a dominant strategy for a high cost firm not to invest. Hence, the only relevant choices are choices made by a firm which learns that its cost is low, or by a firm which still ignores its entry cost. A strategy specifies, after every possible history, a pair of probabilities with which the firm invests when it learns that its cost is low and when it ignores its cost. We consider *perfect equilibrium strategies* which maximize the firm's expected discounted payoff after every possible history.

2.3 Collusive benchmark

We compute the expected profit of a single firm experimenting on the market as:

$$V_O = \frac{\mu}{2(\mu + r)} \left(\pi_m - \underline{\theta}\right).$$

In order to compute the collusive benchmark, we note that the firms have three options. First, they can choose to enter immediately, and earn an expected profit of $\pi_m - \widetilde{\theta}$. Second, they can wait to draw one signal, invest immediately in the project if the cost is low, and invest immediately in the other project (without knowing its cost) if the cost is high, resulting in an expected profit:

$$V_{C1} = \frac{2\mu}{2\mu + r} \left(\pi_m - \frac{\underline{\theta} + \widetilde{\theta}}{2}\right).$$

parameter region	optimal cooperative choice
$\widetilde{\theta} < \pi_m - 2V_O$	invest immediately
$\pi_m - 2V_O \leq \widetilde{\theta} < \pi_m - V_O$	invest immediately after receiving the first signal
$\pi_m - V_O \leq \widetilde{\theta}$	invest only after receiving a low-cost signal

Table 1: Optimal collusive choice

Third, they can wait until they learn whether one of the firm has a low cost and only invest in a low cost firm to obtain an expected profit:

$$V_{C2} = \frac{\mu}{2\mu + r} (\pi_m - \underline{\theta}) \left(1 + \frac{\mu}{2(\mu + r)}\right).$$

Table 1 shows the optimal collusive strategy. After drawing a first negative signal, firms prefer to wait until they learn whether the second firm has a low cost if and only if the benefit of waiting for one signal exceeds the cost, i.e. $V_0 \geq \pi_m - \widetilde{\theta}$. Firms prefer to draw one signal than invest immediately if and only if $2V_0 \geq \pi_m - \widetilde{\theta}$. Hence, the parameter space can be divided into three regions, where the optimal collusive choice corresponds to each of the alternatives.

3 Entry timing

We now analyze the game played by two competing firms. We first compute the profits of the leader and follower firms. Suppose that one firm (the leader) invests first. The second firm (the follower) will only follow suit if it learns that its cost is low. Hence the expected value of the follower is given by

$$V_F = \frac{\mu}{2(\mu + r)} (\pi_d - \underline{\theta}).$$

The leader thus extracts monopoly profit as long as the other firm has not entered, and duopoly profits after entry of the follower. The expected value of the leader is thus given by:

$$V_L = \pi_m - \frac{\mu}{2(\mu + r)} (\pi_m - \pi_d).$$

3.1 Entry timing with public signals

In this subsection, we suppose that the signals received by the two firms during the experimentation phase are *public*. Proposition 1 characterizes the equilibrium of the entry timing game.

Proposition 1 *In the entry timing game with public information, a firm which learns that its cost is low invests immediately. If $V_L - \widetilde{\theta} > V_F$, preemption occurs and (i) a firm which ignores its cost invests with positive probability at any date $t = 0, \Delta, ...$ whenever the other firm has not invested and (ii) a firm invests immediately after it learns that the other firm has a high cost. If $V_L - \widetilde{\theta} < V_F$, firms do not enter unless they learn that their cost is low.*

Proposition 1 shows that the entry timing game is either a preemption game (when $V_L - \widetilde{\theta} > V_F$), or a waiting game (when $V_L - \widetilde{\theta} < V_F$). Notice that $V_L - V_F = \pi_m - V_0$. Hence, there is a parameter region ($\pi_m - 2V_O \leq \widetilde{\theta} < \pi_m - V_O$) where firms prefer to wait in the collusive benchmark but invest immediately in the noncooperative game. Firms invest *too early* in the competitive entry game, and competition results in excess momentum.

3.2 Entry timing with private signals

When signals are private, firms do not learn the cost of their competitor. Each firm holds beliefs $\gamma_t(\theta)$ about the cost of its rival. These beliefs evolve over time given the strategies and the observation of investments. In order to compute the beliefs, we let $G(t, \tau)$ denote the probability that a firm which learns that its cost is low at date τ invests at $t \geq \tau$, with $g(t, \tau)$ the instantaneous probability. We also let $h(t)$ denote the instantaneous probability that a firm which ignores its cost invests exactly at date t. Using Bayes' rule, the beliefs at period t are then given by:

$$
\begin{aligned}
\gamma_t(\underline{\theta}) &= \frac{\int_0^t [1 - G(t, \tau)] \mu e^{-\mu \tau} d\tau}{A(t)}, \\
\gamma_t(\overline{\theta}) &= \frac{1 - e^{-\mu t}}{A(t)}, \\
\gamma_t(\widetilde{\theta}) &= \frac{2[e^{-\mu t} - \int_0^t e^{-\mu \tau} h(\tau) d\tau]}{A(t)}
\end{aligned}
$$

where

$$A(t) = \int_0^t [1 - G(t,\tau)]\mu e^{-\mu\tau} d\tau + 1 - e^{-\mu t} + 2[e^{-\mu t} - \int_0^t e^{-\mu\tau} h(\tau) d\tau].$$

We first establish that a firm which learns that its cost is low has an incentive to invest immediately:

Lemma 1 *In the entry timing game with private signals, a firm which learns that its cost is low has an incentive to enter immediately.*

By Lemma 1, if a firm has not invested at date t, it has either learned that it has a low cost, or has not received a signal. This enables us to simplify the beliefs:

$$\begin{aligned}
\gamma_t(\underline{\theta}) &= 0, \\
\gamma_t(\overline{\theta}) &= \frac{1 - e^{-\mu t}}{1 + e^{-\mu t} - 2\int_0^t e^{-\mu\tau} h(\tau) d\tau}, \\
\gamma_t(\widetilde{\theta}) &= \frac{2[e^{-\mu t} - \int_0^t e^{-\mu\tau} h(\tau) d\tau]}{1 + e^{-\mu t} - 2\int_0^t e^{-\mu\tau} h(\tau) d\tau}.
\end{aligned}$$

It is easy to check that the belief that the other firm has learned that it has a high cost, $\gamma_t(\overline{\theta})$, increases over time. The expected profit of a firm which is the first to invest at date t is given by:

$$V_L(t) = \gamma_t(\overline{\theta})\pi_m + \gamma_t(\widetilde{\theta})V_L = \pi_m - \gamma_t(\widetilde{\theta})\frac{\mu}{2(\mu + r)}(\pi_m - \pi_d).$$

Because $\gamma_t(\overline{\theta})$ is increasing over time, the value of the leader is also *increasing*. No news is good news: as time passes, each firm becomes more convinced that the other firm has received a negative signal, and becomes more optimistic about its own prospects. The value of the leader increases from V_L at $t = 0$ to π_m when t goes to infinity. The value of the follower, V_F, remains independent of time. Figures 1, 2 and 3 illustrate the three possible régimes, ranking the values of the leader and the follower, as a function of the parameters of the model.

Cases 1 and 3 correspond to the preemption and waiting cases in the timing game with public signals. Case 2 exploits the fact that beliefs evolve over

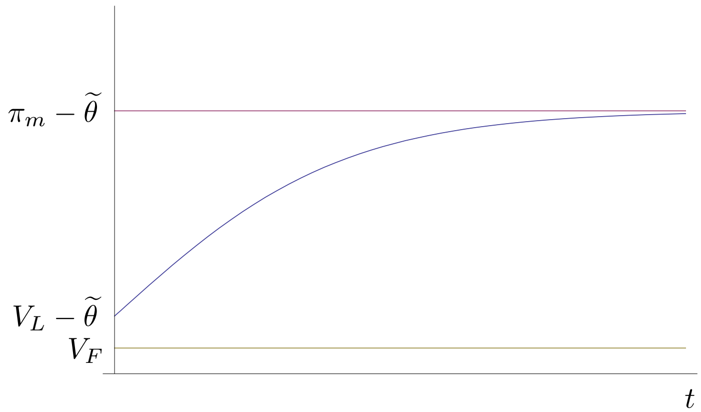

Figure 1: Case 1: $V_L - \widetilde{\theta} > V_F$

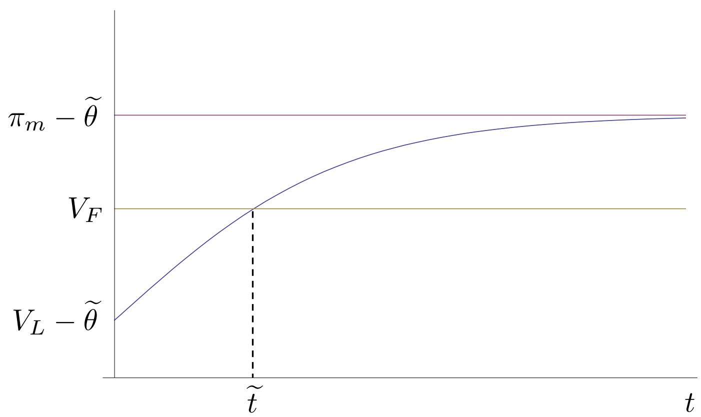

Figure 2: Case 2: $\pi_m - \widetilde{\theta} \geq V_F \geq V_L - \widetilde{\theta}$

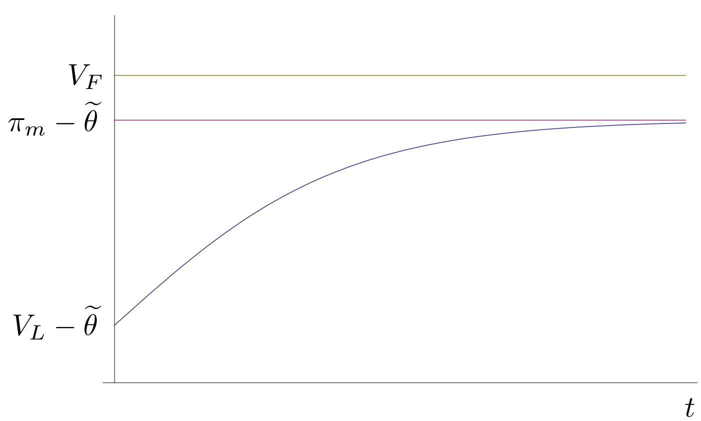

Figure 3: Case 3: $V_F > \pi_m - \widetilde{\theta}$

time, and describes a new situation where preemption occurs at a finite time. In this case, the entry timing game is formally identical to the innovation game studied by Fudenberg and Tirole (1985). The expected payoff of the leading firm is initially lower than the expected payoff of the following firm, but is increasing over time and eventually becomes higher than the payoff of the following firm. As in Fudenberg and Tirole (1985), the unique subgame perfect equilibrium results in rent equalization: the leader invests exactly at the time where the expected payoffs of the leader and follower coincide, $V_L(\tilde{t}) - \widetilde{\theta} = V_F$. Formally,

Theorem 1 *In the entry timing game with private signals, a firm which learns its cost invests immediately. If* $V_L - \widetilde{\theta} > V_F$*, preemption occurs at the beginning of the game and both firms invest with positive probability at time* 0*. If* $\pi_m - \widetilde{\theta} \geq V_F \geq V_L - \widetilde{\theta}$*, in a symmetric equilibrium, rents between the leader and the follower are equalized and each firm invests with probability* $\frac{1}{2}$ *at time* $\tilde{t}$ *such that:* $V_L(\tilde{t}) - \widetilde{\theta} = V_F$*. If* $V_F > \pi_m - \widetilde{\theta}$*, firms do not enter unless they learn that their cost is low.*

Theorem 1 shows that excess momentum is higher with private signals than with public signals. In one configuration of the parameters, when $\pi_m - \widetilde{\theta} \geq V_F \geq V_L - \widetilde{\theta}$, firms wait to learn their costs in the collusive benchmark and when signals are public, but invest at finite time $\widetilde{t}$ when signals are private. This result stands in contrast to Hopenhayn and Squintani (2011) who show that preemption is stronger with public signals than with private signals. This difference is easily explained. In Hopenhayn and Squintani (2011), new information can only signal an improvement in the competitive situation of the firm, so that competition is fiercer when information is public. In our model, private information can only signal a degradation in the competitive situation of the firm, so that competition is fiercer when information is private.

We now focus on Case 2 and perform a comparative static analysis of the effect of changes in the parameters of the model on the preemption time $\widetilde{t}$. The preemption time is implicitly defined as the unique solution to the equation:

$$V_L\left(t\right) - \widetilde{\theta} - V_F = 0. \tag{2}$$

By implicit differentiation of equation (2), we obtain the comparative statics displayed in Table 2.

parameter	comparative static
π_m	$-$
π_d	$+$
$\overline{\theta}$	$+$
$\underline{\theta}$	$+$
r	$-$
μ	$+/-$

Table 2: Preemption time $\tilde{t}$ – Comparative statics

All parameters have the expected effect on the preemption time $\tilde{t}$, except for the intensity of the Poisson process μ.[2] Changes in the Poisson arrival rate μ have ambiguous effects. An increase in the arrival rate accelerates the process by which a firm learns its cost, increasing the value of the follower:

$$\frac{dV_F}{d\mu} = \frac{r}{2\left(\mu + r\right)^2}\left(\pi_d - \underline{\theta}\right) > 0,$$

and decreasing the value of the leader at time zero

$$\frac{dV_L}{d\mu} = -\frac{r}{2\left(\mu + r\right)^2}\left(\pi_m - \pi_d\right) < 0.$$

In addition, an increase in μ increases the speed at which a firm updates its belief about its opponent. hence, the rate at which $V_L(t)$ increases is higher and

$$\frac{dV_L(t)}{d\mu} = (-\frac{r}{2(\mu + r)^2}\gamma_t(\widetilde{\theta}) - \frac{d\gamma_t(\widetilde{\theta})}{d\mu}\frac{\mu}{2(\mu + r)})(\pi_m - \pi_d),$$

where $\frac{d\gamma_t(\widetilde{\theta})}{d\mu} < 0$.

[2] Changes in the Poisson arrival rate μ can be attributed to changes in the screening technology to evaluate projects. It is often argued that because they screen many different projects or because they have privileged access to information regarding the projects, venture capitalists have a better ability to judge the profitability of early-stage ventures than actors in different industries, such as industrial investors or even banks (See, for example, Ueda (2004) and Fabrizi et al. (2011).) This would imply that projects funded by venture capitalists have a higher Poisson arrival rate μ than projects independently run by firms.

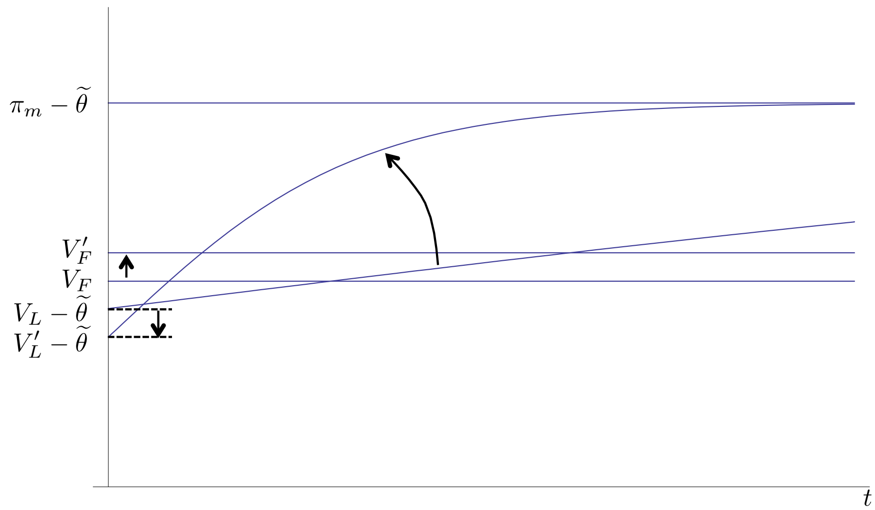

Figure 4: Effect of an increase in μ on $V_L(t)$ and V_F.

Figure 4 shows the effects of an increase in μ on V_F and $V_L(t)$. We conjecture that the effect of a change in μ on the preemption time $\tilde{t}$ is non-monotonic. If μ is low and $\tilde{t}$ is low, an increase in μ will mostly have the effect of increasing V_F and reducing V_L, resulting in an increase in the preemption time. If, on the other hand, μ is high and $\tilde{t}$ is high, an increase in μ will mostly have the effect of increasing $V_L(t)$, reducing the preemption time. This non-monotonicity is illustrated in Figure 5 which shows how $\tilde{t}$ varies with μ when $\pi_m = 0.7$, $\pi_d = 0.3$, $\overline{\theta} = 0.8$, $\underline{\theta} = 0.2$, and $r = 0.05$.

3.3 Efficiency comparison

We now compare the joint profits of the two firms in the collusive benchmark, the equilibrium of the noncooperative game with public signals and with private signals. We distinguish between four parameter regions, depending on the magnitude of the expected entry cost $\widetilde{\theta}$:

1. $\widetilde{\theta} < \pi_m - 2V_O$: immediate entry in the cooperative regime, and preemption at zero in both competitive regimes;

2. $\pi_m - 2V_O \leq \widetilde{\theta} < V_L - V_F = \pi_m - V_O$: delayed entry in the cooperative regime (firms wait for one signal), and preemption at zero in both competitive regimes;

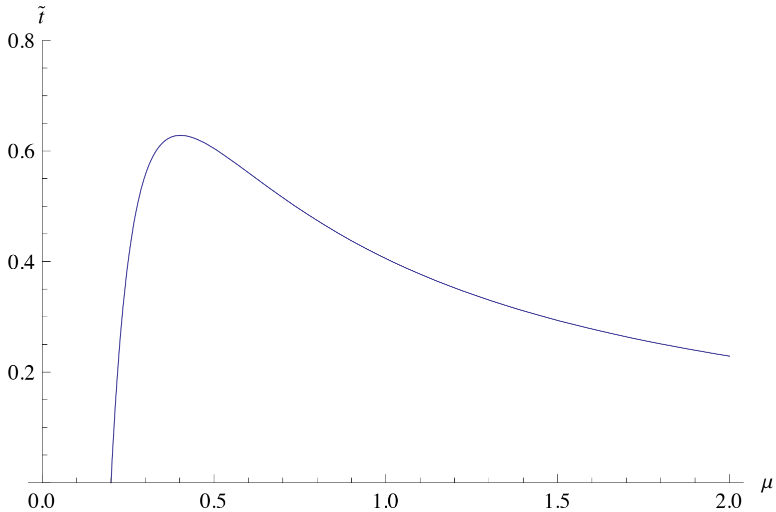

Figure 5: $\tilde{t}$ AS A FUNCTION OF μ FOR $\pi_m = 0.7$, $\pi_d = 0.3$, $\overline{\theta} = 0.8$, $\underline{\theta} = 0.2$, AND $r = 0.05$.

3. $V_L - V_F = \pi_m - V_O \leq \widetilde{\theta} < \pi_m - V_F$: delayed entry in the cooperative regime (firms wait for up to two signals) and in the competitive regime with public signals, preemption at finite time $\tilde{t}$ in the competitive regime with private signals;
4. $\pi_m - V_F \leq \widetilde{\theta}$: delayed entry in the cooperative regime (firms wait for up to two signals) and in both competitive regimes.

We define the industry profits when both firms delay their entry until they learn that their cost is low as:

$$V_S = \frac{\mu}{2\mu + r}\left[(\pi_m - \underline{\theta}) + \frac{\mu}{2(\mu + r)}(2\pi_d - 2\underline{\theta})\right]$$

and the industry profits with preemption at finite time $\tilde{t}$ as:

$$\begin{aligned} V_P &= \left(1 - e^{-(2\mu+r)\tilde{t}}\right)\frac{\mu}{2\mu + r}\left[(\pi_m - \underline{\theta}) + \frac{\mu}{2(\mu + r)}(2\pi_d - 2\underline{\theta})\right] \\ &+ e^{-(\mu+r)\tilde{t}}\left(1 - e^{-\mu\tilde{t}}\right)\frac{\mu}{2(\mu + r)}(\pi_m - \underline{\theta}) \\ &+ e^{-(2\mu+r)\tilde{t}}2\frac{\mu}{2(\mu + r)}(\pi_d - \underline{\theta}) \end{aligned} \quad (3)$$

It is easy to check that $V_{C2} > V_S > V_P > 2V_F$ and $\pi_m - \widetilde{\theta} > 2V_F$. Table 3 lists the joint profits under the three regimes in the four configurations of parameters.

parameter region	cooperative	public	private
$\widetilde{\theta} < \pi_m - 2V_O$	$\pi_m - \widetilde{\theta}$	$2V_F$	$2V_F$
$\pi_m - 2V_O \leq \widetilde{\theta} < V_L - V_F = \pi_m - V_O$	V_{C1}	$2V_F$	$2V_F$
$V_L - V_F = \pi_m - V_O \leq \widetilde{\theta} < \pi_m - V_F$	V_{C2}	V_S	V_P
$\pi_m - V_F \leq \widetilde{\theta}$	V_{C2}	V_S	V_S

Table 3: Efficiency comparisons

Not surprisingly, Table 3 shows that joint profits are always higher when signals are public rather than private. Simple derivations also show that all values V_{C1}, V_{C2}, V_F, V_S are strictly increasing in μ, and numerical computations suggest that V_P is also increasing in μ as well. Hence, better screening technologies or information collection methods always result in higher profits for the two firms. Table 3 also illustrates three sources of inefficiency due to competition. First, by competing on the market, the firms forgo the benefits of market monopolization – the difference between monopoly profits, π_m and the sum of duopoly profits, $2\pi_d$. Second the firms pay twice the entry cost θ, whereas in the collusive benchmark, only one firm enters. Finally competition results in excess momentum, making firms enter the market before they learn their cost, whereas in the collusive benchmark, they prefer to wait until they learn their cost before entering.

4 Collusion and compensating payments

In this section, we analyze compensating payment schemes which allow the firms to achieve the collusive outcome. Compensating payments are paid by one firm in order to compensate the other firm for not entering the market. These transfers can be implemented at three different points in time:

- *ex ante*: payments are made before the firms learn their entry cost;
- *at the interim stage*: payments are made by one firm after it learns its cost, when it ignores the cost of the other firm;
- *ex post*: payments are made after the costs of both firms are common knowledge.

The best timing of compensating payments depends on the specific configuration of parameters and the publicity of signals. If $\widetilde{\theta} < \pi_m - 2V_O$, it is efficient to choose one of the two projects immediately, and compensating payments should be made *ex-ante* in order to prevent inefficiencies due to market competition and duplication of entry costs.

If $\pi_m - 2V_O < \widetilde{\theta}$, the collusive outcome involves experimentation. Whenever it is optimal to experiment, we argue that the best timing for compensating payments is either the *interim stage* (for private signals) or the *ex-post stage* (for public signals). At the ex-ante stage, if one of the firm pays the other firm to leave the market, it loses access to that firm's technology, and may be unable to select the right project.[3] When signals are public, cooperation should happen at the ex post stage, with compensating payments being paid to the follower firm only when it learns that its cost is low. When signals are private and firms cannot credibly convey information about their entry cost, ex-post compensating payments can only be made after the follower firm has entered the market. In that case, the entry cost of the follower firm has been sunk, so that it becomes impossible to alleviate the duplication of entry costs with payments at the ex-post stage and the best timing for compensating payments is the *interim stage*.

We focus on this last situation and analyze compensating payments made at the interim stage when $\pi_m - 2V_O < \widetilde{\theta}$ and signals are private. Individual rationality and incentive compatibility imply that the utility obtained by the leader and the follower when investing at date t must satisfy:

$$U_L(t) \geq V_L(t), \quad (4)$$

$$U_F(t) \geq V_F. \quad (5)$$

The first inequality states that the leader is willing to pay the compensation. The second inequality results from the follower's incentive compatibility and individual rationality constraints. As the follower's type (whether he has dropped from the race or not received a signal) is unknown, the follower must receive a payment at least equal to V_F, the expected continuation value of a follower who ignores his cost. Budget balance implies that the sum of utilities received by the leader and follower are exactly equal to the monopoly profit:

$$U_L(t) + U_F(t) = \pi_m. \quad (6)$$

[3]Even if the firm could buy the technology of its rival, it would still need the collaboration of workers in that firm. This would give rise to agency problems which involve an efficiency loss.

Given inequalities (4), (5) and equation (6), a necessary condition for the existence of a budget balanced, individually rational and incentive compatible transfer scheme is thus

$$\pi_m \geq V_L(t) + V_F.$$

As $V_L(t)$ is increasing, $V_L(0) < \pi_m - V_F$ and $V_L(\infty) > \pi_m - V_F$, there exists a unique date t^*, such that *no budget balanced individually rational compensating payments exist if the first firm enters the market at date $t \geq t^*$.* This remark captures the following simple intuition. As time passes, firms become more optimistic about their prospects. If a firm enters at a late date, it will expect the other firm to have dropped and will not be willing to compensate the other firm at the level V_F, which is the minimal level that a firm which ignores its cost is willing to accept to drop from the market. This remark also shows that *there is no efficient, budget balanced and individually rational collusive mechanism.* To see this, consider a realization of the signals where no firm has learned its cost before t^*. Either the mechanism prescribes that one of the firm invests before t^*, and the mechanism is inefficient because it will result in a high cost firm investing with positive probability, or the mechanism prescribes to wait until one of the firm has learned it has a low cost, and the mechanism is inefficient because there is no budget balanced, individually rational compensating payment which prevents the other firm from entering the race.

The latest point at which firms can collude, t^*, is implicitly determined by

$$\pi_m = V_L(t^*) + V_F. \tag{7}$$

Table 4 shows the other comparative statics of changes in parameters on the date t^*. Notice that a change in the Poisson arrival rate μ, has a clear negative effect on τ^*. When firms learn their costs more quickly, beliefs evolve faster, and the last time at which collusion may occur is reduced.

We now consider the following problem: How should compensating payments be designed in order to guarantee that, whenever one firm learns that it has a low cost before t^*, it is chosen to be the only firm operating on the market?

parameter	comparative static
π_m	$+$
π_d	$-$
$\overline{\theta}$	0
$\underline{\theta}$	$+$
r	0
μ	$-$

Table 4: LATEST TIME FOR COOPERATION t^* – COMPARATIVE STATICS

Proposition 2 *A differentiable compensating payment scheme $U_F(t)$ implements the cooperative benchmark when a firm learns that it has a low cost before t^* if and only if for all $t < t^*$,*

$$\pi_m - \widetilde{\theta} < 2U_F(t) < \frac{2r+\mu}{r+\mu}\pi_m + \frac{U_F'(t)}{r+\mu}.$$

Proposition 2 shows that efficient compensating payment schemes must be designed to satisfy two requirements. First, the payment to the follower must be large enough to prevent early entry by firms which ignore their costs. Second, the payment to the follower should not be too large, in order to give incentives to a firm which learns that its cost is low to enter immediately. These two requirements provide an upper and a lower bound on the expected payoffs of the follower and leader firm and show that the cooperative surplus must be shared in a balanced way between the two firms.

In order to provide additional intuition, we specialize the model by assuming that the compensating payment scheme assigns a fixed bargaining power to the leader and the follower, so that

$$U_L(t) = V_L(t) + \alpha(\pi_m - V_L(t) - V_F), \quad (8)$$
$$U_F(t) = V_F + (1-\alpha)(\pi_m - V_L(t) - V_F). \quad (9)$$

We observe that $U_L(t)$ is increasing and $U_F(t)$ decreasing over time. Figure 6 displays these profits for $\alpha = 0$ and $\alpha = 1$. It illustrates three aspects of the model. First, it displays the t^*, for which $\pi_m - V_L(t^*) - V_F = 0$. Second, Figure 6 shows that payoffs are independent of time if $\alpha = 1$, that is, if all of the bargaining power is given to the leader. In this case, the follower receives his outside utility, $U_F = V_F$, and the leader receives all surplus plus his outside utility, $U_L(t) = \pi_m - V_F$. Third, Figure 6 shows that the gap between the payoff of the leader and follower is increasing in α.

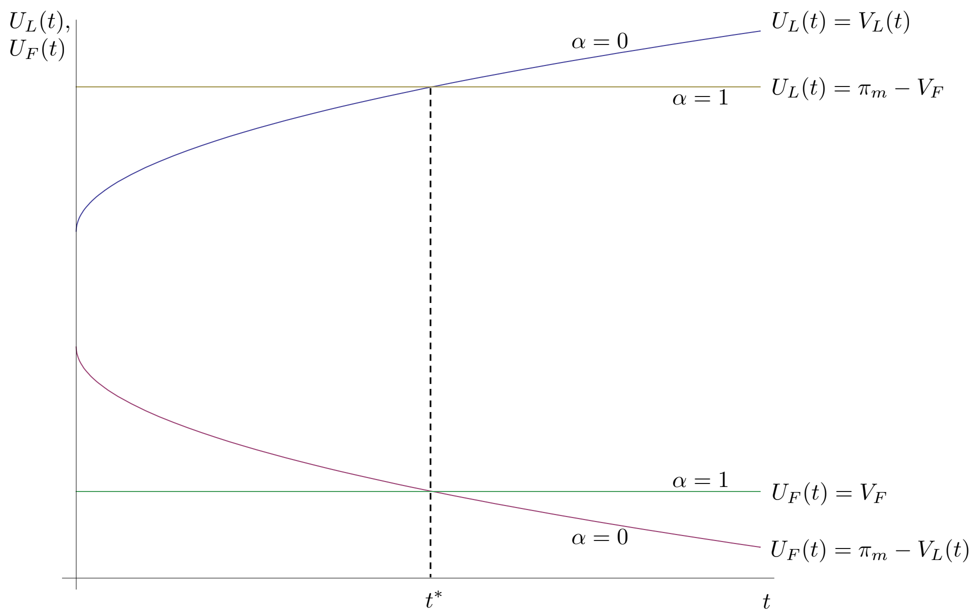

Figure 6: EXPECTED UTILITIES WITH COMPENSATING PAYMENTS

Using (8) and (9), Proposition 2 imposes two restrictions on the share of the surplus that accrues to the leader, α. On the one hand, to prevent firms to invest early when they ignore their cost, α has to be sufficiently low:

$$\alpha < \frac{\pi_m + \widetilde{\theta} - V_L(t)}{2\left(\pi_m - V_L(t) - V_F\right)}. \tag{10}$$

On the other hand, to give a firm that learns that it has low cost incentives to enter immediately, α must be sufficiently high:

$$\frac{2\left(\pi_m - V_L(t)\right) + \frac{V_L'(t)}{r+\mu} - \frac{2r+\mu}{r+\mu}\pi_m}{2\left(\pi_m - V_L(t) - V_F\right) + \frac{V_L'(t)}{r+\mu}} < \alpha. \tag{11}$$

Define

$$\overline{\alpha} \equiv \min\left\{\frac{\pi_m + \widetilde{\theta} - V_L(t)}{2\left(\pi_m - V_L(t) - V_F\right)}, 1\right\}$$

and

$$\underline{\alpha} \equiv \max\left\{0, \frac{2\left(\pi_m - V_L(t)\right) + \frac{V_L'(t)}{r+\mu} - \frac{2r+\mu}{r+\mu}\pi_m}{2\left(\pi_m - V_L(t) - V_F\right) + \frac{V_L'(t)}{r+\mu}}\right\}.$$

Corollary 1 *A compensating payment scheme that assigns a fixed bargaining power to the leader and the follower firm, so that a share* α *of the surplus from cooperation accrues to the leader, implements the collusive outcome when a firm learns that it has a low cost before* t^* *if and only if for all* $t < t^*$,$\alpha \in [\underline{\alpha}, \overline{\alpha}]$.

A necessary condition for implementation of the collusive outcome is that $0 \leq \underline{\alpha} \leq \overline{\alpha} \leq 1$, which is guaranteed if the following conditions on the parameters hold:

$$2V_F \geq \pi_m - \widetilde{\theta}, \tag{12}$$

$$2V_F + 2(1-\alpha)(\pi_m - V_L(0) - V_F) \leq \frac{2r+\mu}{r+\mu}\pi_m - \frac{V_L'(0)}{r+\mu}. \tag{13}$$

Notice that if condition (12) fails, early preemption will occur before t^*. However, the value of α can be designed in order to delay entry of firms which ignore their costs as far as possible. By reducing α, and giving a larger share of the surplus to the follower, the mechanism designer reduces incentives to preempt and delays inefficient entry of firms on the market.[4] The optimal compensating payment mechanism is then given by the *lowest value of* α for which condition (13) holds.

5 Extensions

In this Section, we extend the analysis in two directions. We first investigate the effect of collusion on consumer surplus. We then extend the model to allow for stochastic profits after entry.

5.1 Consumer surplus

Expected consumer surplus is affected both by the market structure and the expected time of entry. Expected consumer surplus is higher in duopoly than monopoly, and decreases with the time of entry. Let CS_m and CS_d denote consumer surplus in a monopoly and a duopoly with $CS_m \leq CS_d$.

[4]The fact that giving a prize to the loser of a contest may be efficient, as it reduces the gap between the winner and the loser and minimizes wasteful expenditures, has long been noted in the literature on contests. See for example Moldovanu and Sela (2001).

We define the expected consumer surplus with preemption at finite time $\widetilde{t}$ as

$$CS_P = \left(1 - e^{-(2\mu+r)\widetilde{t}}\right) \frac{\mu}{2\mu + r} \left[CS_m + \frac{\mu}{2(\mu + r)} (CS_d - CS_m)\right]$$
$$+ e^{-(\mu+r)\widetilde{t}} \left(1 - e^{-\mu\widetilde{t}}\right) \frac{\mu}{2(\mu + r)} CS_m$$
$$+ e^{-(2\mu+r)\widetilde{t}} \left[CS_m + \frac{\mu}{2(\mu + r)} (CS_d - CS_m)\right].$$

Denoting by p the probability with which each firm enters at date $t = 0$, we can write the expected consumer surplus in the equilibrium with preemption at time $t = 0$ as

$$CS_{P0} = p^2 CS_d + \left(1 - p^2\right) \left(CS_m + \frac{\mu}{2(\mu + r)} (CS_d - CS_m)\right).$$

Finally, the expected consumer surplus when both firms delay their entry until they learn that their cost is low can be written as

$$CS_S = \frac{\mu}{2\mu + r} \left[CS_m + \frac{\mu}{2(\mu + r)} (CS_d - CS_m)\right].$$

The ranking of consumer surplus is exactly opposite to the ranking of joint profits: while firms rank $V_S \geq V_P \geq 2V_F$, consumers rank $CS_S \leq CS_P \leq CS_{P0}$. Hence, consumer surplus is always higher in the non-cooperative game with private signals than with public signals, and consumers prefer the competitive outcome to the collusive outcome. As the ranking of consumer surplus and joint profits are exactly opposite, the socially optimal policy depends on the weights ascribed to consumer surplus and joint profits in the social welfare function.

5.2 Market uncertainty

We extend the analysis by supposing that, after investment, the profits of the two firms, π_m and π_d, are random variables rather than deterministic values. More precisely, we consider a model where after investing in the project, the two firms may either encounter success or failure. Furthermore, the market can only sustain one firm: the first firm to succeed obtains a positive value, whereas the other firm receives a zero profit.

This model captures an R&D race with two distinct phases. In the initial *experimentation phase*, as in Section 2, two firms receive signals about the fixed cost of investing in a research project. In the second, *innovation phase*, the firms engage in a development process to obtain an innovation with commercial value normalized to 1. After investing in the research project, firms succeed in finding the innovation according to a Poisson process with intensity λ.

We interpret the monopoly profit, π_m, as the expected profit of a firm when the other firm does not invest: $\pi_m = \int_0^\infty \lambda e^{-\lambda t} e^{-rt} dt = \frac{\lambda}{\lambda + r}$, and the duopoly profit, π_d, as the expected profit of a firm when the other firm has invested: $\pi_d = \int_0^\infty \lambda e^{-2\lambda t} e^{-rt} dt = \frac{\lambda}{2\lambda + r}$.

Notice that $2\pi_d > \pi_m$ in this model. In order to guarantee that firms have an incentive to select one of the two projects rather than running both in parallel, we assume that the entry cost is larger than the difference $2\pi_d - \pi_m$:

Assumption 2 $2\pi_d - \pi_m = \frac{\lambda r}{(2\lambda + r)(\lambda + r)} \leq \underline{\theta}$.

5.2.1 Collusive benchmark

In the collusive benchmark, under assumption 2, a single firm will be chosen to develop the product. Hence, the values of V_O, V_{C1}, and V_{C2} and the analysis of Section 2.3 remain unchanged, once the monopoly profit π_m is replaced by the specific value $\pi_m = \frac{\lambda}{\lambda + r}$.

5.2.2 Investment timing with public signals

In the non-cooperative investment timing game, we need to modify the expressions for the values of the leader and the follower, as the follower will only obtain the duopoly profit π_d if he enters before the leader has succeeded in the innovation phase. Hence, we compute

$$\begin{aligned} V_F &= \frac{\mu}{2(\mu + \lambda + r)}(\pi_d - \underline{\theta}) \quad \text{and} \\ V_L &= \pi_m - \frac{\mu}{2(\mu + \lambda + r)}(\pi_m - \pi_d). \end{aligned}$$

With this modification, Proposition 1 can be extended to

Proposition 3 *In the entry timing game with public information, a firm which learns that its cost is low invests immediately. If $V_L - \widetilde{\theta} > V_F$, preemption occurs and (i) a firm which ignores its cost invests with positive probability at any date $t = 0, \Delta, ...$ whenever the other firm has not invested and (ii) a firm invests immediately after it learns that the other firm has a* *high cost. If $V_L - \widetilde{\theta} < V_F$, firms do not enter unless they learn that their cost is low.*

Notice that, in the market uncertainty model, $V_L - V_F \geq \pi_m - V_O$. Hence, there is a new configuration of parameters, $\pi_m - V_O < \widetilde{\theta}w < V_L - V_F$, for which the two firms choose to wait until they learn that one of the project has low cost in the collusive benchmark ($V_{C2} > V_{C1}$), whereas they invest immediately in the non-cooperative entry game.

5.2.3 Investment timing with private signals

We again need to modify the values of the leader and follower to take into account the possibility that the leader succeeds in the innovation phase before the follower learns his cost.

$$
\begin{aligned}
V_L(t) &= \gamma_t(\overline{\theta})\pi_m + \gamma_t(\widetilde{\theta})V_L \\
&= \pi_m - \gamma_t(\widetilde{\theta})\frac{\mu}{2(\mu+\lambda+r)}(\pi_m - \pi_d).
\end{aligned}
$$

The analysis of section 3.2 can be replicated without any difficulty.

Proposition 4 *In the entry timing game with private signals, a firm which learns its cost invests immediately. If $V_L - \widetilde{\theta} > V_F$, preemption occurs at the beginning of the game and both firms invest with positive probability at time 0. If $\pi_m - \widetilde{\theta} \geq V_F \geq V_L - \widetilde{\theta}$, in a symmetric equilibrium, rents between the leader and the follower are equalized and each firm invests with probability $\frac{1}{2}$ at time $\tilde{t}$ such that: $V_L(\tilde{t}) - \widetilde{\theta} = V_F$. If $V_F > \pi_m - \widetilde{\theta}$, firms do not enter unless they learn that their cost is low.*

An increase in the parameter λ governing the rate of success on the market, intensifies competition between the two firms and reduces the time of preemption $\widetilde{t}$. Furthermore, the expected time before innovation is always smaller in the competitive régime, when firms accelerate market entry to preempt their rival than in the collusive régime where the two firms wait until they learn their cost. In the Appendix, we compute the expected time before innovation in the different régimes for different configurations of the parameters as: Table 5 lists the expected time until an innovation is obtained in the entry game with market uncertainty under the three regimes in the five configurations of the parameters.

parameter region	cooperative	public	private
$\widetilde{\theta} \leq \pi_m - 2V_O$	$E_{C0}(T)$	$E_{P0}(T)$	$E_{P0}(T)$
$\pi_m - 2V_O < \widetilde{\theta} \leq \pi_m - V_O$	$E_{C1}(T)$	$E_{P0}(T)$	$E_{P0}(T)$
$\pi_m - V_O < \widetilde{\theta} \leq V_L - V_F$	$E_{C2}(T)$	$E_{P0}(T)$	$E_{P0}(T)$
$V_L - V_F < \widetilde{\theta} \leq \pi_m - V_F$	$E_{C2}(T)$	$E_S(T)$	$E_P(T)$
$\pi_m - V_F < \widetilde{\theta}$	$E_{C2}(T)$	$E_S(T)$	$E_S(T)$

Table 5: Expected time until an innovation is introduced

where the formulae for expected time before innovation are given by:[5]

$$\begin{aligned}
E_{C0}(T) &= \frac{1}{\lambda}, \\
E_{C1}(T) &= \frac{1}{2}\left(\frac{2}{\lambda} + \frac{1}{\mu}\right) \\
E_{C2}(T) &= \frac{1}{\lambda} + \frac{5}{6\mu} \\
E_{P0}(T) &= \frac{1}{2\lambda} + \frac{\lambda}{2\left(\lambda+\mu\right)^2} + \frac{\mu}{8\lambda\left(\lambda+\mu\right)} + \frac{\mu}{4\left(\lambda+\mu\right)^2} \\
E_S(T) &= \frac{1}{12}\left(\frac{9}{\lambda} + \frac{10}{\mu} + \frac{2\lambda}{\left(\lambda+\mu\right)^2}\right).
\end{aligned}$$

Table 6 displays the magnitudes of the expected times to the introduction of the innovation when the intensity of the Poisson processes λ or μ converges to ∞, namely when either experimentation or innovation are immediate.

Table 6 confirms that the expected time before innovation is lowest when the firms invest immediately (E_{C0} and E_{P0}) and longest when the firms wait until they learn that they have a low cost (E_{C2} and E_S). Because firms enter more rapidly in the competitive régime, and innovation is accelerated when the two firms participate in the patent race, innovation is always faster in the competitive régime, and private signals accelerate innovation. More precisely, when $\mu \to \infty$, $E_{P\cdot}(T) < E_S(T) < E_{C\cdot}(T)$. For $\lambda \to \infty$, $E_{C0}(T) = E_{P0}(T) < E_{C1}(T) < E_S(T) = E_{C2}(T)$.

[5]We omit the formula for $E_P(T)$, expected time before innovation in the finite preemption case as it is very cumbersome.

	$\mu \to \infty$	$\lambda \to \infty$
$E_{C0}(T)$	$\frac{1}{\lambda}$	0
$E_{C1}(T)$	$\frac{1}{\lambda}$	$\frac{1}{2\mu}$
$E_{C2}(T)$	$\frac{1}{\lambda}$	$\frac{5}{6\mu}$
$E_{P0}(T)$	$\frac{5}{8\lambda}$	0
$E_P(T)$	$\frac{5}{8\lambda}$	$\hat{t} \in \left[0, \frac{5}{6\mu}\right]$
$E_S(T)$	$\frac{3}{4\lambda}$	$\frac{5}{6\mu}$

Table 6: LIMITS OF THE EXPECTED TIME UNTIL AN INNOVATION IS INTRODUCED

6 Conclusion

This paper analyzes a model of entry with learning. Two firms contemplate entry into a new market, or the development of a new product and gradually learn about their private entry costs. We show that when signals are public, the model either results in a preemption game or a waiting game, and when signals are private, firms which ignore their cost may choose to enter at a finite time, resulting in the same rent equalization phenomenon as in Fudenberg and Tirole (1985). As opposed to Hopenhayn and Squintani (2011), we find that preemption is greater when signals are private, because firms do not know whether the other firm has given up on entering the market. As compared to the collusive outcome, the equilibrium of the entry timing game exhibits three sources of inefficiencies: dissipation of the monopoly rent, duplication of entry costs and excess momentum. We analyze how compensating payments by one firm to prevent the other firm from entering the market can be implemented. We observe that collusion can only be effective if the first firm enters sufficiently early, and that compensating payments must allocate a significant share of the surplus to the excluded firm. Our model also covers two-stage R&D models, where firms first experiment to learn their cost in the research project, and then enter into a stochastic innovation race.

Our analysis belongs to an emerging literature on innovation races and timing games with private signals. It leaves a number of questions unanswered. What happens if signals are not perfect, and what is the effect of changes in the precision of the signals on preemption? What happens when uncertainty pertains to the common value of the innovation rather than the private value entry cost? What if firms can control the acquisition of information by choosing their level of (costly) experimentation? What happens if the two firms, rather than being independent agents, are two teams in an organization contracting with a principal? We plan to tackle these problems in future research.

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A Proofs

A.1 Proof of Proposition 1

We first note that, if both firms learn that their cost is low, they have no incentive to delay their investment and will both invest immediately. Consider then a situation where firm i has learned that its cost is low and firm j has not learned its cost yet. We will show that it is a dominant strategy for firm i to invest immediately. If firm j invests, firm i obtains $\pi_d - \underline{\theta}$ by investing immediately and $(1 - r\Delta)(\pi_d - \underline{\theta})$, by delaying its investment, and thus prefers to invest immediately. If firm j does not invest, and chooses to invest with probability p at period $t + \Delta$, by delaying its investment until $t + \Delta$, firm i will obtain a payoff:

$$\begin{aligned} W(t+\Delta) &= (1 - r\Delta)[(1-\mu\Delta)[(1-p)V_L + p\pi_d] \\ &\quad +\mu\Delta\frac{\pi_d + \pi_m}{2} - \underline{\theta}]. \end{aligned}$$

Now

$$\begin{aligned} W(t+\Delta) - (V_L - \underline{\theta}) &= -r\Delta(V_L - \underline{\theta}) - \mu\Delta(1 - r\Delta)(V_L - \frac{\pi_d + \pi_m}{2}) \\ &\quad -p(1-r\Delta)(1-\mu\Delta)(V_L - \pi_d) + \mathcal{O}(\Delta^2). \end{aligned}$$

Note that

$$V_L - \frac{\pi_d + \pi_m}{2} = \frac{r}{2(\mu + r)}(\pi_m - \pi_d) > 0,$$

so that $W(t + \Delta) - (V_L - \underline{\theta}) < 0$, establishing that firms invest immediately after they learn that their cost is low.

Next, it is easy to check that a firm invests immediately after it learns that the other firm has high cost if and only if

$$\pi_m - V_O = \pi_m - \frac{\mu}{2(\mu + r)}(\pi_m - \underline{\theta}) = V_L - V_F \geq \widetilde{\theta}.$$

Consider the investment game played by the two firms if none of them has invested up to date t and costs are not known as shown in Table 7, where

$$W(t+\Delta) = (1-r\Delta)[(1-2\mu\Delta)W(t) + 2\mu\Delta\frac{V_L - \underline{\theta} + V_F + \max[V_O, \pi_m - \widetilde{\theta}]}{4} + \mathcal{O}(\Delta^2).$$

	invest	not invest
invest	$(\pi_d - \widetilde{\theta}, \pi_d - \widetilde{\theta})$	$(V_L - \widetilde{\theta}, V_F)$
not invest	$(V_F, V_L - \widetilde{\theta})$	$(W(t+\Delta), W(t+\Delta))$

Table 7: Investment game played by the two firms is none of them has invested up to date t and costs are not known.

We first consider a symmetric equilibrium where both firms invest with positive probability $p \in (0,1)$. In that equilibrium,

$$W(t) = p(\pi_d) + (1-p)V_L - \widetilde{\theta},$$

and

$$W(t) = pV_F + (1-p)(W(t) + \delta).$$

Solving this equation, we find

$$p = \frac{V_L - \widetilde{\theta} - V_F}{V_L - \pi_d},$$

showing that an equilibrium with preemption exists if and only if $V_L - V_F \geq \widetilde{\theta}$.

Next, we consider a symmetric equilibrium in the waiting game when $V_L - V_F \leq \widetilde{\theta}$. Notice that, by delaying investment one period, the firm obtains a payoff:

$$W(t+\Delta) = (1-r\Delta)[(1-2\mu\Delta)(V_L - \widetilde{\theta}) + 2\mu\Delta \frac{V_L - \underline{\theta} + V_F + \pi_m - \widetilde{\theta}}{4} + \mathcal{O}(\Delta^2).$$

Next notice that

$$\pi_m - \underline{\theta} > \pi_d - \underline{\theta}, \quad = \quad \frac{2(\mu + r)V_F}{\mu}.$$

Next, as $V_F > V_L - \widetilde{\theta}$,

$$\begin{aligned} \frac{\mu}{2}\pi_m - \underline{\theta} + V_F + V_L - \widetilde{\theta} &> (\mu + r)V_F + \frac{\mu}{2}(V_F + V_L - \widetilde{\theta}), \\ &> (2\mu + r)(V_L - \widetilde{\theta}) \end{aligned}$$

establishing that $W(t+\Delta) > V_L - \widetilde{\theta}$, so that firms always have an incentive to wait.

A.2 Proof of Lemma 1

Suppose that firm i learns that its cost is low at date t. If firm j invests at t, firm i obtains $\pi_d - \underline{\theta}$ by investing at t and $(1 - r\Delta)(\pi_d - \underline{\theta})$ by delaying investment. As $\pi_d - \underline{\theta} > 0$, it has an incentive to invest. If firm j does not invest, and firm i invests at t, it obtains a discounted expected payoff:

$$W(t) = \gamma_t(\overline{\theta})\pi_m + \gamma_t(\widetilde{\theta})V_L + \gamma_t(\underline{\theta})\pi_d - \underline{\theta}.$$

By delaying investment until time $t + \Delta$, the firm will obtain an expected discounted payoff:

$$\begin{aligned} W(t+\Delta) &= e^{-r\Delta}\Bigg(\Bigg[1 - \int_t^{t+\Delta}\left(\int_0^{t+\Delta} g(\rho,\tau)\frac{\mu}{2}e^{-\mu\tau}d\tau + e^{-\mu\rho}h(\rho)\right)d\rho\Bigg] \\ &\quad \Big[\gamma_{t+\Delta}(\overline{\theta})\pi_m + \gamma_{t+\Delta}(\widetilde{\theta})V_L + \gamma_{t+\Delta}(\underline{\theta})\pi_d - \underline{\theta}\Big] \\ &+ \int_t^{t+\Delta}\left[\int_0^{t+\Delta} g(\rho,\tau)\frac{\mu}{2}e^{-\mu\tau}d\tau + e^{-\mu\rho}h(\rho)d\rho\right][\pi_d - \underline{\theta}]\Bigg). \end{aligned}$$

For small Δ, we have:

$$\begin{aligned} W(t+\Delta) &= [1 - r\Delta - \Delta\int_0^t g(t,\tau)\frac{\mu}{2}e^{-\mu\tau}d\tau - e^{-\mu t}h(t)]W(t) \\ &+ [\pi_d\frac{\partial\gamma_t(\underline{\theta})}{\partial t} + \pi_m\frac{\partial\gamma_t(\overline{\theta})}{\partial t} + V_L\frac{\partial\gamma_t(\widetilde{\theta})}{\partial t}]\Delta \\ &+ \Delta[\int_0^t g(t,\tau)\frac{\mu}{2}e^{-\mu\tau}d\tau - e^{-\mu t}h(t)](\pi_d - \underline{\theta}). \end{aligned}$$

We compute:

$$\begin{aligned} \frac{\partial\gamma_t(\underline{\theta})}{\partial t} &= \frac{\frac{\mu}{2}e^{-\mu t} - \int_0^t g(t,\tau)\frac{\mu}{2}e^{-\mu\tau}d\tau}{A(t)} - \frac{A'(t)}{A(t)}\gamma_t(\underline{\theta}), \\ \frac{\partial\gamma_t(\overline{\theta})}{\partial t} &= \frac{\frac{\mu}{2}e^{-\mu t}}{A(t)} - \frac{A'(t)}{A(t)}\gamma_t(\overline{\theta}), \\ \frac{\partial\gamma_t(\widetilde{\theta})}{\partial t} &= \frac{-\mu e^{-\mu^t} - e^{-\mu t}h(t)}{A(t)} - \frac{A'(t)}{A(t)}\gamma_t(\widetilde{\theta}) \end{aligned}$$

Hence

$$\begin{aligned} W(t+\Delta) - W(t) &= -r\Delta W(t) - \Delta \frac{\mu e^{-\mu t}}{A(t)}[V_L - \frac{\pi_m + \pi_d}{2}] \\ &+ \Delta \int_0^t g(t,\tau)\frac{\mu}{2}e^{-\mu\tau}d\tau \frac{W(t) - \pi_d}{A(t)} \\ &+ \Delta h(t) e^{-\mu t}\frac{W(t) - \pi_d}{A(t)}. \end{aligned}$$

Notice that if $h(t) = g(t,\tau) = 0$, $W(t+\Delta) - W(t) < 0$, so it never pays to delay investment if the other firm does not delay investment.

However, if the other firm delays investment ($g(t,\tau) > 0$ or $h(t) > 0$), a firm may benefit from delaying investment, as delaying will enable it to learn the type of the other firm. We now prove that in fact firms will never face a positive incentive to delay investment in order to learn the type of the other firm. To see this, consider now a firm which has not yet learned its cost and contemplates delaying its investment between t and $t+\Delta$. We compute the discounted expected value of leaving at t as:

$$Y(t) = \gamma_t(\overline{\theta})\pi_m + \gamma_t(\widetilde{\theta})V_L + \gamma_t(\underline{\theta})\pi_d - \widetilde{\theta}.$$

and the discounted expected value of leaving at $t+\Delta$ as:

$$\begin{aligned} Y(t+\Delta) &= e^{-r\Delta}([1 - \int_t^{t+\Delta}(\int_0^{t+\Delta} g(\rho,\tau)\frac{\mu}{2}e^{-\mu\tau}d\tau) + e^{-\mu\rho}h(\rho)d\rho] \\ &\quad [e^{-\mu\Delta}(\gamma_{t+\Delta}(\overline{\theta})\pi_m + \gamma_{t+\Delta}(\widetilde{\theta})V_L + \gamma_{t+\Delta}(\underline{\theta})\pi_d - \widetilde{\theta}) \\ &\quad + \frac{1 - e^{-\mu\Delta}}{2}(\gamma_{t+\Delta}(\overline{\theta})\pi_m + \gamma_{t+\Delta}(\widetilde{\theta})V_L + \gamma_{t+\Delta}(\underline{\theta})\pi_d - \underline{\theta})] \\ &+ (\int_t^{t+\Delta}(\int_0^{t+\Delta} g(\rho,\tau)\frac{\mu}{2}e^{-\mu\tau}d\tau) + e^{-\mu\rho}h(\rho)d\rho) \\ &\quad [e^{-\mu\Delta}(\pi_d - \widetilde{\theta}) + \frac{1 - e^{-\mu\Delta}}{2}(\pi_d - \underline{\theta})]). \end{aligned}$$

We compute:

$$
\begin{aligned}
Y(t+\Delta) - Y(t) &= -r\Delta Y(t) - \Delta \frac{\mu e^{-\mu t}}{A(t)}[V_L - \frac{\pi_m + \pi_d}{2}] \\
&+ \Delta \int_0^t g(t,\tau)\frac{\mu}{2}e^{-\mu\tau}d\tau \frac{W(t)-\pi_d}{A(t)} \\
&+ +\Delta h(t) e^{-\mu t}\frac{W(t)-\pi_d}{A(t)} + \Delta \frac{\overline{\theta} - Y(t)}{2}.
\end{aligned}
$$

Hence,

$$
(Y(t+\Delta)-Y(t)) - (W(t+\Delta)-W(t)) = r\Delta(W(t)-Y(t)) + \Delta\frac{\overline{\theta}-Y(t)}{2}.
$$

As $Y(t) < W(t)$ and $\overline{\theta} - Y(t) > 0$, $Y(t+\Delta) - Y(t) > W(t+\Delta) - W(t)$ for all t, Δ. Hence, a firm always has a stronger incentive to wait when it ignores its cost than when it knows that its cost is low. In particular, this implies that whenever $g(t) > 0$ (so that the firm is indifferent between investing at t and $t+\Delta$ when it knows that its cost is low), then a firm must prefer to wait when it ignores its cost.

Suppose by contradiction that $g(t,\tau) > 0$ for some $t, \tau < t$ and let $t^* = \min\{\tau | g(t,\tau) > 0 \text{ for some } \tau < t\}$ be the earliest date at which one of the firms delays its investment. By the previous argument, at t^*, a firm which ignores its cost must prefer to wait so that $h(t^*) = 0$. Furthermore, by construction, $\int_0^{t^*} g(t^*,\tau)\frac{\mu}{2}e^{-\mu\tau}d\tau = 0$. But, as $V_L - \frac{\pi_m+\pi_d}{2} > 0$, this implies that

$$
W(t^*+\Delta) - W(t^*) < 0,
$$

contradicting the fact that a firm which learns its cost at t^* has an incentive to delay its investment.

A.3 Proof of Theorem 1

As in the proof of Proposition 1, we first note that, if $V_L - \widetilde{\theta} \geq V_F$, there exists an equilibrium where both firms preempt with positive probability at time $t = 0$ and at any time $t > 0$. Suppose next that $V_L - \widetilde{\theta} \leq V_F$ and $\pi_m - \widetilde{\theta} \geq V_F$. Then there exists $\tilde{t} > 0$ such that $V_L(\tilde{t}) - \widetilde{\theta} = V_F$. By investing at $t < \tilde{t}$, a firm either obtains $\pi_d - \widetilde{\theta} < V_F$ (if the other firm invests) or $V_L(t) - \widetilde{\theta} < V_F$ (if the other firm does not invest). By investing at time $\tilde{t}$, the firm obtains V_F. Hence it is a dominated strategy to invest at any time $t < \tilde{t}$. At any time $t \geq \tilde{t}$, there is a preemption equilibrium where both firms invest with positive probability $p(t)$ at date t. As t converges to $\tilde{t}$, the loss due to coordination failures converges to zero, so that at $t = \tilde{t}$, as in Fudenberg and Tirole (1985), rent equalization occurs and both firms receive an expected payoff of $V_L(\tilde{t}) = V_F$.

Finally, suppose that $\pi_m - \widetilde{\theta} < V_F$. We show that, any time t, the firm has an incentive to wait. If the firm waits one period before investing it will obtain a payoff of $V_F > \pi_d - \widetilde{\theta}$ if the other firm invests. If the other firm does not invest, it obtains a payoff of

$$W(t) = V_L(t) - \widetilde{\theta} = \gamma_t(\overline{\theta})(\pi_m - \widetilde{\theta}) + \gamma_t(\widetilde{\theta})(V_L - \widetilde{\theta})$$

by investing, and

$$\begin{aligned} W(t+\Delta) &= (1 - r\Delta)W(t) + \Delta h(t)e^{-\mu t}(V_F - W(t)) + \Delta\frac{\mu}{2}(V_F + V_L(t) - \underline{\theta}) \\ &\quad -3\Delta\frac{\mu}{2}W(t) + \Delta\gamma'(t+\Delta)(\overline{\theta})(\pi_m - V_L(t)) \end{aligned}$$

by waiting until $t + \Delta$, for $\Delta \to 0$. Now $V_F > W(t)$ and $\gamma_t'(\overline{\theta}) > 0$. Furthermore,

$$\begin{aligned} V_L(t) -_u th &> V_L - \underline{\theta} \\ &> \pi_d - \underline{\theta} \\ &> \frac{2(\mu + r)}{\mu}V_F. \end{aligned}$$

Hence,

$$\begin{aligned} V_F + V_L(t) - \underline{\theta} &> \frac{3\mu + 2r}{\mu}V_F \\ &> \frac{3\mu + 2r}{\mu}W(t) \end{aligned}$$

establishing that $W(t+\Delta) - W(t) > 0$, so it is profitable for the firm to wait.

A.4 Proof of Proposition 2

In order to implement the cooperative benchmark, two conditions must be satisfied: (i) no firm must be willing to enter the market at $t < t^*$ if it ignores its cost and (ii) a firm which learns that it has a low cost must be willing to enter the market immediately. The first condition will hold as long as:

$$U_F(t) > U_L(t) - \widetilde{\theta}.$$

As $U_L(t) = \pi_m - U_F(t)$, this results in

$$2U_F(t) > \pi_m - \widetilde{\theta}.$$

For the second condition to hold, we characterize the conditions under which an equilibrium where a firm immediately invests after it observes that its cost is low exists. The discounted expected payoff of investing at period t when the other firm does not invest is:

$$W(t) = U_L(t) - \underline{\theta},$$

whereas by waiting one period the firm will obtain a discounted expected payoff of

$$W(t+\Delta) = e^{-r\Delta}[(1 - \frac{e^{-\mu\Delta}}{2})U_F(t+\Delta) + \frac{e^{-\mu\Delta}}{2}U_L(t+\Delta)].$$

For Δ small enough and assuming that utilities are differentiable,

$$W(t+\Delta) - W(t) = \Delta[(-2r-\mu)U_L(t) + \mu U_F(t) + U_L'(t)]$$

so that the firm has an incentive to enter immediately if and only if:

$$2U_F(t) < \frac{2r+\mu}{r+\mu}\pi_m + \frac{U_F'(t)}{r+\mu}.$$

A.5 Derivation of the expected time until the innovation is introduced in a patent race

$$E_{C0}(T) = \int_0^\infty \lambda t e^{-\lambda t}\, dt = \frac{1}{\lambda}$$

$$E_{C1}(T) = 2\int_0^\infty \mu e^{-2\mu\tau} \int_\tau^\infty \lambda t e^{-\lambda(t-\tau)}\, dt\, d\tau = \frac{1}{2}\left(\frac{2}{\lambda} + \frac{1}{\mu}\right)$$

$$\begin{aligned} E_{C2}(T) &= \frac{4}{3}\left[\int_0^\infty \mu e^{-2\mu\tau} \int_\tau^\infty \lambda t e^{-\lambda(t-\tau)} dt d\tau \right. \\ &\quad \left. + \int_0^\infty \mu e^{-\mu\tau} \int_\tau^\infty \frac{\mu}{2} e^{-\mu\rho} \int_\rho^\infty \lambda t e^{-\lambda(t-\rho)} dt d\rho d\tau\right], \\ &= \frac{1}{\lambda} + \frac{5}{6\mu}. \end{aligned}$$

$$\begin{aligned} E_{P0}(T) &= \int_0^\infty \lambda t e^{-\lambda t} \frac{1}{2}\left(1 + e^{-\mu t}\right) dt \\ &\quad + \int_0^\infty \frac{\mu}{2} e^{-\mu\tau} \int_\tau^\infty \lambda t e^{-\lambda t} e^{-\lambda(t-\tau)} dt d\tau, \\ &= \frac{1}{2\lambda} + \frac{\lambda}{2(\lambda+\mu)^2} + \frac{\mu}{8\lambda(\lambda+\mu)} + \frac{\mu}{4(\mu+\lambda)^2} \end{aligned}$$

$$\begin{aligned} E_S(T) &= \frac{4}{3}\left(\int_0^\infty \mu e^{-\mu\tau} \left(\int_\tau^\infty \frac{1}{2}(1 + e^{-\mu t}) \lambda t e^{-\lambda(t-\tau)}\, dt \right.\right. \\ &\quad \left.\left. + \int_\tau^\infty \frac{\mu}{2} e^{-\mu\rho} \int_\rho^\infty \lambda t e^{-\lambda(t-\rho)} e^{-\lambda(t-\tau)}\, dt\, d\rho\right)\, d\tau\right) \\ &= \frac{1}{12}\left(\frac{9}{\lambda} + \frac{10}{\mu} + \frac{2\lambda}{(\lambda+\mu)^2}\right) \end{aligned}$$