ISSN 1178-2293 (Online)

University of Otago
Economics Discussion Papers
No. 1303

February 2013

On Real Interest Rate Persistence: The Role of Breaks

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On Real Interest Rate Persistence: The Role of Breaks

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This version: February 2013

Abstract

The role of structural breaks in long spans of ex-post real interest rates for ten industrialized countries is studied. First, the persistence of the real interest is assessed with newly proposed low-frequency tests of Müller and Watson (2008). Second, the test of Leybourne *et al.* (2007) for a change in persistence of a time-series is applied to the real interest rate. The results show that real interest rates over the full sample period do not fit a covariance-stationary or unit-root model, nor a fractionally-integrated, near-unit-root or local-level model. Instead, the persistence of real rates changes over time and there are periods when the real rate is covariance stationary and other periods when it follows a unit root process.

JEL Classification: E43, C22

Keywords: Real interest rates, persistence of a time series, breaks in persistence.

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1. Introduction

The real interest rate is a key variable in theoretical models in macroeconomics and finance. The properties of these models generally depend on the time series behavior of the real interest rate (e.g., Neely and Rappach, 2008). However, there is an ongoing controversy in the literature about the time series behavior of the real rate of interest, especially in the long run.

Neely and Rappach (2008) surveyed the literature on the long-run persistence of real interest rates.[1] Empirical research points to considerable persistence of real rates but the form this persistence should take is in dispute. Some studies claim empirical evidence favoring for real interest rates: a unit root process (e.g., Rose, 1988; and Mishkin, 1992), a fractionally integrated process (e.g., Phillips, 1998; Tsay, 2000; Sun and Phillips, 2004; and Karanasos *et al.*, 2006), a non-linear process (e.g., Million, 2004; and Koustas and Lamarche, 2010), or a mean-reverting covariance-stationary process with structural breaks. Structural breaks in the mean of the real interest rate could lead to incorrect inference as to whether a time series is integrated of order one, denoted I(1), or equivalently has a unit root, or instead is I(0), i.e., is covariance stationary (Bai and Perron, 2003).

In order to account for structural breaks, Garcia and Perron (1996) used a Markov-switching model with three possible regimes for U.S. real interest rates. Neely and Rapach (2008) criticized applying the Markov-switching model in the context of breaks because it generally assumes ergodicity. This means that the current state will eventually revert back to a previous state, which is normally not happening with structural breaks in real rates.[2] The break test applied by Celemente *et al.* (1998) allows for standard endogenous breaks, but is limited to two such breaks. On the other hand, Caporale and Grier's (2000) and Bai and Perron's (2003) tests allow for multiple breaks in real rates but require the real rate to be I(0). Rapach and Wohar (2005) also applied Bai and Perron's (1998, 2003) break testing methodology to real rates of 13 industrialized countries from 1960:4 to 1998:3, requiring real interest rates to be I(0). In contrast, Lai (2008) tested for breaks in the possible presence of unit

[1] There is a related literature that studies the relationship between nominal interest rates and inflation, based on the Fisher hypothesis (e.g., Haug *et al.*, 2011). I do not pursue this line of research here.

[2] See also Bai and Perron (2003, fn. 15, p. 17).

roots in real rates, tying in with Clemente *et al.* by not assuming that real rates are I(0).[3] The longstanding debate since Rose (1988) on whether real interest rates are I(0) or I(1) make accounting for breaks an important issue. The literature on real interest rates so far has not applied a break testing methodology that allows for multiple switches (breaks) from I(0) to I(1) regimes and vice versa, at unknown dates. This paper tries to fill that gap.

I use a unique data set, starting in 1880, for ex-post long-term real interest rates for ten industrialized countries.[4] Long spans of annual data generally lead to more powerful tests in the possible presence of unit roots than shorter spans with a higher frequency of observation (Haug, 2002). By contrast, all the above studies on structural breaks in real interest rates used quarterly data starting in 1960 or later.

I argue in this paper that multiple regime shifts in the persistence of real interest rates from I(0) to I(1) and vice versa explain their behavior. First, I examine the low frequency (long-run) properties of real interest rates and show that their behavior is not consistent with various alternative time series models, some of which were previously considered in the literature: I(0), I(1), fractional integration, and a near-unit-root or local-level model. For this purpose, I use the recently proposed low-frequency tests of Müller and Watson (2008). Second, I apply the test of Leybourne *et al.* (2007) for multiple changes in the persistence of a time series at unknown dates in order to identify the various I(0) and I(1) regimes in the sample. Third, I re-apply the tests of Müller and Watson (2008) to the various I(0) and I(1) regimes identified in order to check that they have persistence profiles consistent with an I(0) or I(1) process.

In contrast to previous break tests applied to real rates, the test of Leybourne *et al.* (2007) allows for multiple changes in persistence of the real interest rate, be it from I(0) to I(1) or from I(1) to I(0). Its advantage over other break tests is that it does not require using several different break tests. Testing for a switch from an I(0) regime to an I(1) regime means applying a break test to an I(0) series. However, testing for a switch from an I(1) regime to an I(0) regime means, generally, applying

[3]Lai considered only one endogenous break but studied eight industrialized and eight developing countries.

[4]Fisher (1930, p. 43) argued that the one-for-one relationship of nominal interest rates and (expected) inflation is a long-run process so that long-term real interest rates over long spans would seem appropriate, though I do not directly test the Fisher hypothesis in this paper.

a different break test to an I(1) regime. The reason is that the distribution of a break test is usually different, depending on whether it is applied under the null hypothesis to an I(0) or an I(1) time series (Leybourne *et al.*, 2007). Furthermore, in the case of possibly multiple breaks, testing methodologies require in general a pre-test in order to determine whether there are any breaks at all in a time series (Carrion-i-Silvestre *et al.*, 2009; Perron and Yabu, 2009). On the other hand, the test proposed by Leybourne *et al.* (2007) is applicable to testing for regime changes from I(0) to I(1) regimes and also when regime changes occur from I(1) to I(0) regimes. In addition, the methodology of Leybourne *et al.* (2007) produces consistent estimates of the dates for the regime switches.

2. Testing for structural breaks

Perron's (1989) seminal paper showed the importance of accounting for structural breaks in order to avoid finding a spurious unit root when a time series is a stationary process around a broken deterministic trend line. Leybourne *et al.* (1998) showed that a break can also have the reverse effect when it occurs early in the sample period for an I(1) series with a break: standard unit root tests incorrectly reject a unit root in favor of a mean-reverting I(0) process if the break is ignored. Furthermore, using a break test designed for known breaks in order to search for breaks generally changes the limiting distribution of the test (Carrion-i-Silvestre *et al.*, 2009). On the other hand, imposing candidate break dates for a Chow-type break test can lead to finding spurious breaks (Hansen, 2001). Therefore, break dates should be treated as unknown.

Finally, it is crucial how many breaks a break test allows for. When the true data generating process has multiple breaks, a test that allows only for one break may incorrectly lead to a finding of no structural change. Bai and Perron (2006) demonstrated that multiple breaks lead to low powers of tests for a single break.

Leybourne *et al.* (2007) developed a new procedure that allows sequential testing for multiple changes in the persistence of a time series. Their procedure consistently determines multiple changes from I(0) to I(1) regimes and vice versa. It allows also for the consistent estimation of the break dates. I apply this procedure to determine the breaks in the real interest rate series and the type of regime, I(0) or I(1), that the real interest rate follows in a given time period.

The test of Leybourne *et al.* (2007) is based on the Dickey-Fuller unit root test with local generalized-least-squares demeaning or detrending (*DF-GLS*), suggested by Elliott *et al.* (1996).[5] The test statistic M minimizes the doubly-recursive sequence of the *DF-GLS* statistics for sample observations between λT and τT, where T is the sample size, with $\lambda \in (0,1)$ and $\tau \in (\lambda, 1)$:

$$M \equiv \inf_{\lambda \in (0,1)} \inf_{\tau \in (\lambda,1]} DF\text{-}GLS(\lambda, \tau).$$

The associated local break point estimates are $\widehat{\lambda}$ and $\widehat{\tau}$.

The null hypothesis is that the time series is I(1) throughout the sample. The alternative hypothesis is that there are one or more I(0) regimes in the sample, i.e., there is at least one regime shift between I(1) and I(0). First, the most prominent I(0) regime in the sample is tested for, followed subsequently by reapplication of the test to sub-samples, if a break is found in the first round.

Leybourne *et al.* (2007) derived the limiting distribution of the test and proved test consistency. They did the same for the change-point estimator of the break dates. Furthermore, they provided critical values for the test for finite sample applications and showed in Monte Carlo simulations that the test has good power and size properties in various data generating processes for finite samples. Moreover, Leybourne *et al.* pointed out that other break tests cannot be used in general to consistently separate I(0) from I(1) regimes. Even in the case of a single break only, different test and break-point estimators would be required depending on whether the change is from an I(0) to an I(1) regime or from an I(1) to an I(0) regime. The M-test overcomes this problem with the doubly-recursive process.

One limitation, that is unavoidable when testing for breaks, is that the presence of multiple breaks in a sample may eventually lead in the sequential application of the test to sub-samples that become too small for further reliable inference. I follow Leybourne *et al.* and generally set the minimum sample size according to $\tau T = \lambda T + 0.2T$. This leads, in my application, to sub-samples that are large enough

[5] I apply the demeaned version and set $\bar{c} = -7$, following Elliott *et al.* Also, the lag order for the test is chosen with sequential t-tests and a 10% level of significance, following Ng and Perron (1995).

for the majority of cases. However, for a few cases where this poses a problem, I will use instead a higher value than 0.2 and, in addition, resort to quarterly post-WWII data to make the analysis feasible and reliable, depending on the country in question.

3. Empirical Analysis

3.1 Data

The annual data on the consumer price index (CPI) and long-term interest rates from 1880 to 2001 are from Dewald (2003) and were kindly provided by the author. His data appendix (pp. 52-58) provides details on the data sources. The historical price index used for constructing inflation rates is mostly the CPI, and the historical nominal long-term interest rate is mostly the long-term (10 year) government bond yield. I updated the series with corresponding data from the IMF's online International Financial Statistics to 2006. The post-WWII quarterly data are from the same IMF source.

The endpoint of 2006 was chosen so that data from the recent global financial crisis are not included. The period beyond 2006 is marked by extreme events that lead to unconventional monetary and fiscal policy reactions in most of the countries considered in this study. Also, the available observation beyond 2006 would be too few in order for the test used to detect a break, say in 2007, because truncation at the sample endpoints is necessary in order to implement the testing.

The ex-post real interest rate is the nominal interest rate less the ex-post inflation rate. Allowing for the construction of the inflation rate, as the first difference of natural logarithms of the CPI multiplied by 100, means that this series starts in 1881 and not in 1880, as does the real interest rates series. The ten industrialized countries covered in my analysis are listed in Table 1.

3.2 Assessing the persistence of real interest rates

In order to assess the persistence profiles of the real interest rate series of the ten countries, I use the low-frequency analysis developed by Müller and Watson (2008). Their methodology considers various alternative models for time series: the I(0), I(1), fractionally integrated, near-unit-root (local-to-unity), and local-level specifications. The behavior at low frequencies, below the business cycle frequency, i.e., at cycles longer than 32 quarters, characterizes these alternative specifications. I apply the $LFST$-, $LFUR$-, S-, and H-tests in order to test whether real interest rate behavior is consistent with an I(0) or I(1) specification over the full sample period. I also estimate 95% confidence bands for the parameters that describe a fractional, near-unit-root and local-level model for the S- and H-tests. These allow me to assess the suitability of such specifications for the real interest rate time-series process.

The $LFST$-test is a low frequency test with the null hypothesis of an I(0) process that maximizes power against a point-alternative hypothesis of a local-level model. The local-level model consists of an I(1) component with permanent effects and a noise component with temporary effects. The weight of the I(1) component is denoted by g. The local-level model is non-stationary.

The $LFUR$-test is also a low-frequency test but with the null hypothesis of a unit root that maximizes power against a point-alternative hypothesis of a local-to-unity or near-unit-root model, with local-to-unity parameter c. Following Müller and Watson (2008), I set g=10 and c=14 so that a 5% level test has approximately 50% power at the alternative for which it is optimal.

The S- and H-tests are designed to test for misspecified persistence and misspecified low-frequency heteroscedasticity. I apply these tests to the I(0) and I(1) models of the real interest rate. Again, the tests are set up so that a 5% level test maximizes power at 50% at the alternative for which it is optimal.

Table 1 reports results for the above tests for the period 1881 to 2006 for the ten countries. The $LFST$-test does not reject the I(0) specification for all countries, except for France ($p = 0.02$).[6] According to the S-test, the I(0) model seems to capture the low-frequency persistence well, except again for France ($p = 0.00$). Canada passes the H-test but barely so ($p = 0.08$). On the other hand, the real interest rate exhibits too much low-frequency heteroscedasticity in order to be consistent with the behavior of an I(0) model for the other nine countries, based on the H-test. The H-test rejects the null hypothesis of no excessive heteroscedasticity for all countries, except Canada. Therefore, an I(0) specification is only supported for Canada, with the H-test being somewhat of a borderline case.

[6]I use a 5% level of significance throughout the paper.

The $LFUR$-test strongly rejects the I(1) model for the real interest rates of all countries. The S- and H-tests for the I(1) model lead to the same result, except for the S-test for France that is a borderline case ($p = 0.07$). Overall, the I(0) and I(1) specifications do not fit the data for real interest rates for nine of the ten countries. An I(0) specification for Canada may be an acceptable description for real interest rate behavior, if the I(0) process is stable over time.

Next, I explore whether a fractionally integrated, a near-unit-root or a local-level model can possibly provide a better approximation to the time series behavior of real interest rates over the long span, as compared to an I(0) or I(1) model. Table 2 reports the 95% confidence intervals for the parameters d (for fractional integration), c (for near-unit roots) and g (for local-levels) calculated from inverted S- and H-tests.

The inverted S-test confidence band for the fractional model includes zero ($d = 0$) for all countries except for France, where the I(1) specification ($d = 1$) is within the confidence band. There is less persistence in real interest rates for countries other than France. However, the fractional model does not fare well for the inverted H-test confidence bands. An outright rejection of the fractional model occurs for France, Italy and Switzerland. The confidence bands for all countries, except for Canada, do not include the I(0) model ($d = 0$). Worse still, the overlap of the confidence bands for the inverted S- and H-tests is an empty set, except for Canada ($-0.04 \leq d \leq 0.08$) and Sweden ($-0.50 \leq d \leq -0.38$). As argued before, Canadian data seem to fit the I(0) model ($d = 0$) and Swedish data may fit a stationary fractional model with a negative value of d.

The near-unit-root model is soundly rejected based on the inverted H-test confidence bands. The same holds true for the local-level model, except for Canadian data where zero ($g = 0$) is included in the interval, which is the I(0) case. The inverted S-test results are largely consistent with these two models, except for three countries where the near-unit-root model is rejected outright. However, the inverted S-test and H-test confidence bands taken together only support a local-level model for Canada, whereas a near-unit root model is rejected. To summarize, Table 2 supports an I(0) model for Canada (when $g = 0$) and possibly a fractionally integrated model for Sweden. However, it may be that the result in favor of a fractional model is caused by structural breaks. I explore this issue in the next section.

3.3 Testing for structural change in the persistence of real interest rates

The M-test of Leybourne *et al* (2007) for multiple structural changes in the persistence of a time series is applied to the real ex-post long-term interest rate of each of the ten industrialized countries. I start with the full sample from 1881 to 2006. If the null hypothesis of an I(1) regime throughout the sample cannot be rejected at the 5% level of significance, then I conclude that there are no significant breaks in the persistence of the real interest rate. On the other hand, if the null hypothesis is rejected, the dates of the I(0) regime are estimated. For the subperiod(s) outside the I(0) regime, I repeat the application of the M-test. If the null hypothesis cannot be rejected, there is no change in persistence detected by the test. If the null hypothesis is rejected, dates for the I(0) regime are determined. The M-test is again applied to the subperiod(s) outside the I(0) regime, and so on. The test results are reported in Table 3. Whenever the sample size becomes too small, no test is carried out and "na" is reported in Table 3.[7] If the sample with insufficient observations falls in the post-WWII period, I use quarterly data in those cases to calculate the M-test. The last two columns of Table 3 list quarterly results for sub-samples adjacent to detected I(0) regimes.

It is evident from the results in Table 3 that there are several regime changes for each country. Also, countries differ in the pattern of how their real interest rates change persistence over time. However, one result that stands out is that the most recent period from the early 1980s or mid-1980s to 2006 is characterized by an I(1) regime for all countries except for the Netherlands from 2001Q1 to 2005Q3 when it is an I(0) regime and for Switzerland that follows an I(0) regime. The I(1) regime in Italy and Denmark started already in 1920 and 1971, respectively, and not in the 1980s as for the other countries. The change-over from an I(0) to an I(1) regime in those other countries, including the UK and USA, coincides with the change in monetary policy in the USA for the period from 1979 to 1982, as documented for example by Sims and Zha (2006), among others. This I(1) regime preceded the global financial crisis of 2007-08. It is interesting to note that Switzerland is not part of the post-1980s I(1)

[7] The parameter $\tau = 0.20$, except for the two smallest samples when $T = 29$ for Sweden for the period 1922-1950 and $T = 36$ for Denmark for the period 1971-2006, for which $\tau = 0.35$ and 0.30.

group. Switzerland, one of the safe-haven countries during the recent financial and sovereign debt crises, has had an I(0) regime from 1940 to 2006 for the real interest rate. This is likely a reflection of the monetary policy of the Swiss National Bank.

The period after WWII till the early- to mid-1980s is more or less a period with I(0) regimes in most countries, except for Italy that has an I(1) regime. The I(0) regime in Norway and the UK started already before WWII in 1902 and 1933, respectively. This likely reflects the monetary policy under the post-WWII Bretton Woods fixed exchange rate system that ended in 1973, followed by an adjustment period of monetary policies in the aftermath of the oil price shocks in the 1970s that lead to the above I(1) regimes from the 1980s onwards.

The pre-WWI period is characterized by the gold standard and the interwar period between WWI and WWII saw countries abandoning the gold standard after WWI, followed by unsuccessful attempts to re-introduce it in some form in the period 1925 to 1931.[8] The period from the 1880s to the start of WWII is a period with mixed regimes across countries. An I(1) regime prevailed in Denmark, the Netherlands, the UK and the USA. On the other hand, Canada, France, Italy, Norway and Switzerland faced I(0) regimes (with a delayed start in 1902 for Norway). Sweden switched in 1922 from an I(0) regime to and I(1) regime.

As a robustness check for the I(0) and I(1)regimes uncovered in Table 3, I applied the low-frequency tests of Müller and Watson (2008) to the sample period of each regime, as long as the sample size was not too small. The results, not reported to conserve space, generally support the detected I(0) and I(1) specifications in Table 3, with only a few borderline cases for the H-test. This means that it is unnecessary to resort to fractionally integrated, non-linear or other models referred to in the introduction. Such models may incorrectly approximate breaks with, for example, non-linear forms or a higher order of integration.

In order to determine whether the changes in persistence of the ex-post real interest rates is likely due to changes in monetary policy, I applied the M-test to inflation rates and nominal interest rates, following Rapach and Wohar (2005). The results, not reported, clearly show that the breaks mostly align with breaks in the inflation rates and much less so with breaks in nominal interest rates, which often do

[8]See, for example, Mitchener and Weidenmier (2010).

not show breaks in persistence. This result gives an indication that likely monetary policy regime changes are responsible for the breaks in real interest rate, though other explanations are possible. Also, this result is consistent with the findings of Rapach and Wohar (2005), as far as the cause of breaks in real rates is concerned.

4. Conclusion

This paper contributes to the empirical literature on ex-post long-term real interest rates by applying new tests in order to identify the persistence behavior and multiple breaks in persistence for real interest rates. Real interest rates for ten industrialized countries over a long time span starting in 1881 are analyzed. The low frequency (long-run) tests of Müller and Watson (2008) reveal that real interest rates over the full sample period have persistence profiles that are in general not consistent with I(0), I(1), fractionally integrated, near-unit root or local-level models.

It is possible that changes in monetary or fiscal policies over time lead to structural break in the behavior of real interest rates. Therefore, I applied break test of Leybourne *et al* (2007) for unknown break dates. In contrast to previously used break tests, their M-test allows for multiple changes in the persistence of a time series, from I(0) to I(1) regimes and also from I(1) to I(0) regimes. The test results show that real rates in all countries are affected by breaks in persistence.

The findings in this paper are that the period since the 1980s is generally well described by real long-term interest rates that follow an I(1) regime. The post-WWII to early 1980s period is dominated by I(0) regimes for real rates across the ten countries. The pre-WWII period shows a mixed pattern of I(0) and I(1) regimes across countries. Reapplying the tests of Müller and Watson (2008) lends support to the uncovered I(0) and I(1) regimes because the persistence profiles of these regimes are generally consistent with the detected I(0) and I(1) behavior.

The empirical results demonstrate that real long-term interest rates change persistence over time. Real interest rates are a crucial determinant of investment, savings and intertemporal economic decisions. The findings in this paper provide empirical evidence on the properties of long-term real interest rates, which has implications for theoretical macroeconomic modelling.

Acknowledgements

Most of the research for this paper was carried out while the author was visiting the Econometrics Institute at the Warsaw School of Economics in Warsaw, Poland. The author wishes to thank the Institute for its kind hospitality and seminar participants at the Institute for their helpful comments.

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Table 1
Low-frequency tests of Müller and Watson (2008) for I(0) and I(1) models

Country	*LFST*-test	*S*-test for I(0) model	*H*-test for I(0) model	*LFUR*-test	*S*-test for I(1) model	*H*-test for I(1) model
Canada	0.53	1.86	1.62	84.86	158.74	100.78
	(0.26)	(0.08)	(0.08)	(0.00)	(0.00)	(0.00)
Denmark	0.45	0.44	19.13	78.60	11.86	530.45
	(0.32)	(0.49)	(0.00)	(0.00)	(0.01)	(0.00)
France	5.46	16.38	287.04	37.50	2.05	368.92
	(0.02)	(0.00)	(0.00)	(0.00)	(0.07)	(0.00)
Italy	0.61	0.55	586.54	67.57	6.14	996.08
	(0.23)	(0.36)	(0.00)	(0.00)	(0.02)	(0.00)
Netherlands	1.01	0.72	5.38	75.22	22.33	260.98
	(0.13)	(0.26)	(0.02)	(0.00)	(0.00)	(0.00)
Norway	0.40	0.91	28.41	72.80	6.03	143.56
	(0.34)	(0.20)	(0.00)	(0.00)	(0.02)	(0.00)
Sweden	0.24	0.18	176.51	115.24	255.17	703.07
	(0.57)	(0.97)	(0.00)	(0.00)	(0.00)	(0.00)
Switzerland	0.33	0.23	710.34	100.22	119.38	149.07
	(0.43)	(0.89)	(0.00)	(0.00)	(0.00)	(0.00)
UK	0.28	0.31	76.16	86.32	11.81	296.63
	(0.51)	(0.69)	(0.00)	(0.00)	(0.01)	(0.00)
USA	0.52	0.84	17.17	73.33	24.18	422.07
	(0.27)	(0.22)	(0.00)	(0.00)	(0.00)	(0.00)

Note: The values in parentheses are *p*-values. A recorded value of 0.00 means a value below 0.005.

Table 2
95% confidence bands for model parameters calculated from inverted *S*- and *H*-tests of Müller and Watson (2008) for fractional (d), near–unit-root (c) and local-level (g) models

Country	Factional *S*-test (d)	Fractional *H*-test (d)	Near-unit-root *S*-test (c)	Near-unit-root *H*-test (c)	Local-level *S*-test (g)	Local-level *H*-test (g)
Canada	-0.04, 0.44	-0.50, 0.08	--	--	0.0, 30.0	0.0, 0.8
Denmark	-0.20, 0.78	-0.50, -0.34	16.5, 30.0	--	0.0, 30.0	--
France	0.16, 1.04	--	0.0, 30.0	--	10.0, 30.0	--
Italy	-0.14, 0.86	--	11.0, 30.0	--	0.0, 30.0	--
Netherlands	-0.12, 0.68	-0.50, -0.16	24.0, 30.0	--	0.0, 30.0	--
Norway	-0.14, 0.84	-0.50, -0.46	13.0, 30.0	--	0.0, 30.0	--
Sweden	-0.50, 0.48	-0.50, -0.38	--	--	0.0, 30.0	--
Switzerland	-0.42, 0.48	--	--	--	0.0, 30.0	--
UK	-0.28, 0.76	-0.50, -0.44	19.5, 50.0	--	0.0, 30.0	--
USA	-0.14, 0.66	-0.50, -0.28	26.5, 30,0	--	0.0, 30.0	--

Table 3
Testing for structural breaks in the persistence of real interest rates: detected I(0) and I(1) regimes and time periods

Country	Annual data							Quarterly data	
Canada	na 1881-1895	I(0) 1896-1922	na 1923-1947	I(0) 1948-1965	na 1966-1975	I(0) 1976-1987	na 1988-2006	I(0) 1971Q2-1982Q1	I(1) 1982Q2-2006Q4
Denmark	I(1) 1881-1938	I(0) 1939-1970	I(1) 1971-2006						
France	I(0) 1881-1938	na 1939-1953	I(0) 1954-1980	na 1981-2006				I(1) 1981Q1-2006Q4	
Italy	na 1881-1882	I(0) 1883-1919	I(1) 1920-2006						
Netherlands	I(1) 1881-1944	I(0) 1945-1984	na 1985-2006					I(1) 1985Q1-2000Q4	I(0) 2001Q1-2005Q3
Norway	na 1881-1901	I(0) 1902-1981	na 1982-2006					I(1) 1982Q1-2006Q4	
Sweden	na 1881-1883	I(0) 1884-1921	I(1) 1922-1950	I(0) 1951-1981	na 1982-2006			I(1) 1982Q1-2006Q4	
Switzerland	I(1) 1881-1909	I(0) 1910-1923	na 1924-1939	I(0) 1940-2006					
UK	I(1) 1881-1930	I(0) 1931-1984	na 1985-2006					I(1) 1985Q1-2006Q4	
USA	I(1) 1881-1943	I(0) 1944-1982	na 1983-2006					I(1) 1983Q1-2006Q4	

Note: Results are based on a 5% significance level for the *M*-test of Leybourne *et al.* (2007). An entry of "na" means that the sample size was too small to carry out the test.