Propagation of an Ice Shelf Water Plume beneath Sea Ice in McMurdo Sound, Antarctica

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Abstract

A cold water mass, termed Ice Shelf Water, appears to exist for much of the year beneath the sea ice cover in western McMurdo Sound, Antarctica, yet it is present for only a few months in the east. In an east–west transect taken 3 km in front of the McMurdo Ice Shelf edge during spring tide in late November 2011 and repeated during neap tide in early December, this water mass was observed throughout the entire water column at the two, of four, westernmost sites. In situ supercooling was observed at all sites and, at the coldest site, was measured to depths of 60–73 m. Ice Shelf Water alters the sea ice fabric through the introduction of millimetre-sized ice crystals, termed frazil ice, that grow in supercooled water. Four first-year sea ice cores from the transect are analysed to determine the extent of the altered sea ice fabric, platelet ice, to provide a time-history of oceanographic conditions during the 2011 austral winter. The onset of platelet ice is delayed to greater depths in the core with distance eastward along the transect, which suggests that the lateral extent of Ice Shelf Water flowing into McMurdo Sound from beneath the ice shelf expands from the west throughout winter.

A steady-state, one-dimensional Ice Shelf Water plume model is adapted for McMurdo Sound to predict the evolution of this supercooled water emerging from beneath the McMurdo Ice Shelf at the site where the coldest water was observed. A third oceanographic transect following the likely direction of this supercooled water provides initial model conditions and, in conjunction with historical data, downstream validation. Application of the plume model under sea ice is reliant on the addition of an ambient current, which moves parallel to the plume and accounts for currents that are not driven by thermohaline processes within the ice shelf cavity.

The RMS tidal velocity, the ambient current velocity, the drag coefficient and parameters affecting the nucleation of frazil ice each affect the size distribution of suspended frazil ice crystals. These parameters are the key physical controls on the survival of in situ supercooled water as it travels northwards away from the ice shelf. This survival is predicted from the average of 26 different model runs along the approximately 250 km path between the McMurdo Ice Shelf and the Drygalski Ice Tongue. Starting at 65 m, the thickness of the in situ supercooled layer beneath the ice–ocean interface decreases to 11 ± 6 m and 4 ± 3 m at distances from the ice shelf of 100 km and 200 km, respectively.
First off I would like to thank my supervisor Pat Langhorne for giving me the opportunity to do field work in Antarctica, which is a place I had always wanted to go and won’t ever forget. (I’m not normally one to take many photos, yet somehow I came back with several hundred.) Also, my work greatly improved thanks to your continual constructive feedback on anything from which study to look at to answer one of my questions, how to best frame my results, or what grammar and punctuation to use to best get the point across.

Thanks to my second supervisor, Greg Leonard. Field work in McMurdo Sound was a whole lot easier thanks to your expertise. I’m still confused as to how you always immediately knew which direction to park the Hägglund so that it pointed into the wind. Or is that away from the wind? I’m sure my thesis would have been missing some vital oceanographic data had it not been for your on-site checks making sure the CTD profiler had worked properly. Thanks for your prompt feedback on my drafts and always making the long walk right across campus so that we could have a meeting and bounce ideas around.

Pat, Greg and I were joined on the ice by six further people in 2011 for Event K063. Thanks to Dan Price, Wolfgang Rack, Christian Haas, Kelvin Barnsdale, Justin Beckers and Alex Gough not only for a great trip, but also for each playing a role in taking the ice thickness measurements that I use in Chapter 4. These measurements were very helpful when it came to understanding and explaining my oceanographic data and modelling predictions.

The topic of thesis wasn’t set in stone at the start and I want to thank Inga Smith and Alex Gough from Otago and Mike Williams, Craig Stevens and Natalie Robinson from NIWA for all of your ideas right back at the start when I trying to figure out what my project was. My hard work near the end was made easier by having someone else in my office who was just as committed, so thanks for that Pat Wongpan.

To my family and friends, thanks for taking an interest wherever possible, despite it being likely that you are in no way affected by, say, the finer points of the physics of ice–ocean interactions.

I was lucky enough to receive both the University of Otago Master’s Scholarship and the Kelly Tarlton’s Antarctic Scholarship (part of the Antarctica New Zealand Scholarship programme). Not having to worry about money along the way allowed
me to just get on with the work that I wanted to do.

Fortunately, I didn’t have to start from scratch when it came to my modelling study. Greg passed on to me the original code for Lars Smedsrud and Adrian Jenkins’s plume model. Although I’ve not met these two people I owe them a lot, for their well-commented MATLAB code saved me what would have been several weeks at least trying to get my head around the model.

Finally, thanks to my girlfriend Erin for all of your support throughout my whole project. You were always happy to listen to my boring stories of, say, finding a typo in my code and then fixing it. And thanks for always meeting me for lunch to help break up the day.
Figure i – The author enjoying one of many sunny days on the ice, and one of many chances to use a very large drill. Photo: Pat Langhorne.
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Nomenclature

Table i – Parameters and symbols. Units may differ in presentation of data.

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Nomenclature

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<td>$\Theta'$</td>
<td>dimensionless</td>
<td></td>
</tr>
<tr>
<td>Mass diffusivity of salt in seawater</td>
<td>$\kappa_s$</td>
<td>m$^2$s$^{-1}$</td>
<td>$8.0 \times 10^{-10}$</td>
</tr>
<tr>
<td>Thermal diffusivity of seawater</td>
<td>$\kappa_T$</td>
<td>m$^2$s$^{-1}$</td>
<td>$1.4 \times 10^{-7}$</td>
</tr>
<tr>
<td>Kinematic viscosity/momentum diffusivity</td>
<td>$\nu$</td>
<td>m$^2$s$^{-1}$</td>
<td>$1.95 \times 10^{-6}$</td>
</tr>
<tr>
<td>Density of plume</td>
<td>$\rho$</td>
<td>kg m$^{-3}$</td>
<td></td>
</tr>
<tr>
<td>Density of ambient seawater</td>
<td>$\rho_a$</td>
<td>kg m$^{-3}$</td>
<td></td>
</tr>
<tr>
<td>Density of ice</td>
<td>$\rho_i$</td>
<td>kg m$^{-3}$</td>
<td></td>
</tr>
<tr>
<td>Density of seawater</td>
<td>$\rho_w$</td>
<td>kg m$^{-3}$</td>
<td></td>
</tr>
<tr>
<td>Shear stress</td>
<td>$\tau$</td>
<td>Pa</td>
<td></td>
</tr>
</tbody>
</table>

**Table ii – Acronyms.**

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>AABW</td>
<td>Antarctic Bottom Water</td>
</tr>
<tr>
<td>AASW</td>
<td>Antarctic Surface Water</td>
</tr>
<tr>
<td>CDW</td>
<td>Circumpolar Deep Water</td>
</tr>
<tr>
<td>DISW</td>
<td>Deep Ice Shelf Water</td>
</tr>
<tr>
<td>HSSW</td>
<td>High-Salinity Shelf Water</td>
</tr>
<tr>
<td>ISW</td>
<td>Ice Shelf Water</td>
</tr>
<tr>
<td>MCDW</td>
<td>Modified Circumpolar Deep Water</td>
</tr>
<tr>
<td>SISW</td>
<td>Shallow Ice Shelf Water</td>
</tr>
</tbody>
</table>
1.1 Sea Ice

Sea ice, a thin, frozen layer floating atop the polar oceans, exhibits a massive variation in extent in response to the seasonal variation in temperature. During winter, sea ice effectively doubles the size of the Antarctic continent (Dodds, 2012). Its growth is largely driven by a transfer of heat out of the ocean, to the atmosphere. However, around Antarctica a number of different mechanisms may occur: precipitation of small ice crystals suspended in the underlying ocean, heat flux to the ocean below and solidification of a surface snow layer. These each have varying levels of importance depending on the location.

Despite being only metres thick at most, sea ice provides a very influential barrier between ocean and atmosphere. An ice cover reflects far more sunlight relative to an uncovered ocean due to its higher albedo and so a positive feedback results, whereby an increasing sea ice extent leads to a cooler ocean. Sea ice also reduces the wind-driven momentum transfer at the ocean surface.

Sea ice growth and decay both redistribute heat and salt around the ocean. The bulk salinity of sea ice is a small fraction of that of seawater, as salt ions are not easily incorporated into the molecular structure of ice. Consequently, ice growth leads to the creation of highly saline brine at the sea ice base. This brine is denser than the underlying seawater, allowing it to sink to depths in the ocean that would otherwise be unaffected by surface mixing processes. Later in the year, landfast sea ice may break up and be transported away by winds and surface currents, providing a source of relatively fresh water at a different location.

Current sea ice research spans a range of scales, as demonstrated in the following examples. At a small scale, understanding of turbulence and heat and mass transfer at the ice–ocean interface is needed for better estimates of sea ice thickness (e.g., McPhee et al., 2008), which is a difficult property to measure by remote methods (Haas, 2003). In a field environment, accurate in situ measurements of mechanical and thermodynamical properties of sea ice, and how these differ under various conditions and throughout the year, are needed to allow current climate models to make more accurate predictions (e.g., Pringle et al., 2007; Timco and Weeks, 2010). At a planetary scale, satellite measurement of sea ice extent helps monitor current climatic conditions.
and interannual changes (e.g., Liu et al., 2004).

1.2 Ice Shelves

Ice shelves are the floating extension, often hundreds to thousands of metres thick, of the grounded Antarctic ice sheets. Ice shelves are common in Antarctica and account for 11% of the ice sheet area (Fox and Cooper, 1994) or 44% of the coastline (Drewry et al., 1982) (see Figure 1.1). Ice sheets form over millennial time scales by compaction of snow with the result being a combination of pure ice and air bubbles. After passing the grounding line* and becoming afloat, these massive ice volumes can accelerate to speeds of up to a few kilometres per year (Rignot et al., 2011), as well as becoming subject to oceanic processes that can cause basal accumulation and/or ablation.

An ice shelf affects the ocean in many ways. First, it provides a physical barrier with a depth scale similar to the depth of surface ocean currents. Consequently, the effects of wind-driven currents and waves are greatly reduced or removed within an ice shelf cavity and the advection of relatively warm surface currents may be blocked. Second, ice shelf basal melting at depth can induce a sub-ice shelf thermohaline

*Boundary between the grounded ice sheet and the floating ice shelf.
1.3 Supercooling

A buoyant water mass resulting from an admixture of meltwater at depth can rise up the ice shelf base and promote freezing and act as a source of dense water elsewhere (e.g., MacAyeal, 1984; Hellmer and Olbers, 1989; Smelshruud and Jenkins, 2004). Third, the production of icebergs at the front of the ice shelf, a process known as calving, has a dramatic yet erratic effect on the ocean. For example, grounded icebergs modified the circulation and sea ice cover of McMurdo Sound for several years (Robinson and Williams, 2012).

1.3 Supercooling

Water in the metastable state of having its temperature below its freezing point is said to be supercooled. Observations of supercooling in the ocean surrounding Antarctica are common, but seldom exceed 70 mK. For comparison, the accuracy of both modern instrumentation and the definition of the freezing point for a certain pressure and salinity (Millero, 1978) allow supercooling to be measured to an accuracy of ±5 mK or better.

Supercooled water found near Antarctica usually occurs for one of three reasons (Martin, 1981): (i) ice formation may occur too slowly to account for rapid heat loss at the surface in regions of low ice cover, (ii) the diffusivity of salt in seawater is much smaller than that of heat or (iii) the freezing point of seawater decreases with increasing pressure.

For the first reason (i), supercooling can be encountered at the ocean surface in polynyas, which are regions of persistent open water where sea ice would be expected (see Section 2.2.2).

The second reason (ii) causes supercooling in the following way. Many ice–ocean interaction processes in Antarctica occur at an equilibrium temperature, i.e., the freezing point, a salinity-dependent temperature. The different diffusivities of heat and salt disturb this equilibrium. The result is volumes of water of similar temperature but differing salinity. The lower salinity region with its corresponding higher freezing point may become supercooled.

Considering the third reason (iii), water set at its in situ freezing point at depth will become supercooled if raised adiabatically because the freezing point decreases with depth at a rate of $0.761 \times 10^{-3} \text{ K m}^{-1}$. Melting of an ice shelf at depth commences a process often referred to as an “ice pump” (Lewis and Perkin, 1983, 1986). The meltwater is believed to rise up the ice shelf base as a turbulent current entraining the underlying warmer, saltier water as it rises. This creates a water mass known as Ice Shelf Water (ISW), which is sometimes laden with small, suspended ice crystals, termed frazil ice. Precipitation of this suspended ice as well as direct basal freezing produces marine ice, which is distinct from glacial ice due to its formation in the ocean. This redistribution of ice, which is simply a result of the depth dependence of the freezing point, can become significant: marine ice layers in excess of 100 m thick
4 Introduction

I S W

HSSW

Sub-Ice Platelet Layer

Brine Rejection

/ Heat Flux to Atmosphere

Sea Ice

Figure 1.2 – The production of buoyant Ice Shelf Water (ISW) from meltwater at depth, as well as terminology and other processes that are discussed in this thesis. HSSW is High-Salinity Shelf Water (see Section 2.2.1).

have been observed in certain locations (e.g., Craven et al., 2009). Measurements and models suggest that if ISW continues beyond the ice shelf edge it can enhance sea ice growth by 10–20 cm over one growth season (Hellmer, 2004; Purdie et al., 2006; Gough et al., 2012b). The creation, transport and fate of this very cold water is summarised in Figure 1.2.

1.4 Polar Ocean Modelling

Experimentally based research in the polar regions requires overcoming a number of inherent challenges. For example, many regions are difficult to reach and inhospitable, accessible regions are typically expensive to get to, scientific work is often slowed and/or delayed due to weather, or the ocean to be studied may inconveniently lie below an ice shelf hundreds of metres thick.

Like many areas of science involving difficulties in observation, much of the understanding comes from numerical modelling. Polar ocean models have been used for a number of reasons: to quantify the influence of ice shelves on sea ice thickness (e.g., Beckmann and Goosse, 2003; Hellmer, 2004), to predict the fate of an ice shelf facing an ocean temperature increase (e.g., Jenkins, 1991; Olbers and Hellmer, 2009), to explain the presence of layers of marine ice beneath ice shelves (e.g., Holland et al., 2003; Smersrud and Jenkins, 2004), or to understand the seasonal response in water mass production and distribution on the continental shelf (e.g., Hellmer and Jacobs, 1995; Assmann and Timmermann, 2005). Recently there has been a move toward research using autonomous underwater vehicles (e.g., Nicholls et al., 2006; Queste et al., 2012). This would act as a step toward providing the data necessary to complement the ever-increasing sophistication of polar ocean models.
1.5 Aim and Thesis Outline

The aims of this thesis are to examine the distribution of ISW near the McMurdo Ice Shelf edge and predict the spatial extent of the supercooled water that flows northward out of McMurdo Sound (see Figure 1.1 for location). The primary drivers and key parameters that explain the creation, sustainment and depth of this very cold water are to be determined through both analysis and modelling. This work is motivated by the need to link simultaneous measurements of ocean and ice properties at multiple sites throughout the Sound—especially the lesser studied western side—with a modelling component intended to elucidate and extrapolate results from said measurements. Further, the model developed here will quantify the effect of the growth of suspended frazil ice whilst beneath sea ice, a process not often incorporated into large-scale numerical models.

Chapter 2 outlines the current understanding of the oceanography and ice formation processes in McMurdo Sound that has arisen from numerous experiments conducted in the area. Processes both local to and beyond the bounding regions of the western Ross Sea and the McMurdo Ice Shelf are described. Application of numerical models of various levels of sophistication has augmented the understanding attained through experiments. A review of the physics underlying these models is presented in Chapter 3, followed by a discussion of their application to different locations and success, or lack thereof, in reproducing various measurements.

A new set of oceanographic measurements were recorded by a team including the author in 2011. These comprised two transects, one very close and parallel to the ice shelf edge and the other following the likely direction of the coldest water in the Sound. These data, along with analysis of four first-year sea ice cores, are presented in Chapter 4.

The modelling aspect of this thesis aims to extend the one-dimensional, steady-state Ice Shelf Water plume model published by Smedsrud and Jenkins (2004), hereafter termed the SJ04 model. This model, originally developed to predict the creation of marine ice layers beneath the Filchner-Ronne Ice Shelf, predicts the evolution of a turbulent, frazil-laden, buoyancy-driven plume with differential growth and precipitation of the suspended frazil crystals. It is possible to apply the plume model beyond the ice shelf edge and predict ocean and ice properties beneath sea ice by adapting a number of the existing processes. The addition of an ambient current and a heat flux to the atmosphere are two examples of significant alterations made in this work. The former accounts for the effect of other water mass transport processes that are otherwise excluded from the extended model, and the latter is a process that is significant when concerned with sea ice, as opposed to an ice shelf. All of the changes from the SJ04 model are discussed in Chapter 5 along with their mathematical formulations.

The extended model is applied to McMurdo Sound in an attempt to reproduce the
2011 oceanographic measurements. These data, in conjunction with historical data, span a distance of approximately 60 km and provide an estimate of the validity of the model output, which will be used to predict surface ocean and ice growth properties over a larger spatial scale. The model output is described in Chapter 6, along with a comparison to measurements within the Sound. Comparison of the extended model to other applicable models and a prediction of the fate of the supercooled water are discussed in Chapter 7. Finally, Chapter 8 emphasises the key findings from field measurements and application of an extended plume model to McMurdo Sound.
Chapter 2
Aspects of Ice and Ocean in McMurdo Sound

2.1 Location

McMurdo Sound, situated in the southwestern Ross Sea, is approximately 80 km long and 60 km wide and centred at 77°30′S, 165°E (see Figure 2.1). It consists of a deep (600–800 m) basin in the east which slopes upward to ~200 m in the west. Ross Island and Victoria Land establish the eastern and western boundaries, respectively. The McMurdo Ice Shelf provides a thin, floating, southern boundary, but the Sound is connected oceanographically to the combined McMurdo and Ross Ice Shelf cavities via Haskell Strait (Robinson and Williams, 2012).

Within 100 km it is possible to find a range of geographical features such as fast ice, pack ice, ice shelves, the Antarctic mainland, perpetually ice-free valleys, volcanic cones, glacier ice tongues and open ocean (Figure 2.1). These features coupled with the proximity of two logistic bases on Ross Island, Scott Base (NZ) and McMurdo Station (USA), make it one of the best researched regions of Antarctica, although the work has a significant spring and summer bias. See Figure 2.2 for the locations of previous studies that are cited in this chapter.

2.2 Ross Sea Oceanography

2.2.1 Water Masses in the Ross Sea

Water masses, bodies of water with distinct properties such as salinity and temperature, found in polar regions are often quite close to their surface freezing point of approximately −1.9°C. Water above this temperature is restricted to the surface, seldom exceeding 200 m in depth, and is a consequence of summer warming. In contrast, seawater in the winter or at depth may even be colder than the surface freezing point; the low temperature being attributed to either sea ice formation, convective heat loss to the atmosphere or contact with ice at depth.

As shown in Figure 2.3, Antarctic Surface Water (AASW) and Circumpolar Deep Water (CDW) intrude onto the continental shelf, mixing to become Modified Circumpolar Deep Water (MCDW) (Jacobs et al., 1985; Russo et al., 2011). These waters provide the initial source from which other water masses in McMurdo Sound
are derived. Seasonal warming or cooling and salinification or freshening by various ice, ocean and atmospheric interactions results in the creation of the two water masses most pertinent to McMurdo Sound, High-Salinity Shelf Water (HSSW) and Ice Shelf Water (ISW). In a closing of the system the latter two water masses provide some of the ingredients needed for Antarctic Bottom Water (AABW), a dense water mass that flows down the continental shelf (Neal et al., 1976; Jacobs et al., 1970; Baines and Condie, 1998). The *potential temperature* (i.e., the temperature of water if it is raised adiabatically to the sea surface) and salinity of water masses residing on the continental shelf are shown in the potential temperature–salinity diagram of Figure 2.4. It is important to note that there is variation between studies in the names given to
water masses. For example, HSSW has been given different names, such as Western Shelf Water or Ross Sea Shelf Water, depending on its location. Similarly, Warm Deep Water, Warm Core Water (WMCO) and MCDW all refer to a warm water mass that intrudes onto the continental shelf (Jacobs et al., 1985).
2.2.2 Water Mass Conversion

Polynyas play a dominant role in water mass conversion. These are predictable areas, usually coastal and up to thousands of square kilometres in area, in which there is little to no ice cover (Smith et al., 1990). This lack of persistent ice cover does not imply a lack of growth. Rather, in many polynyas, sea ice often grows quickly due to a large heat flux to the atmosphere. Drift of the newly formed ice due to winds and currents maintains the ice deficit. The largest Antarctic polynya is the Ross Sea Polynya (RSP), with a mean size of 25,000 ± 5,000 km² (Martin et al., 2007). It resides to the east of Ross Island, adjacent to the Ross Ice Shelf and is nearly ten times larger than the nearby Terra Nova Bay Polynya, immediately north of the Drygalski Ice Tongue (Martin et al., 2007). Both polynyas can be identified, in part, in Figure 2.1.

Ice growth in these polynyas, as well as other areas in the Ross Sea, drive the formation of High-Salinity Shelf Water (Jacobs and Giulivi, 1998; Dinniman et al., 2007; Robinson and Williams, 2012), the most abundant water mass in McMurdo Sound (Barry, 1988). Its name is derived from both its high salinity* of 34.6 or greater and its residence on the continental shelf. Its temperature, nearly isothermal at the surface freezing point, combined with its high salinity makes it the densest water on the continental shelf (Jacobs et al., 1985; Assmann and Timmermann, 2005).

The other water mass significant to McMurdo Sound is Ice Shelf Water (ISW). Being formed by contact of HSSW with ice at depth, its temperature is below the

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*In this thesis, salinity will follow the Practical Salinity Scale 1978 (PSS-78) (UNESCO, 1980). Salinity is a dimensionless quantity, derived from electrical conductivity, but its numerical value is very similar to the mass of salt in grams in one kilogram of seawater.
2.3 Seasonal Ocean Cycle

The ocean in McMurdo Sound undergoes a seasonal cycle that is manifest in the numerous oceanographic measurements taken in the region. The proximity of Scott...
Aspects of Ice and Ocean in McMurdo Sound

Base and McMurdo Station means there is a strong bias on measurements toward the eastern Sound. The data recorded in this area will be discussed first, followed by consideration of studies undertaken in the western Sound.

2.3.1 Eastern Sound

The ocean in McMurdo Sound is often described in terms of two oceanographic seasons: summer and winter (Littlepage, 1965; Leonard et al., 2011; Mahoney et al., 2011; Robinson and Williams, 2012). The shorter summer period, approximately December to April, begins abruptly. Rapid warming of surface waters accompanies the break up and drift of sea ice that has been weakened by warmer ocean temperatures and a few months of perpetual sunlight. The top of the water column becomes stratified, with relatively high temperatures and low salinities appearing in the top few hundred metres. There is some variability to the time of strongest stratification, with previous studies indicating a range between mid-January and late-February (Tressler and Ommundsen, 1962; Mahoney et al., 2011; Robinson and Williams, 2012).

In contrast to the well-defined onset of summer, the end of summer is gradual (Mahoney et al., 2011). The air temperature in McMurdo Sound drops following its maximum in December/January (Wexler, 1959; Littlepage, 1965) and a steadily cooling water column results. Initial sea ice formation occurs about March (see Section 2.6) and is followed by the disappearance of the sun in late April. These events mark the beginning of oceanographic winter. Salinities increase throughout due to sea ice formation. Leonard et al. (2011) and Mahoney et al. (2011) both calculated that local sea ice formation could account for only half of the measured salinity increases in 2003 and 2009, respectively, indicating that advection of HSSW from north of the Sound is likely an important part of the salt budget in McMurdo Sound. Leonard et al. (2006) measured an increase in surface salinities from 34.1 to 34.7 between mid-March and mid-September, and their value is indicative of what others have measured or modelled (Littlepage, 1965; Tressler and Ommundsen, 1962; Assmann et al., 2003).

The most important consequence of the cooling and salinification is the move toward a nearly homogeneous water column. The formation of a mixed layer, i.e., a surface layer with nearly uniform temperature and salinity, affects the dynamics of mixing processes and the communication of surface and deep waters. Any water near its freezing temperature with a salinity greater than 24.7 will increase in density with an increase in salinity (Weeks, 2010). This means brine, produced at the sea ice base, can sink through the mixed layer to the pycnocline, the region of greatest vertical density gradient. Brine creation is therefore linked to the mixed layer depth, which increases throughout winter (McGuinness et al., 2009).

Measuring temperatures near Scott Base, Gilmour et al. (1962) found temperatures close to freezing in water above 110 m at the end of April. By June the whole water column was at the surface freezing temperature. Recent studies suggest there is variability in the time at which the water column loses its signature of summer
warming. For example, Leonard et al.’s (2006) oceanographic measurements of the upper 275 m showed an abrupt change in mid-May, when the water column became essentially isothermal. In contrast, Mahoney et al. (2011) observed a three-layer structure, with a warm intermediate layer, that persists until July.

The lack of stratification in the surface water allows supercooled water and its suspended frazil ice to reach, and persist at, the surface. Here it provides a growth enhancement to the sea ice due to both the introduction of frazil and a significant negative oceanic heat flux (Dempsey et al., 2010; Gough et al., 2012b). Despite measuring water below its freezing temperature on a number of occasions throughout the year, an early study in eastern McMurdo Sound disregarded supercooling as a feasible explanation, incorrectly suggesting that the water needed to be pure to sustain supercooling (Littlepage, 1965). Modern equipment has allowed recent studies to measure the development of supercooling throughout the winter to high accuracy. There is always much variability in the values over time, between positive and negative, but smoothed data suggest slight trends towards increasing supercooling as winter progresses (Leonard et al., 2011; Mahoney et al., 2011).

2.3.2 Western Sound

Oceanographic data for the western Sound are relatively sparse due to its distance from Scott Base and McMurdo Station. Gilmour (1975) cited rough, impractical and/or dangerous travel and unpredictable conditions as further reasons for the lack of studies undertaken on this side. In the last 30 years it has been shown that the western side of the Sound is an extremely important part of McMurdo Sound’s oceanography, acting as a significant outflow region for Ice Shelf Water (Lewis and Perkin, 1985; Robinson and Williams, 2012).

The western Sound, which is ice covered for a large part, if not all, of the year is subject to less variation in the summer than the east. Barry (1988) suggested that northward advection of cold water from beneath the McMurdo Ice Shelf would persist year-round. There appear to be no studies to date to indicate this to be incorrect and the mechanism he suggested for production of this very cold water, which was the contact with ice at depth, is backed up by more recent studies.

Water advected from beneath the McMurdo and Ross Ice Shelves is not the only source for the western Sound. Summer nutrient and oxygen data have indicated water from the eastern Sound also acts as a source (Barry, 1988). Some of the southward flow of water in the east swings westward at Cape Royds (Littlepage, 1965; Lewis and Perkin, 1985) with more deflected westward at the ice shelf edge (Robinson et al., 2010).

The first real understanding of the oceanography of the Sound as a whole, and hence the western side, came from the work of Lewis and Perkin (1985). Their helicopter-aided, month-long study in October/November, i.e., at the end of “oceanographic winter”, consisted of a cross-sound transect, a long-sound transect and a small number
of other sites. Their suite of oceanographic measurements allowed them to indicate the water movement within the vicinity of the ice shelf and map surface supercooling to an accuracy of approximately 0.01°C. Their Figures 8 and 13, shown here combined (Figure 2.5), are particularly useful in illustrating both the transport of supercooled water out of the Sound and the winter circulation.

Excluding the study described above, understanding of winter oceanography in the western Sound comes from either modelling or inferences from sea ice cores (see Sections 2.6 and 2.7). A number of regional ocean modelling studies that have sufficient resolution to capture flows in McMurdo Sound have indicated a flow of water on the western side out of the ice shelf cavity (Assmann et al., 2003; Holland et al., 2003; Dinniman et al., 2007). It is not possible to deduce much more than the mean flow direction given that results are presented in terms of either flows for selected times of the year, selected depths or depth-integrated values.
2.4 Currents

Water movement in McMurdo Sound can be considered as the superposition of mean circulation, the topic of this section, and oscillatory flows with no net transport (Section 2.5). Several processes, both local and from outside the Sound, interact to produce ocean currents. These are described first, followed by a summary of previous current measurements.

2.4.1 Processes

During spring and summer, oceanic processes conspire to produce a clockwise circulation in both surface and deep water (Barry and Dayton, 1988; Robinson and Williams, 2012). The southward flow of warm water in the eastern Sound is likely the continuation of a coastal current, which flows westward across the front of the Ross Ice Shelf, and is diverted around Cape Bird, the northernmost part of Ross Island (Keys and Fowler, 1989; Barry, 1988). As described in the previous section, the northward flow in the west has a source from beneath the ice shelf, but also mixes with a westward flow across the front of ice shelf (Robinson et al., 2010).

Much of the actual transport of water in McMurdo Sound is geostrophic flow, which occurs when a horizontal pressure/density gradient is present. Water moves from higher to lower sea level or equivalently from lower to higher density, as a lower density results in a higher sea level. However, because the Earth is rotating, water movement is acted on by an apparent force due to the Coriolis effect. This “force” deflects the water to the left in the southern hemisphere, resulting in flows approximately parallel to isopycnals, i.e., lines of constant density. A local density gradient across McMurdo Sound (Lewis and Perkin, 1985) and a large-scale density gradient in the Ross Sea (Assmann et al., 2003) set up the pressure gradients that lead to clockwise circulation within the Sound. True geostrophic flow only occurs in an unbounded region. In McMurdo Sound, the complex topography plays a significant role in redirecting the flow.

Water movement in McMurdo Sound is also caused by local and large-scale wind-driven circulation and tidal rectification (Barry and Dayton, 1988; MacAyeal, 1985b). Wind-driven circulation is relatively straightforward: a momentum transfer from the wind to the ocean drives a current. However, the two are not in the same direction. Rather, due to the Coriolis effect, the surface current moves at an angle to the left (in the southern hemisphere) of the wind. The magnitude of the current attenuates with depth, while the angle increases. The (theoretical) result is a net transport of water 90° to the left of the wind.

Tidal rectification is not as simple and the details are beyond the scope of this thesis. However, the process can be summarised as the production of a time-independent, residual current through the interaction of tidal flows with topography. Modelling by MacAyeal (1985b) suggests tidal rectification causes a net southward flow of
approximately 0.5 cm s\(^{-1}\) through the centre of the Sound. Slower flows with variable direction are suggested either side of the centre. However, his work included the entire Ross Sea, and consequently a coarse depiction of McMurdo Sound’s bathymetry.

### 2.4.2 Measurements

A number of methods have been used to measure currents in the last 50 years. In early studies, currents were derived from electrical potentials of flowing seawater or measured with a mechanical current meter. Inferences from sea ice and iceberg drift tracks (e.g., Keys and Fowler, 1989) provide a simple indication of net surface currents. However, wind plays a large part so only general results may be deduced. More recent studies rely on Acoustic Doppler Current Profilers (ADCP), tools capable of accurately measuring current as a function of depth over approximately 100 m (e.g., Leonard et al., 2006; Robinson et al., 2010).

In the eastern Sound during the summer, all studies report net flows into the ice shelf cavity. Examples of residual currents measured close to the Hut Point Peninsula are an easterly flow of approximately 25 cm s\(^{-1}\) (Gilmour, 1963), a southerly flow of 0–5 cm s\(^{-1}\) (Gilmour, 1975) or a southeastward flow of 10–20 cm s\(^{-1}\) (Robinson, 2004). Net transport has been estimated as an inflow to the ice shelf cavity of \(\sim1.8\) Sv (1 Sv = 10\(^6\) m\(^3\) s\(^{-1}\)) (Robinson et al., 2010).

In an early study, Gilmour (1963) measured a residual northward flow over four days in the central/western Sound of 7.6 cm s\(^{-1}\) at 50 m deep and decreasing below. By itself this value would suggest a very strong flow out of the Sound, in this area. However, residual currents measured at other sites within 10 km in either direction were either much smaller in magnitude or in a different direction. In a later study, Gilmour (1975) calculated a small, 0.04 Sv, northward flow of “exceptionally cold water”. His stations, however, reached only half way across the Sound, suggesting his value was a significant underestimate. A more complete estimate by Robinson (2012) suggests \(0.100 \pm 0.015\) Sv of water exits on the western side. The water separating the eastern and western sides of the Sound generally exhibits moderate speeds but little net movement (Barry and Dayton, 1988).

There is growing evidence for a switch in net circulation on the eastern side of the Sound, sometime during the beginning to middle of winter, associated with the appearance of ISW. The net surface flow in McMurdo Sound changes from the relatively well understood, summer regime of flow into the ice shelf cavity to a winter regime of net northward transport. Lewis and Perkin (1985) indicate net northward flow throughout most of the Sound during October and November with a mean speed of \(\sim5\) cm s\(^{-1}\). The progressive vector plot by Barry et al. (1990) shows the current direction at 180 m deep in the eastern Sound as northeastward from March to August, but changing to northward until November. This occurs despite a continued southward flow at 570 m depth throughout winter. This corresponds well with the measurements of Mahoney et al. (2011) who, in a similar location, found that the current at 170 m
changed to a northward flow (from an eastward flow) at the start of July, despite deeper flow remaining toward the ice shelf cavity.

Current measurements are known to be affected by both their depth and distance offshore of Ross Island (Heath, 1971). Coupled with interannual variability (see Section 2.9), this leads to variation in the reported date that the change in circulation occurs. For example, Leonard et al. (2006) found that the net transport switched from being southwestward to northeastward during April, whereas Littlepage (1965) described the same SW to NE change, but as commencing in October.

2.5 Tides

Tidal measurements of both height and current velocity have been published in a number of studies (e.g., Gilmour et al., 1962; Padman et al., 2003; Robinson, 2004; Leonard et al., 2006) and tidal flow has been described as dominating the currents in the Sound (Barry and Dayton, 1988). The most comprehensive study of the tides in the Sound comes from a 471-day analysis by Goring and Pyne (2003) and forms the basis of the summary in the following paragraph.

The three largest tidal constituents, $K_1$, $O_1$ and $P_1$, are all diurnal. Their periods are 23.93 h, 25.82 h and 24.07 h, respectively, and the amplitudes of the first two constituents are three times larger than the third. During spring tide these constituents conspire to produce a tidal amplitude of approximately 0.6 m. However, every 13.66 days the amplitude drops to nearly zero, corresponding to the moon crossing the equator. Two semidiurnal constituents ($M_2$ and $S_2$) and a terdiurnal constituent ($SK_3$) are the only others to have a significant amplitude.

The freely available Tide Model Driver software (Padman and Erofeeva, 2005) will be used in this thesis to predict tidal speeds in and beyond McMurdo Sound. A number of different models are available depending on what output is required. Here only root mean square (RMS) speed will be required. For McMurdo Sound the optimal model is $Ross_{VMADCP}_{9cm}$. This is based on assimilation of ADCP data from three cruises in the Ross Sea (Erofeeva et al., 2005) and is described as an excellent model for velocities in the western Ross Sea* (Padman and Erofeeva, 2005). The predicted RMS speed from this model is shown for McMurdo Sound and surrounding areas in Figure 2.6 to give an indication of the magnitude of tidal currents in comparison to residual currents described in the previous section.

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*RMS tidal speed beneath the Ross Ice Shelf will be required in (Chapter 5). The $Ross_{Inv}_{2002}$ will be used instead, although predicted velocities in the cavity are very similar for both models.
Figure 2.6 – Root mean square tidal speed in McMurdo Sound and surrounding areas, predicted using the Tide Model Driver software with the Ross_VMADCP_9cm model (Padman and Erofeeva, 2005).

2.6 Sea Ice in McMurdo Sound

2.6.1 Annual Cycle

As outlined in Section 1.1, sea ice grows primarily by a loss of heat to the atmosphere. However, in McMurdo Sound, solidification of flooded snow, a heat flux to the ocean and precipitation of crystals from the underlying water column are each significant to sea ice growth. In most years, much of the ice cover in the Sound will melt and break up during the summer, as early as November in the northeast but otherwise around late-January, to be transported away by wind and currents the following month (Heath, 1971; Jeffries et al., 1993). It is not long before ice growth recommences, with initial formation occurring in late March/early April, but possibly not remaining fast until May (Littlepage, 1965; Falconer and Pyne, 2004; Gough et al., 2012b). Some ice may survive the summer with growth continuing, albeit more slowly, the following winter. This ice, termed multi-year sea ice, primarily forms a belt bordering the southern and western edges of the Sound (Gilmour, 1975; Barry and Dayton, 1988; Gow et al., 1998). Multi-year ice can be in excess of 5 m thick, compared to approximately 1.7–2.3 m for first-year ice (Jeffries et al., 1993; Jones and Hill, 2001).

2.6.2 Crystallography and Composition

Properties of sea ice can be broadly classified into four areas: mechanical, thermodynamical, compositional and crystallographic. This thesis will be most concerned with the last of these as it provides an indication of oceanic conditions during the
The crystal structure of sea ice is often elucidated by shining light through a thin section, i.e., a piece of ice approximately 10 cm in two of its dimensions, thinned to less than a millimetre in the third. Each ice crystal alters the polarisation of incoming light by an amount dependent on its orientation and the thickness of the section. If the ice is placed between two crossed polarisers the individual crystals are easy to discern by their differences in colour (see Figure 2.7). Preparation of a thin section of sea ice is relatively straightforward in comparison to rocks or other minerals. Its transparency means it can be an order of magnitude thicker than a typical rock thin section, adhesion of the ice to the glass slide by its own refrozen melt means no glue is required and the individual crystals are large enough to be recognised and measured by the naked eye.

Most studies in McMurdo Sound identify three distinct sea ice textures: granular, columnar and platelet (e.g., Jeffries et al., 1993; Jones and Hill, 2001). Further classification can be made to the latter two types depending on alignment and/or shape of the crystals (e.g., Dempsey et al., 2010; Gough et al., 2012b). Typical examples of columnar and platelet ice are shown in Figure 2.7. Sea ice accommodates a layer of snow above it and sometimes an open-textured, mechanically weak aggregation of ice crystals on the underside termed the sub-ice platelet layer (Dayton et al., 1969; Smith et al., 2001), which is described in detail in Section 2.8.

Sea ice is not completely solid; a small fraction of liquid brine is trapped during growth. A consequence is that the ice attains a bulk salinity, commonly between 5 and 10, with the high end of this range being exhibited at the top and bottom. A large amount of salt is trapped at the top of the sea ice during the rapid growth of the initially thin ice, whereas the shorter period of time available for desalination by brine drainage and the high liquid fraction are the causes of the high salinity at the bottom. The result is the C-shaped salinity profile often measured in first-year sea ice (e.g., Smith et al., 2001; Pringle et al., 2007; Dempsey et al., 2010).
Figure 2.8 – (a) Schematic diagram of the orientation of the basal plane and c-axis for growing columnar ice crystals. (b) The salinity curve produced by an advancing sea ice interface. (c) Constitutional supercooling resulting from the discrepancy between the salinity-dependent freezing point curve and the temperature curve. Image concept from figures in Langhorne and Robinson (1986) and Eicken (2003).

(2012a) show that an additional factor is that platelet ice contains a higher bulk salt content, relative to columnar ice growing at the same rate.

2.6.3 Growth Processes

The growth of granular ice occurs at the top of sea ice and is composed of small frazil crystals that are formed either near the surface in slightly supercooled water or above the surface due to splashing, wind spray or bursting air bubbles (Weeks, 2010). Although the physical processes occurring at this early stage are of little importance to this thesis, a small amount of granular ice will be evident in sea ice cores shown in Chapter 4, where it can be identified by its fine, randomly-orientated grains (Jones and Hill, 2001).

After a layer of granular ice has formed, columnar ice will grow below, with the structure being dictated by geometric selection. Crystals initially growing with vertical basal planes will impede the growth of less favourably orientated grains by obstructing them. This process follows from a number of thermodynamic advantages attained by crystals with their basal planes orientated vertically or equivalently c-axes orientated horizontally (see Figure 2.8). First, ice crystal growth is kinetically favoured in the basal plane (Weeks and Ackley, 1982; Holland and Feltham, 2005; Dempsey and Langhorne, 2012). Second, crystals with vertical basal planes extend further into a small region of constitutionally supercooled water that exists slightly ahead of the ice–ocean interface, a consequence of the differing diffusivities of heat and salt in seawater (Figure 2.8). Third, the release of latent heat tends to warm the water through which the basal plane has already passed thus reducing c-axis growth even further (Lock, 1990).

On the eastern side of McMurdo Sound, approximately the top metre of first-year sea ice is primarily composed of columnar ice. This value varies significantly by location and year (Gow et al., 1998; Jones and Hill, 2001; McGuinness et al., 2009),
and a variation between studies in the method used to quantify or define distinct textures further complicates the picture. If a directionally consistent current exists below growing columnar ice, then the crystals will align with their $c$-axes parallel to the flow to form “aligned columnar ice”. Langhorne and Robinson (1986) found that this is because the flow, which is disturbed at the interface, results in grains with $c$-axes parallel to flow growing slightly more rapidly than those that are perpendicular. Further, they show that a persistent current with a magnitude of the order of 10 mm s$^{-1}$ or larger will influence the sea ice fabric.

Direct measurement of the $c$-axis angle of numerous individual crystals provides a time-history of the surface current direction during sea ice growth. This process has been shown to be useful in McMurdo Sound. Gow et al. (1998) and Jones and Hill (2001) each measured $c$-axis alignment on a number of cores with both studies describing the $c$-axis directions as corresponding well with measured circulation patterns. Furthermore, Jones and Hill (2001) measured a rotation of alignment with depth in cores at a number of sites throughout the Sound. This is consistent with the changing currents described in Section 2.4.

## 2.7 Platelet Ice

In recent times platelet ice has been the source of some controversy, both in its definition and its growth mechanism (Smith et al., 1999). In this thesis the term “platelet ice” is used to define ice that has formed by the precipitation of frazil crystals from the underlying water column, which continue to grow while adhered to the sea ice base and then become frozen into the ice cover by the solid ice–water interface advancing through the seawater-filled interstices. Confusion sometimes arises when “platelets” is used to describe small ice crystals suspended in the water column. Here the term “frazil” is used to describe these suspended crystals; individual crystals attain the term “platelets” only after they precipitate. Note the potential for further confusion as precipitation does not preclude further movement of the ice crystals. Unconsolidated platelets can become resuspended or drift along the underside of the sea ice. The other part of the platelet debate is the question of how much growth a crystal attains while in the water column compared to how much it attains while adhered to the sea ice base. Smith et al. (2001) argue the latter is more important with more recent studies having added weight to that argument. A number of studies suggest frazil crystals with a diameter in excess of a few millimetres would precipitate out of the water column very quickly (Smedsrud and Jenkins, 2004; Holland and Feltham, 2005; McGuinness et al., 2009), yet crystals in platelet ice are commonly several times this size (Gough et al., 2012b; Dempsey and Langhorne, 2012). Furthermore, if crystals reached their full size before precipitating, they would be expected to lie with their basal planes vertical, not randomly orientated as observed (McGuinness et al., 2009).

Like the formation of sea ice, the appearance of platelet ice may initially be episodic
Figure 2.9 – Relative platelet ice abundance. Figure from Dempsey et al. (2010). The relative abundance was calculated by comparing platelet ice thickness from each core in a certain study to that of the mean of all cores in the study. Contours indicate the number of standard deviations from the mean. Darker contours indicate areas with a higher volume of platelet ice. Markers indicate sites and sources from which measurements were taken.

before becoming persistent later in the winter. A thin layer of platelet ice before a transition back to columnar ice has been measured on some occasions (Jeffries et al., 1993; Leonard et al., 2006; Gough et al., 2012b). Alternatively, the abrupt transition from columnar to platelet ice over 20–30 mm (Dempsey et al., 2010) may be linked with the complete absence of columnar structure in the ice below.

The abundance of platelet ice exhibits significant variability from year to year, yet the spatial distribution is well behaved. Based on various observations, Barry (1988) noted that the highest volumes of platelet ice and ice on mooring lines were located nearest the ice shelf in the middle and western Sound. Dempsey et al. (2010) built on this idea, using three datasets from different years, to generate a “relative platelet abundance” map (Figure 2.9). The relative abundance concept removes interannual variability as platelet ice percentages are only compared to others from the same study. The result is a tongue of high platelet ice concentration in the centre of the Sound, which generally decreases with distance from the ice shelf. As noted in some of the original studies used to generate the map (e.g., Jeffries et al., 1993), the locations of cores with a high percentage of platelet ice corresponds well with the regions of highest surface supercooling measured by Lewis and Perkin (1985) (see Figure 2.5).
2.8 Sub-ice Platelet Layer

The sub-ice platelet layer is a region below sea ice of unconsolidated platelets (Figure 2.10), which are individual crystals up to 150 mm in diameter and 5 mm in thickness (Crocker and Wadhams, 1989; Jeffries et al., 1993). The layer forms by precipitation of frazil crystals from the water column and their subsequent growth. A sub-ice platelet layer is present if the unconsolidated ice crystals accumulate downward more quickly than the solid edge of the congelation growth (Dempsey et al., 2010). This is quite common in McMurdo Sound, where it has been measured to be up to 7.5 m thick (see Chapter 4). Not surprisingly, the thickest sub-ice platelet layers have been measured in the same regions that exhibit the highest platelet ice percentages (Crocker and Wadhams, 1989; Dempsey et al., 2010).

An early study suggested that contact of supercooled water with the sea ice base results in a sub-ice platelet layer (Crocker and Wadhams, 1989). Although they recognised the possibility of ice precipitation, they found no evidence to suggest that crystals originated in the water column. However, recent studies have shown that small ice crystals exist in the water column. Leonard et al. (2006) conclusively linked an increase in ADCP signal strength to the presence of scatterers (frazil ice crystals) and Gough et al. (2012b) produced video evidence of suspended frazil crystals beneath a sub-ice platelet layer to a depth of 23 m. Dempsey et al. (2010) suggested that the development of a sub-ice platelet layer is dependent on the solid fraction of the layer, as well as the ratio of the upward flux of frazil ice to the downward advance of the solid ice interface. By quantifying each of the distinct heat fluxes present at the ice–ocean interface, Gough et al. (2012b) determined that a sub-ice platelet layer would form as soon as the magnitude of the downward oceanic heat flux was greater than the upward conductive heat flux multiplied by the factor of $\beta/(1 - \beta)$, where $\beta$...
is the fraction of solid ice in the layer, which they calculated to be $0.25 \pm 0.06$.

The sub-ice platelet layer has two important consequences for ice–ocean interactions: it provides a buffer against sea ice melting in the summer (Dempsey et al., 2010) and it causes a significant reduction to current velocities in the upper water column compared to far-field. Its apparent “slushiness” means this velocity reduction is even greater than a prediction based on its actual roughness (Robinson, 2012). The slushiness of the layer also has some other more awkward consequences. Lewis and Perkin (1985) mentioned that five minutes of continuous bailing of loose ice crystals from a hole was often required before lowering an instrument through the ice; a significant obstacle when considering the number of sites they used. In addition, Crocker and Wadhams (1989) cited the problem of the sub-ice platelet layer adding an unacceptable uncertainty to their measured sea ice thickness values. Its unconsolidated state also complicates the task of remotely measuring sea ice thickness.

### 2.9 Interannual Variability

As alluded to in much of this chapter, ocean and ice conditions in McMurdo Sound are subject to significant interannual variability. In recent years much of this variability was due to the calving of large icebergs from the Ross Ice Shelf from 2000 to 2005. These affected McMurdo Sound by blocking the drift of sea ice in the summer, redirecting surface currents, redistributing freshwater sources and affecting polynya formation (Robinson and Williams, 2012). Together, these processes form a complex system of interactions that is hard to predict.

In contrast to iceberg calving events, which occur sporadically and affect processes over various time scales, large-scale environmental events play a continuous role. El Niño Southern Oscillation, the Southern Annular Mode and other global-scale processes are some of the causes cited as playing a part (e.g., Barry and Dayton, 1988; Jacobs and Giulivi, 1998; Liu et al., 2004, and references cited therein). Again, the interactions between processes are complex and these are beyond the scope of this thesis. Instead, spatial variability among data from the same year is emphasised.

One long-term change that is occurring at a consistent rate is the decreasing salinity of shelf waters in the Ross Sea. These have declined at a steady rate of $0.03 \text{ decade}^{-1}$ (Jacobs et al., 2002; Jacobs and Giulivi, 2010). This decline, measured over five decades, is linked to changes in large-scale atmospheric circulation. A similar salinity decline for Modified Circumpolar Deep Water ($0.04 \text{ decade}^{-1}$) and cooling of $\sim 0.5^\circ C$ from the late 1970s to 2007 was also described in the latter study. This represents a significant change to the definition of a water mass that has been associated with ice shelf basal melting at mid-depth (Section 2.2.1). These changes cannot be ignored when comparing new and historical datasets given that the salinity change over many decades may exceed the entire range of salinity values measured throughout the Sound (e.g., Lewis and Perkin, 1985).
2.10 Summary

The spring and summer oceanography of McMurdo Sound is relatively well understood, with warmer water from the northwestern Ross Sea advected southward into the eastern Sound and a current of cold water exiting from beneath the McMurdo Ice Shelf on the western side that is believed to persist year-round. There is growing evidence of a switch in surface current direction in the eastern side of the Sound, sometime during winter, which leads to a net transport of water out of the ice shelf cavity in the upper ocean. Superimposed on these net currents are strong tidal currents, with diurnal components $K_1$ and $O_1$ dominating but also combining to form a tidal amplitude that drops to nearly zero every 13.66 days.

Sea ice forms in March and April but may not persist until May. Over the winter the ice grows approximately 2 m thick, with this value being increased in the presence of frazil precipitating from the water column and a heat flux to the ocean in late winter. Platelet ice, a consequence of the suspended frazil, is exhibited in numerous cores from studies of the crystal structure of McMurdo Sound sea ice. The locations of cores exhibiting high percentages of platelet ice indicate regions that experience significant flows of supercooled water. Finally, any loose platelets, which are not incorporated into the sea ice by the advancing solid interface, form the sub-ice platelet layer at the base of sea ice.
3.1 Introduction

An ice shelf cavity interacts with water on the continental shelf predominately through thermohaline-driven circulation. Observations at the ice shelf edge allow some of the processes that occur inside the cavity to be inferred (Hellmer and Jacobs, 1995). However, due to the inherent difficulties of experimental investigation in these regions, they remain extremely under-sampled in comparison to most of the world’s oceans. The importance of using coupled ice–ocean models to overcome this lack of data was introduced in Section 1.4. The purpose of this chapter is to review the formulation and features of a particular subset of these models.

The first half of this chapter reviews the underlying concepts of heat and mass transfer. The “ice pump mechanism” underpins many models, but quantifying its effect requires a mathematical formulation of ice–ocean interactions. This begins with a description of the turbulent transfer of heat and salt that allows the heat and salt budgets at the interface to be formulated. Combining these with a pressure- and salinity-dependent freezing point produces the so-called “three-equation formulation”. The growth of frazil ice is formulated with similar, but simpler, thermodynamics. A description of ice crystal production and precipitation is then given to fully describe frazil ice dynamics.

In this chapter much reference will be made to the “ISW plume models”, which collectively describe the evolving set of models developed by Jenkins (1991), Jenkins and Bombosch (1995), Smedsrud and Jenkins (2004) and Holland and Feltham (2006). A background for these models forms the second half of this chapter, starting with a derivation of a simple, inclined plume. An entrainment formulation is then combined with several conservation equations, and the ice–ocean interactions described above, to derive the SJ04 model, i.e., the one-dimensional, steady-state plume model that is to be adapted in Chapter 5. The derivation is complemented by a discussion of the salient features and sensitivity of ISW plume models. An outline of the current state of knowledge in complementary areas of research—two-dimensional, time-dependent plume modelling and large-scale ocean modelling—ends the chapter and helps explain some of the processes that cannot be captured in simpler models.

Much of the previous work to be discussed in this chapter uses notation with a bias
toward melting. For example, freezing is considered as negative melting. In keeping with this work, much of the same notation will be used here. However, in keeping with the theme of this thesis, concepts will be explained with a bias toward freezing processes.

### 3.2 The Ice Pump

The depression of the freezing point with depth means that a water parcel in thermal equilibrium with ice at depth can become supercooled if it rises. Foldvik and Kvinge (1974) considered this depth-dependence in the context of seawater in the vicinity of an ice shelf and were able to estimate the rate of ice crystal production in the supercooled water as seawater was raised adiabatically to the surface. Lewis and Perkin (1983) built on this work, coined the term “ice pump” to describe the process and suggested ice with a draft of 10 m is sufficient to produce measurable supercooling.

“Ice pumping” can occur in an isolated system, i.e., it can generate its own circulation without external addition of energy (Figure 3.1a), although it may proceed much faster when coupled with other oceanic regimes such as tidal mixing (Lewis and Perkin, 1986). Other factors, however, limit the rate of ice transfer as shown in Figure 3.1b. Two key examples are the stable layer of meltwater produced beneath an ice shelf and the turbulent transfer of frazil away from the interface due to its concentration gradient (Holland and Feltham, 2005).
3.3 Turbulent Transfer

Turbulent transfer is the transfer of a quantity along a gradient by eddies of a range of sizes. Unlike the simpler process of molecular diffusion, it is not easily predictable. Turbulence is characterised by randomness, nonlinearity, fluctuating vorticity, dissipation and intermittency (Kundu and Cohen, 2008), and the energy in a turbulent flow exists at a continuum of scales. However, it is important when describing and quantifying processes at the ice–ocean interface. A parameterisation is a simple way to quantify the complicated effect of turbulence, with the emphasised term describing a mathematical relationship between the quantity of interest and the dependent physical variable(s). It may be derived either empirically or theoretically and need not be exact. Indeed, simpler parameterisations are often favourable as they increase model tractability.

Turbulence in an ice–covered ocean is neither homogeneous nor isotropic, which makes it particularly troublesome to understand (McPhee, 2008). The best approach is therefore to rely on empirical or semi-empirical parameterisations. They may not elucidate the underlying physical process, but they will quantify it.

3.3.1 Turbulent Momentum Transfer

Before discussing turbulent transfers of heat and salt, it is useful to review the physics and terminology of turbulent wall flow—following the explanation given by Kundu and Cohen (2008). Turbulent flow parallel to a wall causes a momentum transfer. The magnitude and mechanism of the transfer depends on the hydrodynamic roughness of the wall, which is determined by the size of the wall’s roughness elements relative to the height of the viscous sublayer (see next paragraph). Growing ice creates a rough surface, whereas ablating ice is generally suggested or assumed to exhibit a smooth surface (Scheduikat and Olbers, 1990; Jenkins, 1991; Smelserud and Jenkins, 2004; Holland and Feltham, 2006). It is worth noting that a recent study that used an underwater autonomous vehicle suggested that the bulk roughness of an ice shelf base is greater than previously thought (Nicholls et al., 2006).

Immediately next to a smooth wall the velocity goes to zero due to viscous forces. At a distance where these become negligible, shear stresses dominate and the velocity is higher. An idealised three-layer structure results: next to the wall is the viscous sublayer where velocity increases linearly with distance from the wall; away from the wall the velocity increases with a distance in a logarithmic relationship, the so-called “logarithmic layer”; between these two layers is a buffer layer where shear and viscous stresses have a similar magnitude. Order of magnitude layer thicknesses at a smooth ice–ocean interface are shown in Figure 3.2.

At a rough surface, viscosity is no longer important. Instead, momentum transfer is dominated by drag induced by the individual roughness elements. Close to the wall the velocity distribution is no longer simple because wakes are created behind
3.3.2 Heat and Salt Transfer

A popular parameterisation of turbulent heat and mass transfer comes from a semi-empirical formulation by Kader and Yaglom (1972). The general form of the equations is derived from theory, and experimental data is used to determine the coefficients. The parameterisations are written in terms of the “heat transfer velocity” and “salt transfer velocity” denoted $\gamma_T$ and $\gamma_S$ respectively, both measured in m s$^{-1}$.

$$\gamma_T = \frac{C_d^{1/2} U}{2.12 \ln \left( C_d^{1/2} \text{Re} \right) + 12.5 \text{Pr}^{2/3} - 9}$$  \hspace{1cm} (3.1)$$

$$\gamma_S = \frac{C_d^{1/2} U}{2.12 \ln \left( C_d^{1/2} \text{Re} \right) + 12.5 \text{Sc}^{2/3} - 9}$$  \hspace{1cm} (3.2)$$

$C_d$ is a quadratic drag coefficient with its associated velocity $U$, $\text{Re}$ is the Reynolds number, $\text{Pr}$ is the Prandtl number and $\text{Sc}$ is the Schmidt number. The Reynolds number is the ratio of inertial to viscous forces; a measure of a flow’s turbulence level. The Prandtl number is the ratio of momentum diffusivity* to thermal diffusivity, and its value in seawater is 13.8. The Schmidt number is the mass analogue of the Prandtl number, i.e., the ratio of momentum diffusivity to mass diffusivity. For salt in seawater

*Momentum diffusivity and kinematic viscosity are alternate names for the same quantity.
its value is 2432. The term containing either Pr or Sc dominates the denominator and accounts for the effect of molecular diffusion through the interfacial sublayer (Holland and Jenkins, 1999; Jenkins et al., 2010). Heat and salt transfer velocities are sometimes made dimensionless (by removing velocity from the numerators of Equations 3.1 and 3.2 and termed heat and salt transfer coefficients. This method is not followed here to save confusion with similarly named coefficients introduced later.

The turbulent heat and salt fluxes, $Q_T$ and $Q_S$, at the interface become

\[
Q_T = \rho_w c_w \gamma_T (T - T_b) \tag{3.3}
\]

\[
Q_S = \rho_w \gamma_S (S - S_b) \tag{3.4}
\]

where $\rho_w$ is the density of seawater, $c_w$ is the specific heat capacity of seawater and $T_b$ and $S_b$ are the temperature and salinity of water at the base of the ice. See Table i (page xiii) for the values of all constants.

These two parameterisations (Equations 3.1–3.4) were invoked in all of the ISW plume models, with some slight variations. Although it was recognised in the early studies that they strictly apply for only a smooth surface (melting regime), they have always been applied during freezing as well. The later plume studies suggest model output during freezing would be insignificantly affected, as direct basal freezing is a relatively inefficient ice formation process in comparison to frazil precipitation. However, in a sea ice context, the problem cannot be ignored.

Parameterisations appropriate for growing sea ice are recommended by McPhee et al. (2008). They are simpler than those described above, but by no means less accurate. As described by these authors, the following relationships “provide a reasonable description of the present observational state of knowledge, even if lacking aesthetic elegance”. They are

\[
Q_T = \alpha_h u_* \rho_w c_w (T - T_b) \tag{3.5}
\]

\[
Q_S = \alpha_s u_* \rho_w (S - S_b) \tag{3.6}
\]

where $\alpha_h$ and $\alpha_s$ are the heat and salt exchange coefficients, which are both set as 0.0057 for growing sea ice, and $u_*$ is the friction velocity: a measure of shear stress in velocity units. Denoting the shear stress as $\tau$,

\[
u_* = \sqrt{\frac{\tau}{\rho_w}} \tag{3.7}
\]

An approximation of Equation 3.7 is often used when the velocity of the mixed or plume layer is known (Holland and Jenkins, 1999; Jenkins et al., 2010),

\[
u^2_* = C_d (U^2 + U_T^2) \tag{3.8}
\]

where $U_T$ is the RMS tidal velocity, which increases the level of interfacial friction.
In this thesis, $u_*$ will always be evaluated at the interface. In other works this is sometimes indicated with a subscript 0, but this would be unnecessary here. Given that the drag coefficient $C_d$ is of the order $10^{-2}$ for the conditions addressed herein (e.g., Lu et al., 2011), as a rule of thumb the friction velocity is one-tenth of the actual velocity.

### 3.3.3 Double Diffusion during Freezing

Part of the motivation for the study cited earlier (McPhee et al., 2008) was to develop a parameterisation such that numerical models would not exhibit a phenomenon that is not seen in reality; that of extensive supercooling resulting from sea ice growth, which may be predicted if heat and salt transfer velocities are used.

Transfer velocities (Equations 3.1 and 3.2) suggest that heat transfer is 20–30 times faster than that of salt; not an unreasonable suggestion. However, a significant upward heat flux exists above a freezing sea ice–ocean interface, transferring heat out of the mixed or plume layer into the atmosphere. Rejected salt depresses the freezing point somewhat, but far too slowly to account for the heat loss, and a supercooled layer is predicted as a result. In general, if a sufficiently large upward heat flux exists and $\gamma_T > \gamma_S$ or $\alpha_h > \alpha_s$, supercooling will be predicted. This explains why $\alpha_h$ and $\alpha_s$ must be set as equal when sea ice is growing.

This may appear to contradict constitutional supercooling as described in Section 2.6.3. The difference, however, is in the size scale considered: constitutional supercooling occurs on a much smaller spatial scale. The lack of supercooling measured on a large scale is attributed to convection within the mushy layer of ice and brine at the sea ice base relieving any double diffusive tendencies (Feltham et al., 2006; McPhee et al., 2008). These tendencies are the result of differing transfer rates of heat and salt and do occur during melting, where heat transfer may occur 35–70 times faster (Notz et al., 2003). Double diffusion has been linked to the stable layer of meltwater beneath an ice shelf (Figure 3.1b), “false bottoms” beneath melting sea ice and anomalously long survival of sea ice above warm water (Notz et al., 2003; McPhee, 2008).

Given the reasons above, transfer velocities are appropriate for models in which melting is predominant and direct basal freezing is either minor or non-existent. Nevertheless, there appears to be a move away from this formulation even where theory suggests that it may suffice. Other formulations using exchange coefficients, i.e., those similar to Equations 3.5 and 3.6, have been favoured by, among others, Notz et al. (2003), Jenkins et al. (2010) and Jenkins (2011).
3.4 Heat Budget

3.4.1 Formulation

The full heat budget at the interface is most easily formulated by considering upward heat fluxes into or out of a thin control volume straddling the ice–ocean interface and containing a latent heat source term as shown in Figure 3.3a. In keeping with the literature, the velocity of the interface is defined in terms of the melt rate \( m' \), whereby freezing is simply negative melting.

In an idealised case, the fraction of total ice thickness below the seawater surface is \( \rho_i/\rho_w \). The quantity \( m' \) measures the rate of change of this fractional thickness, which is approximately nine-tenths of the total thickness. This method simplifies conservation equations, and is often referred to as measuring ice growth in terms of a seawater density. Other primed variables defined later (\( f' \) and \( p' \)) are also expressed in terms of the thickness of seawater, not ice, undergoing phase change.

Three components constitute the heat balance at the interface: the turbulent heat flux in the ocean (Section 3.3.2), the conductive heat flux through the sea ice and the production/removal of latent heat.

The conductive heat flux through the sea ice \( Q_c \) is given as

\[
Q_c = -K_i \left. \frac{dT_i}{dz} \right|_b
\]

(3.9)

where \( T_i \) is the ice temperature and the subscript \( b \) indicates that the derivative is to be evaluated at the base of the ice. The vertical coordinate \( z \) increases upward, i.e., \( Q_c \) is positive for an upward heat flux. The value of the thermal conductivity \( K_i \) differs depending on ice properties. For fresh ice at 0°C and atmospheric pressure it is 2.16 W m\(^{-1}\) K\(^{-1}\). For sea ice, however, it is a function of temperature, bulk salinity
and density of the sea ice ($T_i$, $S_i$ and $\rho_i$). Following Pringle et al. (2007), it is given as

$$K_i = \frac{\rho_i}{917} \left(2.11 - 0.011 T_i + 0.09 \frac{S_i}{T_i}\right) \quad (3.10)$$

The 2007 study notes that there is some discrepancy between model and data, especially at temperatures near freezing, but this relationship is amply accurate for its purpose in this thesis.

The latent heat flux $Q_l$ produced by sea ice growing at a rate $m'$ is

$$Q_l = -\rho_w m' L_i \quad (3.11)$$

where the negative sign is needed to make $Q_l$ positive (a source term) for freezing ($m' < 0$). The latent heat of ice $L_i$ when pure is $3.35 \times 10^5$ J kg$^{-1}$. Again, following Pringle et al. (2007), for sea ice it is given as

$$L_i = 4187 \left(79.68 - 0.505 T_i - 0.0273 S_i + 4.3115 \frac{S_i}{T_i} + 0.0008 S_i T_i - 0.0009 T_i^2\right) \quad (3.12)$$

The final two terms are negligible and will be ignored hereafter.

The fluxes can now be equated. The divergence of heat flux balances the source (sink) of latent heat due to freezing (melting) (Holland and Jenkins, 1999).

$$Q_c - Q_T = Q_l \quad (3.13)$$

Substituting Equations 3.5, 3.9 and 3.11 gives

$$-K_i \frac{dT_i}{dz} \bigg|_b - \alpha_h u_s \rho_w c_w (T - T_b) = -\rho_w m' L_i \quad (3.14)$$

This is often written as

$$\frac{K_i}{\rho_w c_w} \frac{dT_i}{dz} \bigg|_b + \alpha_h u_s (T - T_b) = \frac{m' L_i}{c_w} \quad (3.15)$$

which is termed the “kinematic” form, i.e., divided by $\rho_w c_w$ with units of K m s$^{-1}$ (McPhee, 2008).

### 3.4.2 Temperature Gradient Estimation

In the heat budget, the basal temperature gradient is an input quantity. Unfortunately, it is quite variable. For an ice shelf there is no simple way to estimate its value without knowing the melt rate (Williams et al., 1998). For sea ice its value changes as air temperature and snow cover changes and sea ice thickens.

The ice shelf problem is less significant because the conductive heat flux is at least
two orders of magnitude smaller than the latent heat flux (Determann and Gerdes, 1994). Nevertheless, it is often estimated in some way. A common technique (e.g., Holland and Jenkins, 1999; Smedsrud and Jenkins, 2004; Jenkins, 2011) is to use the approximate relationship

\[ -K_i \frac{dT_i}{dz} \bigg|_b \approx \begin{cases} 
\rho_m c_i (T_b - T_s) & \text{if melting} \\
0 & \text{if freezing}
\end{cases} \]  

(3.16)

where \( c_i \) is the specific heat capacity of ice and \( T_s \) is the surface temperature of the ice shelf. This is derived by assuming constant vertical advection and vertical diffusion of heat into the ice shelf, then linearising the solution. The reason for the asymmetrical solution appears through consideration of what must happen to the temperature gradient if ice at its formation temperature is either removed (melting) or added (freezing) as shown in Figure 3.4.

Where sea ice is considered, a different method must be utilised as it is not thick enough to employ the method above. A common technique is to assume a linear temperature profile through the ice with the basal temperature equal to the freezing point. The problem is then reduced to estimating the temperature at the ice surface, but this is not easily deduced. The main inaccuracies associated with the linear method are due to changes in the temperature profile at the upper and lower edges of the sea ice.

At the top, snow is the biggest problem. Its thermal conductivity can range over one order of magnitude over its density range (Leppäranta, 1993). Hence, it provides varying levels of insulation. Brine is present throughout sea ice and, especially near the bottom, can alter the level of heat transfer. The simplest assumption is that the conductive heat flux at the interface is the same as it is at a point above the layer where convective heat transfer can occur. At this point, the temperature gradient and thermal conductivity are more easily estimated.
3.5 Salt Budget

The salt budget must be formulated differently to that of the heat budget, as shown in Figure 3.3b, because there is no analogue to the latent heat term. At the interface, during freezing, the excess salt at the base due to salt rejection—minus the salt trapped in the ice—must be removed by turbulent transfer. Equivalently, during melting, the basal salinity must be sustained by turbulent transfer overcoming the addition of meltwater.

Consider the control volume in Figure 3.3b as moving downward due to ice growth. As with the heat budget, there is a turbulent transfer term due to the difference in properties across a thin interfacial layer. Because of the downward movement of the control volume, two advection terms are needed to maintain the sea ice salinity at $S_i$ and the interfacial salinity at $S_b$. The interfacial salt balance is written by equating the net advection of salt to the rate of turbulent transfer.

$$\alpha_s u_*(S - S_b) = m'(S_b - S_i)$$ (3.17)

If the freezing rate is low, such as basal freezing beneath ice shelves, the bulk salinity of the ice is generally low and $S_i$ can be ignored (e.g., Jenkins, 1991; Holland and Jenkins, 1999; Holland and Feltham, 2006).

3.6 The “Three-Equation Formulation”

The heat and salt budgets (Equations 3.15 and 3.17) both contain the melt rate, basal temperature and basal salinity as unknowns. Hence, there are two coupled equations with three unknowns. By assuming all processes at the interface occur at the pressure- and salinity-dependent freezing temperature, a third equation (3.18) relating $T_b$ and $S_b$ allows determination of these unknowns (Holland and Jenkins, 1999). This is referred to as the “three-equation formulation”. To facilitate formation of a quadratic equation in $m'$, the relationship for the freezing point at the ice shelf base at a depth $z_b$ must be linearised.

$$T_b = a_{fp} S_b + b_{fp} + c_{fp} z_b$$ (3.18)

Table i (page xiii) gives the values of the constants $a_{fp}$, $b_{fp}$ and $c_{fp}$.

First, Equation 3.17 is solved for $S_b$,

$$S_b = \frac{m'S_i + \alpha_s u_* S}{m' + \alpha_s u_*}$$ (3.19)

By substituting $T_b$ from Equation 3.18 into Equation 3.15, a second relationship for
3.6 The “Three-Equation Formulation”

Sea ice

\[ T_i \]

\[ T_b \]

\[ T \]

\[ S_i \]

\[ S_b \]

\[ S \]

Figure 3.5 – Temperature and salinity profiles at a sea ice–ocean interface for (a) an ocean above its surface freezing point (b) a supercooled ocean. In both cases, the basal salinity exceeds the far-field salinity. The salinity difference \((S - S_b)\) is dependent on the freezing rate, which itself is dependent on the divergence of heat flux at the interface. Basal properties are related by a freezing point relationship and are assumed to convert to their far-field values across a thin layer (above the dashed line). Image concept from Jenkins and Bombosch (1995).

\[ S_b \] can be obtained.

\[ S_b = \frac{K_i}{\rho_w c_w} \frac{dT_i}{dz} \bigg|_b - \frac{m'}{c_w} + \alpha_h u_s \left( T - \left( b_{fp} + c_{fp} z_b \right) \right) \]

Equating the two terms for \( S_b \) and rearranging gives

\[ \left( -\rho_w L_i \right) m'^2 + \left( K_i \frac{dT_i}{dz} \bigg|_b + \alpha_h u_s \rho_w c_w \left( T - T_f(S_i) \right) - \alpha_s u_s \rho_w L_i \right) m' + \alpha_s u_s \left( K_i \frac{dT_i}{dz} \bigg|_b + \alpha_h u_s \rho_w c_w \left( T - T_f(S) \right) \right) = 0 \]

Equation (3.21)

where \( T_f(S_i) = a_{fp} S_i + b_{fp} + c_{fp} z_b \) and \( T_f(S) = a_{fp} S + b_{fp} + c_{fp} z_b \). This is straightforward to solve given the appropriate input variables and software.

The relationship between the unknown variables \((m', T_b \text{ and } S_b)\) is shown in Figure 3.5 for growing sea ice above both an ocean above its surface freezing point and a supercooled ocean. It is assumed that there is a thin interfacial layer in the ocean over which the temperature and salinity at the base of the ice change to their far-field values. In both cases, the basal salinity is higher than the corresponding far-field value due to salt rejection at the interface. In the first case, in which the far-field temperature exceeds the basal temperature, the positive oceanic heat flux reduces the ice growth rate. In the supercooled ocean case, the negative oceanic heat flux enhances both the ice growth rate and the basal salinity.
As described in Section 2.6.2 and shown in Figure 3.5, the bulk salinity of ice is determined to some extent by the freezing rate. This is not considered in the three-equation formulation. Instead, the bulk salinity of the sea ice is an input variable.

### 3.7 Frazil Ice Processes

Frazil ice is important to the formation of platelet ice and the sub-ice platelet layer (Sections 2.7 and 2.8) and modifying the ice pump mechanism (Section 3.2). In this section, the growth, nucleation and precipitation of suspended frazil ice is formulated mathematically. Flocculation (the joining of individual crystals) is not considered as it is insignificant in seawater (Martin, 1981).

Thorough reviews of frazil processes and their numerical treatment have been given by Daly (1984, 1994), Martin (1981) and Svensson and Omstedt (1994), representing a variety of possible formulations. This section will be restricted to analysis of the formulations used in the ISW plume models, in which each frazil crystal is treated as a circular disc with a fixed aspect ratio.

#### 3.7.1 Growth

It is possible to apply the three-equation formulation (Section 3.6) to frazil ice growth provided turbulent transfer parameterisations and ice properties are suitably adapted. Such a method was used by Jenkins and Bombosch (1995). However, they restricted their treatment to frazil crystals of fixed sized and averaged a number of important quantities over depth, thereby reducing the sophistication of their frazil ice treatment. Their approach contrasts with that followed in most studies, in which frazil growth follows the “two-equation formulation” (e.g., Smedsrud and Jenkins, 2004; Holland and Feltham, 2005, 2006).

The two-equation formulation does not explicitly involve the calculation of a quantity equivalent to $S_b$. Rather, salt’s rate-limiting effect is incorporated by using an “effective heat exchange coefficient”. One benefit of this simpler formulation is that an equation of equivalent complexity to Equation 3.21 need not be solved for every crystal size at every grid point and/or time step in a numerical routine. Furthermore, direct measurement of the salt fluxes at the edge of the crystals for either creation or validation of a salt transfer parameterisation would be nearly impossible. Indeed, salt flux measurements are considered difficult even at a metre scale (McPhee et al., 2008). At most, there would be only a small increase in accuracy achieved by using the three-equation formulation.

Ice grows most rapidly on the basal plane. A simplification often used in the literature is to assume that the heat transfer from a growing frazil ice crystal to the surrounding water occurs at only the disc’s edges. The rate of heat transfer $q_i$ from a
single crystal of radius \( r_i \) and thickness \( t_i \) to the surrounding seawater is

\[
q_i = \rho_w c_w \kappa_T \left( \frac{\Delta T}{l} \right) \frac{2 \pi r_i t_i}{l} \tag{3.22}
\]

where \( \kappa_T \) is the thermal diffusivity of seawater \((1.4 \times 10^{-7} \, \text{m}^2 \, \text{s}^{-1})\) and the bracketed term accounts for turbulence. \( \text{Nu} \), the Nusselt number, is the ratio of the actual heat transfer normal to the boundary to that by conduction alone. This is referenced to a temperature difference \( \Delta T \) over a characteristic length scale \( l \).

In the ISW plume models, \( \Delta T \) is the depth-averaged supercooling.

\[
\Delta T = T_f - T \tag{3.23}
\]

where \( T \) is the temperature of the plume and \( T_f \) is the freezing temperature averaged over the plume’s depth, which equals the freezing point at mid-depth due to the linear dependence of pressure on the freezing point. A relationship for \( T_f \) similar to Equation 3.18 provides the second equation in the two-equation formulation.

Estimation of \( q_i \), and hence the growth rate of a frazil crystal, is now reduced to the estimate of an appropriate length scale and Nusselt number relationship. Holland et al. (2007a) suggested future studies employ the relationship given in Daly (1984). They also noted that the relationship given in the 1984 study was presented in a possibly confusing way, which led to errors in several journal articles. Their revised presentation is given below.

The characteristic length scale of the thermal boundary layer should be the disc radius. This is scaled with respect to the Kolmogorov length scale \( \eta \), which is a measure of the length scale at which viscosity dominates turbulent mixing. It is defined as

\[
\eta = \left( \frac{\nu^3}{\varepsilon} \right)^{1/4} \tag{3.24}
\]

where \( \varepsilon \) is the turbulent dissipation rate and \( \nu \) is the kinematic viscosity of seawater \((1.95 \times 10^{-6} \, \text{m}^2 \, \text{s}^{-1})\).

The Nusselt number is given as

\[
\text{Nu} = \begin{cases} 
1 + 0.17 m^* \Pr^{1/2} & \text{if } m^* \leq \Pr^{-1/2} \\
1 + 0.55 m^{2/3} \Pr^{1/3} & \text{if } \Pr^{-1/2} < m^* \leq 1 \\
1.1 + 0.77 \alpha_T^{0.035} m^{2/3} \Pr^{1/3} & \text{if } m^* > 1 
\end{cases} \tag{3.25}
\]

where \( m^* = r_i / \eta \) is the ratio of the disc radius to the Kolmogorov length scale and \( \alpha_T \) is the turbulent intensity as described by Daly (1984, 1994). Given that \( \alpha_T \) only weakly affects \( \text{Nu} \), \( \alpha_T \) can safely be taken as constant \((0.2)\). A fourth case, for a high-turbulence regime, is given in Holland et al. (2007a) but is not needed here.

Jenkins and Bombosch (1995) suggested \( \eta \sim 1 \, \text{mm} \) for ISW flow, which suggests a
Figure 3.6 – The Nusselt number as a function of frazil crystal radius for three likely estimates of the Kolmogorov length scale $\eta$. Formulation from Daly (1984) and Holland et al. (2007a).

The increase in Nu with an increase in $m^*$ represents the increased levels of turbulence experienced by larger crystals. Small crystals feel turbulence as a varying, laminar flow field, whereas turbulence around larger crystals enhances mixing of the thermal boundary layer (Jenkins and Bombosch, 1995; Crook, 2010).

Salt’s rate-limiting impact on growth was noted earlier, but has yet to be quantified. Smedsrud and Jenkins’s (2004) approach was to mimic the rate-limiting impact by using an effective value for Nu of less than one which, by definition, is its lower limit. Reduction by a factor anywhere between 1/1.6 and 1/5.7 was suggested. Their argument was based on work by Holland and Jenkins (1999) who showed that the rate-limiting effect of salt at the base of an ice shelf can be quantified by a single parameter that depends on the ratio of turbulent transfers of heat and salt. Smedsrud and Jenkins (2004) treated the changes to Nu as sensitivity tests and ultimately used $\text{Nu} = 1$ as their standard value, which suggests salt has no effect. The choice of $\text{Nu}$ altered the plume’s supercooling but typical values for the depth-averaged supercooling were less than 1 mK even when Nu was set at its lower limit. This level of supercooling is much lower than that typically experienced by a growing frazil crystal.

The “effective Nusselt number” method is common in the literature and will be used in Chapter 5. However, a more robust formulation of the depth-averaged supercooling will be devised.

It remains to be shown how the heat transfer from an individual crystal relates to
the change in frazil population. When it was first incorporated into a plume model by Jenkins and Bombosch (1995), frazil ice was restricted to a single size class—a major simplification in comparison to reality. However, this provides a good starting point for the derivation of the rate of change of frazil ice concentration.

Frazil concentration $C_i$ is defined as the volume of frazil ice suspended in a unit volume $\Delta V$ and so has “units” of $\text{m}^3 \text{m}^{-3}$. The volume of a single frazil ice crystal is denoted as $v_i = \pi r_i^2 t_i$, so $C_i$ can be written as

$$C_i = \frac{N v_i}{\Delta V} \quad (3.26)$$

where $N$ is the number of frazil crystals in volume $\Delta V$.

Equating the heat flux from $N$ crystals to the latent heat required to produce an ice mass growth rate of $dm_i/dt$ gives

$$L \frac{dm_i}{dt} = N q_i \quad (3.27)$$

The frazil ice formed is assumed to be pure, i.e., all salt is rejected from its structure. To save confusion with the latent heat of sea ice ($L_i$), the latent heat of pure ice will always be written without the subscript $i$.

The ice mass can be rewritten as

$$\frac{dm_i}{dt} = \rho_i \frac{dN v_i}{dt} = \rho_i \Delta V \frac{dC_i}{dt} \quad (3.28)$$

Hence, Equation 3.27 becomes

$$L \rho_i \Delta V \frac{dC_i}{dt} = N q_i \quad (3.29)$$

$q_i$ is given in Equation 3.22 (with $l = r_i$ and $\Delta T = T_f - T$) and $N$ is given in Equation 3.26. Therefore

$$L \rho_i \Delta V \frac{dC_i}{dt} = \left( \frac{\Delta V C_i}{v_i} \right) \left( \rho_w c_w \kappa_T \frac{Nu}{r_i} \frac{T_f - T}{2 \pi r_i t_i} \right) \quad (3.30)$$

$$L \rho_i \frac{dC_i}{dt} = C_i \frac{C_i}{\pi r_i^2 t_i} \rho_w c_w \kappa_T \frac{Nu}{(T_f - T) 2 \pi t_i} \quad (3.31)$$

$$\frac{dC_i}{dt} = \frac{\rho_w c_w \kappa_T}{L} \frac{Nu}{(T_f - T) 2 \pi r_i^2} C_i \quad (3.32)$$

Heat diffusion into the crystal is negligible and all salt is assumed to be expelled on freezing. Because the crystal radius is fixed, frazil growth manifests itself as an increase in the number of crystals. Equation 3.32 should be the same as Smedsrud and Jenkins’s (2004) Equation 11, yet it differs by a factor of $\rho_w/\rho_i$. An explanation
of why requires the definition of the “frazil melt rate” \( f' \), the frazil ice analogue of \( m' \),

\[
 f' = -D \frac{\rho_i}{\rho_w} \frac{dC_i}{dt} \quad (3.33)
\]

where \( D \) is the thickness of the plume layer. Like \( m' \), \( f' \) is negative for ice growth. The density ratio transforms \( f' \) into an equivalent seawater transport rate.

Simple inspection shows that the \( \rho_w/\rho_i \) factor in Equation 3.32 will be cancelled by its reciprocal in Equation 3.33. In their formulation, Smedsrud and Jenkins (2004) removed both factors, and as a result their output was not affected. However, the author considers their approach confusing. It means concentration is defined in terms of an ice volume, but its rate of change is defined in terms of an equivalent seawater volume. In two of their studies, Holland and Feltham (2005, 2006) cite Smedsrud and Jenkins’s (2004) confusing term for \( dC_i/dt \) and consequently end up including an extra factor of \( \rho_w/\rho_i \) in their equation for \( f' \).

Equations 3.32 and 3.33 still apply if the model contains a range of crystal sizes (e.g., Smedsrud and Jenkins, 2004; Holland and Feltham, 2006). \( C_i, f' \) and \( r_i \) are simply replaced by their counterparts for every size class \( k \), i.e., \( C_i(k), f'(k) \) and \( r_i(k) \). The concept of growth, however, becomes more complicated. As shown in Figure 3.7, the concentration of a particular size class decreases when its own crystals grow because they are transferred to the size class above, and its concentration increases when crystals grow from the size class below. Although the number of crystals can increase through secondary nucleation (see Section 3.7.2), this is no longer a requirement for manifestation of growth. Instead, the transfer of specific numbers of crystals from each size class to the one above will result in an increase in total concentration.

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**Figure 3.7 –** Schematic diagram of a multiple-size-class frazil ice model. Size classes, \( k \), are numbered 1 to \( N_{\text{ice}} \). The radius increases discretely, i.e., it is fixed for each size class. The enlarged shaded square shows that the frazil concentration in its own size range can increase due to growth of crystals from below, and decrease due to growth of its own crystals, precipitation and secondary nucleation. The shaded square is representative of all size classes except the first and last. Crystals in the largest class cannot grow, while secondary nucleation is a source instead of a sink for crystals in the smallest size class. Image concept from Svensson and Omstedt (1994).
Growth across the size classes can be written as

\[ f'(k) = -D \frac{\rho_i}{\rho_w} \left( \frac{dC_i(k-1)/dt}{\Delta v_i(k-1)} - \frac{dC_i(k)/dt}{\Delta v_i(k)} \right) v_i(k) \] (3.34)

where \( \Delta v_i(k) = v_i(k+1) - v_i(k) \). The first term in the brackets is the number density increase in class \( k \) due to a transfer of crystals from the class below. The second term is the equivalent for a transfer out of class \( k \) to the class above. In this formulation growth cannot occur for the largest size class, which is not a problem provided that the largest size class represents, at most, only a small percentage of the total frazil volume.

The importance of having a model with multiple size classes is shown by comparison of the results of Jenkins and Bombosch (1995) with that of Smedsrud and Jenkins (2004). The former study had only a single size class which led to an unphysical prediction of oscillating precipitation rates. This prediction disappeared in the latter study because differently-sized crystals precipitate at different rates. A fixed-crystal-size formulation also has implications for predicted growth rates, as growth is strongly dependent on crystal radius (Equation 3.32).

3.7.2 Nucleation

The frazil growth formulation relies on seed crystals being present, but their genesis was not explained. Unfortunately, this process is not well understood. Homogeneous nucleation—the spontaneous formation of ice crystals in supercooled water without the aid of impurities—is impossible because it requires supercooling of a magnitude far greater than that observed in the ocean (Forest, 1994; Martin, 1981). Heterogeneous nucleation—formation of ice crystals at impurities—is also ruled out for the same reason (Daly, 1984; Jenkins and Bombosch, 1995).

For modelling purposes, the actual physical mechanism(s) need not be understood. In nature, seawater seldom exceeds supercooling of more than 0.1 K, suggesting seed crystals must be readily available (Martin, 1981; Eicken, 2003; Holland and Feltham, 2006). In the ISW plume models a low concentration of seed crystals is added at the moment the plume becomes supercooled. An argument for this method is that basal freezing—resulting from supercooling—produces dendritic structures, i.e., fragile ice crystals, at the ice–ocean interface that can then be removed and suspended by turbulence (Jenkins and Bombosch, 1995).

Secondary nucleation—the creation of new seed crystals from collisions between existing crystals and detachment of surface irregularities—is also not properly understood (Daly, 1994). However, it is possible to estimate the number of seed crystals produced given the concentration and size distribution of the crystals. In the ISW plume models, secondary nucleation is based on work by Svensson and Omstedt (1994) and Smeldsrud (2002). The derivations in each paper were only outlined. Here a number of intermediate steps are added for clarity.
Following Svensson and Omstedt (1994), a single crystal of radius $r_i(k)$ moving at a velocity $U_r$ relative to the fluid sweeps out a volume $V_i(k)$ at a rate

$$\frac{dV_i(k)}{dt} = U_r \pi r_i(k)^2$$

(3.35)

$U_r$ incorporates both the rise velocity and turbulent intensity and is given by

$$U_r(k) = \sqrt{\frac{\varepsilon}{15\nu}} (2r_i(k))^2 + w_i(k)^2$$

(3.36)

where $w_i$ is the crystal’s rise velocity (see Section 3.7.3).

Smedsrud (2002) suggested that the radius in the previous two equations should be replaced by the effective radius $r_{ie}$ to account for the nonsphericity of the frazil crystals. The effective radius is the radius of a sphere with the same volume as a disc. Therefore,

$$r_{ie} = \left(\frac{3}{2} a_r\right) \frac{1}{3} r_i$$

(3.37)

where $a_r$ is the disc’s aspect ratio (thickness/diameter).

The likelihood of a collision between a single crystal and any other crystal must be proportional to the total number of other crystals. Consider a crystal number density defined as

$$\pi_i = \sum_{k=1}^{N_{ice}} \frac{C_i(k)}{v_i(k)}$$

(3.38)

where $N_{ice}$ is the number of frazil size classes. If the total volume is divided evenly by the total number of crystals then each crystal occupies a volume of $1/\pi_i$.

Let $\Delta t(k)$ denote the time it takes for all of the crystals in size class $k$, denoted $N(k)$, to sweep out a total volume of $1/\pi_i$.

$$N(k) \frac{dV_i(k)}{dt} \Delta t(k) = \frac{1}{\pi_i}$$

(3.39)

The frequency of collisions, $f_{coll}$, is therefore

$$f_{coll}(k) = \frac{1}{\Delta t(k)} = \pi_i N(k) \frac{dV_i(k)}{dt}$$

(3.40)

By assuming all collisions result in a new crystal belonging to the smallest size class, the collision frequency can be related to the rate of seed crystal production.

$$\frac{dN(k = 1)}{dt} = \sum_{k=2}^{N_{ice}} f_{coll}(k)$$

(3.41)

This can be converted to a concentration rate of change by employing Equation 3.26.
and assuming every collision results in a crystal belonging to size class $k = 1$.

$$\frac{dN(k = 1)}{dt} = \sum_{k=2}^{N_{ic}} \tilde{m}_i N(k) \frac{dV_i(k)}{dt}$$ (3.42)

$$\frac{\Delta V}{v_i(k = 1)} \frac{dC_i(k = 1)}{dt} = \sum \tilde{m}_i \left( \frac{\Delta V C_i(k)}{v_i(k)} \right) \left( U_r(k) \pi r_{ie}(k)^2 \right)$$ (3.43)

$$\frac{dC_i(k = 1)}{dt} = \sum \pi \tilde{m}_i \left( \frac{U_r(k)}{r_{ie}(k)} \right) r_{ie}(k = 1)^3 C_i(k)$$ (3.44)

$$\frac{dC_i(k = 1)}{dt} = \sum \pi \tilde{m}_i \frac{U_r(k)}{r_{ie}(k)} r_{ie}(k = 1)^3 C_i(k)$$ (3.45)

Unlike $dC_i/dt$ calculated for growth (Equation 3.32), the result above does not differ from the equivalent equation in Smedsrud and Jenkins (2004). Indeed, there is a slight inconsistency between the formulations given in Smedsrud (2002) and Smedsrud and Jenkins (2004): for growth, $dC_i/dt$ differs by a factor of $\rho_w/\rho_i$, but there is no such difference for secondary nucleation.

The loss from all other classes is simply

$$\frac{dC_i(k)}{dt} = -\pi \tilde{m}_i \frac{U_r(k)}{r_{ie}(k)} r_{ie}(k = 1)^3 C_i(k) \quad (2 < k < N_{ic})$$ (3.46)

Experimental results show that secondary nucleation is far less efficient than suggested by the formulation above. This is not surprising given that it assumes every collision results in a new crystal and that the motion of individual crystals is in no way affected by the presence of other, nearby crystals. In order to tune the formulation to experimental data, $\tilde{m}_i$ is given an upper limit of $1.0 \times 10^4 m^{-3}$, hereafter denoted $\tilde{m}_{max}$. Physically, this is equivalent to limiting the number of other crystals (potential collision sites) “seen” by each individual crystal. For typical frazil concentrations, the upper limit on $\tilde{m}_i$ is less than the value calculated from Equation 3.38 by orders of magnitude. Hence, the choice of the upper limit significantly affects the predicted efficiency of secondary nucleation.

### 3.7.3 Precipitation

Estimates of the vertical transport and precipitation of frazil ice are not straightforward. Daly (1994) cites a number of unknowns that must be determined in order to accurately assess the vertical transport: size distribution, shape and buoyancy of the crystals and a description of the turbulent flow. Here, however, the use of empirical parameterisations largely overcomes such problems. The resulting mathematical treatment is simpler than that of growth and secondary nucleation as it is assumed that there are no interactions between different size classes.

In the ISW plume literature the formulation of frazil rise velocity and precipitation has not changed since it was first introduced by Jenkins and Bombosch (1995). The
rise velocity $w_i$ in stagnant water is derived from a simple force balance given by Gosink and Osterkamp (1983).

$$w_i = \sqrt{\frac{4(\rho_w - \rho_i) g a_ir_i}{\rho_w c_d}}$$

(3.47)

where $g$ is the acceleration due to gravity and $c_d$ is the disc’s drag coefficient—not to be confused with $C_d$, the drag coefficient for the ice–ocean interface. The value for $c_d$ cannot be known without knowing the rise velocity and so must be found iteratively using an empirical relationship involving $Re_d$, the disc Reynolds number.

$$\log_{10}(c_d) = 1.386 - 0.892 \log_{10}(Re_d) + 0.111 (\log_{10}(Re_d))^2$$

(3.48)

$$Re_d = \frac{2w_ir_i}{\nu}$$

(3.49)

As shown in Figure 3.8a, the empirical relationship for $c_d$ is a very good fit. This would suggest rise velocities calculated from good estimates of $c_d$ should match well with experimental data. However, this is not necessarily the case, as shown in Figure 3.8b. The predicted rise velocity for two different aspect ratios is compared with field and laboratory measurements, which show much variation. There are a number of reasons for this: frazil crystals of a range of aspect ratios would have been present in the experiments; velocities and crystal diameters were estimated by eye by comparison with graduations on a transparent cylinder; and the proximity of other crystals and the cylinder walls may have affected the rise velocities.

To calculate actual precipitation rates, the rise velocity is incorporated into a formulation involving an estimate of the critical velocity below which frazil may precipitate. Jenkins and Bombosch (1995) again followed a semi-empirical approach by others, this time the work of McCave and Swift (1976). The precipitation rate $p'$ is measured as an equivalent seawater thickness rate of change and is negative for a loss of ice from the plume. It is given by

$$p' = -\frac{\rho_i}{\rho_w} C_i w_i \cos(\vartheta) \left( 1 - \frac{U^2}{U_c^2} \right) \text{He} \left( 1 - \frac{U^2}{U_c^2} \right)$$

(3.50)

where $U$ is the plume velocity, $U_c$ is a frazil-size-dependent critical velocity above which precipitation cannot occur (see Equation 3.51) and $\vartheta$ is the ice shelf basal gradient (in a direction parallel to the velocity). The inclusion of the Heaviside function (He) indicates that erosion of deposited frazil cannot occur.

Equation 3.50 can be extended to multiple frazil size classes by simply making $C_i$, $w_i$, and $U_c$ a function of $k$. $U^2$ may be replaced with either its two-dimensional equivalent (Holland and Feltham, 2006) or $U^2 + U_T^2$.

Given that $\vartheta$ is an input and $U$ is predicted by a plume model (see Section 3.10), the only quantity remaining to be found is the critical velocity. Jenkins and Bombosch’s
(1995) approach involved the Shields criterion, which is usually applied in a sediment erosion context. Taking the result of Jenkins and Bombosch’s derivation, the critical velocity as a function of crystal radius is given by

\[ U_c^2 = \frac{2\vartheta_{sc} (\rho_w - \rho_i) g r_i}{\rho_w C_d} \]  

(3.51)

where \( \vartheta_{sc} \) is the critical Shields criterion, a non-dimensionalised shear stress taken to have a constant value of 0.05.

Their derivation was based on the grain Reynolds number \( \text{Re}_g \), defined as

\[ \text{Re}_g = \frac{2u^* r_i}{\nu} \]  

(3.52)

which is different to the disc Reynolds number defined earlier.

Despite recognising that their relationship was valid for only \( \text{Re}_g \) greater than \( \sim 1 \) (\( r_i \sim 1 \text{ mm} \)), they applied it to all crystal sizes and this has not changed in subsequent versions of the plume model, despite crystals with radii less than 1 mm often constituting the majority of the total frazil concentration. In seeking to fix this inconsistency, Crook (2010) considered what change was needed for \( \text{Re}_g < 1 \), where the critical Shields criterion increases significantly. He suggests the best estimate for
Figure 3.9 – Critical velocity for precipitation as a function of frazil crystal radius. The formulation by Jenkins and Bombosch (1995) is used for all crystal sizes in all versions of the ISW plume models. The formulation in Crook (2010) is a suggested amendment for \( \text{Re}_g < 1 \). An aspect ratio of 0.02 and a drag coefficient of \( 2.5 \times 10^{-3} \) were used.

\[ \vartheta_{sc} = 0.075 \text{Re}_g^{-0.41} \]  

(3.53)

Crook (2010) also considered a more recent study by Cao et al. (2006), which has previously been applied to frazil modelling in McMurdo Sound (McGuinness et al., 2009), but he suggested this overestimates the critical velocity. The critical velocity as a function of crystal radius (Equation 3.51) is shown in Figure 3.9 for both a constant Shields criterion of 0.05 as used in the plume models and the alternative formulation given in Equation 3.53 for the low \( \text{Re}_g \) regime. An aspect ratio of 0.02 and a drag coefficient of \( 2.5 \times 10^{-3} \) were used for evaluation. Overall, the change in the critical velocity between the two formulations is small. The difference is only important for sub-millimetre-radius frazil crystals in a situation where the plume and/or tidal velocities are less than approximately 0.1 m s\(^{-1}\).

In an attempt to explain the absence of platelet ice in the top metre of McMurdo Sound ice cores, McGuinness et al. (2009) hypothesized that brine rejection would augment tidally-driven turbulence in slowing precipitation. However, cores shown in Chapter 4 do not display this absence. Also, Gough et al. (2012b) suggest the brine rejection hypothesis is unlikely. Hence, this idea is not considered any further.

### 3.8 Derivation of a Simple Inclined Plume

*Plumes* are parcels of buoyant (negatively buoyant) fluid rising (falling) through and entraining denser (less dense) ambient fluid. They are common in a number of environmental conditions: gas and ash from erupting volcanoes, hot water rising above hydrothermal vents, and thermals in the atmosphere (List, 1982; Turner, 1986). When
a plume is constrained to travel primarily in a horizontal direction, the term *gravity current* is often used. Examples include katabatic winds, freshwater discharging into seawater, turbidity currents and the flow of Antarctic Bottom Water down the continental shelf (Ellison and Turner, 1959; Simpson, 1982; Baines and Condie, 1998). In this thesis, the two emphasised terms are effectively interchangeable. A derivation of a steady-state, positively-buoyant, inclined plume without any ice–ocean interactions is the purpose of this section, providing a foundation for the formulation of a steady-state ISW plume in Section 3.10.

Consider the long-sectional view of a plume in Figure 3.10a. It rises up the inclined base as a turbulent flow, entraining the ambient fluid beneath it, which causes the plume to thicken, slow and become denser. The following derivation, using notation shown in Figure 3.10b, will assume the plume is in a well-developed steady state, i.e., plume properties vary spatially, but not temporally. Starting plumes are more complicated as vortex-mixing processes at the head of the current need to be considered (Turner, 1962).

Plume properties (density $\rho$, velocity $\mathbf{u}$ and depth $D$) are determined by mass and momentum conservation. Assuming steady state and ignoring vorticity and Earth’s rotation, the conservation equations are given by

\begin{align}
\nabla \cdot (\rho \mathbf{u}) &= 0 \quad (3.54) \\
\mathbf{u} \cdot \nabla (\rho \mathbf{u}) &= -\frac{\rho C_d U^2}{D} \mathbf{s} + \rho g' \mathbf{z} \quad (3.55)
\end{align}

where $\rho_a$ is the ambient fluid’s density, $\mathbf{s}$ and $\mathbf{z}$ are unit vectors in the along-plume and vertical directions, respectively, and $g'$ is the reduced gravity defined as

$$g' = \frac{\rho - \rho_a}{\rho} g$$

\[3.56\]
The terms on the right of the Equation 3.55 represent the force per unit volume due to drag and buoyancy. The quadratic drag term is an approximation of the stress at the upper surface, which should not, in general, be written as a force per unit volume. However, as Equation 3.55 will later be depth-averaged, it produces the same result as the more formal method of using the divergence of the Reynolds stress tensor (e.g., Jenkins and Bombosch, 1995). A pressure gradient term should also appear on the right side of Equation 3.55, but it is negligible in comparison to the other terms.

Equations 3.54 and 3.55 can be simplified by applying the Boussinesq approximation, i.e., density differences are negligible unless in terms multiplied by \( g \). Hence, \( \rho \) can be taken out of the derivative. Rewriting the equations without vector notation, and considering only the along-plume component of the momentum balance, gives

\[
\frac{\partial U}{\partial s} + \frac{\partial W}{\partial n} = 0 \tag{3.57}
\]

\[
\frac{\partial U^2}{\partial s} + \frac{\partial (UW)}{\partial n} = -\frac{C_d U^2}{D} + g' \sin(\vartheta) \tag{3.58}
\]

where \( n \) is the coordinate perpendicular to the basal surface.

The mass conservation equation can be depth-averaged as follows.

\[
\int_{-D}^{0} \left( \frac{\partial U}{\partial s} + \frac{\partial W}{\partial n} \right) \, dn = 0 \tag{3.59}
\]

\[
\frac{\partial}{\partial s} \int_{-D}^{0} U \, dn = -[W]_{-D}^{0} \tag{3.60}
\]

\[
\frac{\partial (DU)}{\partial s} = e' \tag{3.61}
\]

where \( e' \), the entrainment velocity, is the velocity normal to and at the base of the plume. It is the rate at which ambient fluid is entrained into the plume. A full discussion of a parameterisation for \( e' \) is deferred until the following section. For now it is simply asserted that \( e' \) generally increases with velocity.

The momentum conservation equation can be similarly depth averaged. The result is

\[
\frac{\partial (DU^2)}{\partial s} = -C_d U^2 + Dg' \sin(\vartheta) \tag{3.62}
\]

It is easy to see from Equations 3.61 and 3.62 that the plume’s depth and velocity are coupled variables. Closer inspection shows an important feedback. An increase in the plume’s velocity causes a decrease in its depth, but this increase is counteracted by the increase in both drag and entrainment. Entrainment counteracts the velocity increase because it causes an increase in mass without a corresponding increase in momentum.
3.9 Entrainment

Entrainment describes the mixing of ambient fluid into a plume (or jet) which, for geophysical-scale flows, will become turbulent at a small distance from its source (Turner, 1986). The entrainment mechanism of ISW flow is not well understood and it would be extremely difficult to observe directly. Its formulation in ISW plume models is based on its similarity to other plumes/Gravity currents.

Entrainment is generally considered to be a function of the Richardson number, $Ri$ (Ellison and Turner, 1959; List, 1982; Hunt and van den Bremer, 2011), which is the ratio of potential energy to kinetic energy. For the plume described in the previous section, it becomes

$$Ri = \frac{g'd}{U^2} \cos(\vartheta)$$

A $\cos(\vartheta)$ term should be included if the slope is significant, but here it is assumed $\cos(\vartheta) \approx 1$.

Entrainment is often set to be linearly dependent on the plume’s velocity. Turner (1986) termed this proportionality the “entrainment assumption”. For the simple case of a plume rising through a homogeneous ambient fluid without drag at the boundary, the assumption leads to a solution which predicts that the plume’s width increases linearly with distance from the buoyancy source.

The surface slope may also affect the entrainment rate, as it affects both the stability of the interface and flow speed (Jenkins, 1991). Bo Pederson (1980) suggests that entrainment mechanism of interfacial wave breaking is more effective with steeper slopes. For sufficiently shallow slopes—comparable to those in the ocean—Baines (2001a) suggests that fluid also detrains from the main flow since fluid in the outer part is not well mixed. This can be seen in photographs from his laboratory experiments (shown here in Figure 3.11). Also evident is the change of intensity of entrainment with a change in slope. The experiments were on negatively-buoyant (high-density) flows, which can be considered as inverted, positively-buoyant flows.

![Figure 3.11](image-url)
Given the physical behaviour described so far, entrainment can be written as

\[ e' = f(Ri) U \quad (3.64) \]

where the \( f \) is a function that may relate the Richardson number to the slope. In the ISW plume models, a simple relationship suggested by Bo Pederson (1980), originally for dense bottom currents on shallow slopes \((< 10^{-2})\), is used. The entrainment rate is expressed as

\[ e' = E_0 \sin(\theta) U \quad (3.65) \]

where \( E_0 \) is a constant termed the entrainment coefficient with a value of 0.036. Its value is half that originally set by Bo Pederson (1980). The decrease compensates for the neglect of the Coriolis effect, which would otherwise deflect the plume such that it flows along a shallower slope (MacAyeal, 1985a; Jenkins, 1991). More complex entrainment formulations have been used in recent ISW plume models (e.g., Holland and Fèltham, 2006), but the predicted entrainment rate is not significantly altered.

Detrainment is not considered in the ISW plume models. Therefore, the bottom of the plume acts as a one-way gate: ambient fluid is allowed in, but the plume-layer fluid is not allowed out. A tidally-driven, mixed-layer model by Scheduikat and Olbers (1990) incorporated detrainment, allowing a more realistic interaction between the mixed layer and the layer below, and detrainment phases were predicted during periods of reduced tidal flow. The distinction between entrainment and detrainment was important as it lead to a change in the predicted melt rate (Williams et al., 1998).

### 3.10 The SJ04 ISW Plume Model

The SJ04 model, i.e., the steady-state ISW plume model by Smedsrud and Jenkins (2004), which is to be adapted in Chapter 5, contains a number of processes: interactions at the ice shelf base (Sections 3.3–3.6); growth, secondary nucleation and precipitation of frazil ice (Section 3.7); mass and momentum conservation (Section 3.8); and entrainment (Section 3.9). The addition of heat, salt and ice conservation equations (and the effects of ice growth/decay on mass conservation) completes the SJ04 model. The plume itself is characterised by its depth-averaged velocity, temperature, salinity and frazil concentration as shown in Figure 3.12.

Two terms are added to the mass conservation equation: advection of water out of (into) the plume due to freezing (melting) and precipitation of frazil. Following the process described by Equations 3.59–3.61, mass conservation for the mixture of seawater and frazil ice can be written as

\[ \frac{\partial (DU)}{\partial s} = e' + m' + p' \quad (3.66) \]

Note that \( p' \) is always a negative quantity. If considering only the water fraction of
3.10 The SJ04 ISW Plume Model

Figure 3.12 – Schematic diagram of the SJ04 model showing the parameters and frazil processes treated in the one-dimensional, steady-state model. Figure from Smedsrud and Jenkins (2004).

the plume, precipitation need no longer be considered. However, frazil growth ($f'$) now acts as a "mass sink". Conservation of the water mass fraction becomes

$$\frac{\partial (DU)}{\partial s} = e' + m' + f'$$

(3.67)

Strictly speaking, there should be a $1 - C_i$ term inside the derivative. Here and elsewhere, it is assumed to equal one since the frazil ice concentration is very small. This appears to make Equations 3.66 and 3.67 inconsistent. However, this is not a problem as each equation is used for a different purpose. The plume’s momentum is dependent on the total mass of the seawater/ice mixture, meaning the momentum conservation equation must be coupled with Equation 3.66. Conversely, heat and salt are assumed to be stored in only the water fraction, suggesting heat and salt conservation equations must be coupled with Equation 3.67.

Ice mass (in each size class) increases by frazil growth ($f' < 0$), but decreases if precipitation occurs ($p' < 0$). Given that both $p'$ and $f'$ are expressed as a water equivalent density, ice conservation becomes

$$\frac{\partial (DU \rho_i C_i(k))}{\partial s} = -\rho_w (f'(k) - p'(k))$$

(3.68)

Now consider heat conservation. Freezing (melting) at the interface advects seawater of temperature $T_b$ out of (into) the plume. Consider an arbitrary reference temperature of zero such that water at $T_b$ has an associated thermal energy per unit volume of $\rho_w c_w T_b$. By this method, freezing leads to a heat flux of $-\rho_w c_w m'T_b$, or simply $-m'T_b$ when written in kinematic form. The equivalent quantity added through entrainment is $e'T_a$, where $T_a$ is the temperature of the ambient seawater.
The heat flux to the plume’s water fraction due to frazil ice growth is

\[(\rho_w c_w \bar{T}_f - \rho_w L) f'\]  

(3.69)

The first term is the removal of thermal energy of seawater (at temperature \(\bar{T}_f\)) from the water fraction. The second, more-significant term is the production of latent heat from growing frazil ice.

Heat conservation in the plume layer is completed by adding the conductive and latent heat fluxes at the ice–ocean interface (Equations 3.9 and 3.11).

\[
\frac{\partial(DUT)}{\partial s} = e'T_a + m'T_b + \left(\bar{T}_f - \frac{L}{c_w}\right) f' + \frac{K_i}{\rho_w c_w} \left.\frac{dT_i}{dz}\right|_b - \frac{m'L_i}{c_w} 
\]  

(3.70)

When the heat transfer velocity (Equation 3.1) is used, Equation 3.70 is often written in an alternative form by replacing the last two terms by \(-\gamma_T(T - T_b)\), which comes from the heat balance at the interface (similar to Equation 3.15). Indeed, this would return the equation to the form given by Smedsrud and Jenkins (2004).

Salt conservation is simpler than heat conservation. Seawater of salinity \(S_a\) is entrained from below and some salt becomes trapped in ice of salinity \(S_i\).

\[
\frac{\partial(DUS)}{\partial s} = e'S_a + m'S_i 
\]  

(3.71)

The salinity, temperature and ice concentration all alter the buoyancy of the plume. A dimensionless density contrast \(\Delta \rho\) between the plume and the ambient seawater can be written as

\[
\Delta \rho = \beta_S (\overline{S}_a - S) - \beta_T (\overline{T}_a - T) - C_i \left(\frac{\rho_i - \rho_w}{\rho_w}\right) 
\]  

(3.72)

where \(\beta_S\) and \(\beta_T\) are constants defined in Table i (page xiii) and \(\overline{S}_a\) and \(\overline{T}_a\) represent averages of the ambient temperature and salinity over the plume’s depth, if the plume was not present. Equation 3.62 then needs to be altered by replacing \(g'\) with \(\Delta \rho g\).

### 3.11 Traits of ISW Plume Models

Modelled ISW plume velocities are typically several centimetres per second (several km day\(^{-1}\)), with corresponding depths are several tens of metres. Frazil concentrations seldom exceed 1 g L\(^{-1}\), and precipitation rates are of the order of 1 m yr\(^{-1}\) (Smedsrud and Jenkins, 2004).

All ISW plume models lack realistic treatment of the effects of tidal flow. Increased turbulence from tidal flow has been accounted for in the treatment of drag, frazil precipitation and interfacial heat transfer. However, the plume’s actual path is not affected. In reality, the plume is likely to exhibit a net motion superimposed on a
back-and-forth motion caused by tidal flow.

Plume models also neglect seafloor effects, which overlooks the possibility of confinement of circulation inside the ice shelf cavity. Both tidal models and models with a vertical dimension have suggested that an overturning circulation cell is possible in the long, narrow cavity beneath an ice shelf (Williams et al., 1998). This is not possible in a plume model, where there is effectively a semi-infinite domain, i.e., there is an upper surface, but no lower surface.

Williams et al. (1998) suggested that the bias toward models with thermohaline forcing—a category to which plume models belong—may have come from conclusions drawn by MacAyeal (1984, 1985b). These earlier works indicated that, although important, tidal processes were incapable of ventilating the ice shelf cavity on observed time scales.

Lane-Serff (1995) demonstrated that properties of the ambient water were the most influential input parameters in a plume model, with ambient temperature being the primary control on the distribution of melting and freezing. Jenkins (1991) tested the sensitivity of his plume model to an ambient temperature increase from $-1.9^\circ C$ to $-1.3^\circ C$. This resulted in a mean melt rate increase from $0.6 \text{ m yr}^{-1}$ to $2.6 \text{ m yr}^{-1}$. In contrast, the melt/freeze distribution is insensitive to the absolute ambient salinity. However, its gradient is important because it controls the density gradient, which determines where the plume becomes neutrally buoyant (Jenkins and Bomboesch, 1995).

The depth of the grounding line and the slope also influence the melt/freeze distribution. The former determines the effective amount of sensible heat available to melt ice at depth. The latter determines how quickly the plume rises back to the surface, which then influences both the efficiency of turbulent transfer and where frazil precipitates.

The presence of frazil ice alters a plume model in two ways. First, ice can grow more efficiently due to the increase in the total available ice surface area. This results in an increase to the total basal accumulation rate. Second, frazil ice decreases the plume’s density. Without frazil, ice formation occurs at only the ice–ocean interface, which results in brine rejection and therefore a density increase.

Two insignificant input parameters in plume models are the initial meltwater flux and the ice shelf temperature. It is convenient that the initial meltwater flux is unimportant because processes at the grounding line are complex and predicting the flux is a challenge (e.g., Jenkins, 2011). The ice shelf temperature is unimportant because conduction into the ice shelf is only a small part of the heat budget (Section 3.4.2). Smedsrud and Jenkins (2004) found that frazil concentration was insensitive to the effective Nusselt number, i.e., the rate of heat transferred away from frazil crystals. However, as described in Section 3.7.1, the average supercooling experienced by the frazil crystals was unrealistically small.
3.12 Complex Plume Models

The Coriolis effect was not considered in plume models until the work by Holland and Feltham (2006). They noted that, up until then, the dynamics of plume models were limited in that the path must be chosen beforehand. In fact, Smedsrud and Jenkins (2004) prescribed their plume paths based simply on tracks where basal elevation data were available.

The main effect of Coriolis is to deflect plumes such that they no longer travel upslope, but rather in a direction that is primarily parallel to the isobaths of the ice shelf base. As a result, plumes tend to travel more slowly and will not become supercooled unless steered upslope by an obstruction. By letting the plume impinge on an idealised coastline, Holland and Feltham (2006) found that a plume would bank up and became a very narrow boundary current, slowed by the drag at the wall.

As well as being two-dimensional, their model was unsteady, allowing treatment of both transience and turbulent horizontal diffusion. The drawback of their unsteady model was a significant increase in computational effort. Model runs were up to one year long, but required a 10 s time step because of the short time-scales associated with frazil ice processes.

Using the model just described, Holland et al. (2007b) attempted to reproduce the observed marine ice distribution beneath the Filchner-Ronne Ice Shelf. They were successful in regions where multiple plumes joined together and caused sufficient fluxes of ISW, and consequently sufficient levels of frazil deposition. The lack of success in other regions was attributed to either the effect of nonplume flows or simply that the plume concept is inappropriate in those regions.

3.13 Large-Scale Ocean Modelling

Regional-scale models have shown McMurdo Sound to be an important component in the ISW cycle, an assertion that agrees with observations (Robinson et al., 2010). The observed outflow of low-salinity water in the western Sound has been predicted in a number of models (Assmann et al., 2003; Holland et al., 2003; Dinniman et al., 2007). Furthermore, despite a coarse depiction of Ross Island, Assmann (2004) showed that McMurdo Sound is also an important inflow region for HSSW.

Much of the Southern Ocean is similar to McMurdo Sound in that its properties are influenced by the proximity of ice shelves and their effect has been suggested to extend for hundreds of kilometres. Using models for the Southern Ocean, Beckmann and Goosse (2003) and Hellmer (2004) both tested the effect of including ice shelf cavities, and the corresponding fluxes of cold, low-salinity seawater, on the growth of sea ice surrounding the continent. Their results are compared in Figure 3.13. Neither of the models contained frazil ice. Hence, if predicted, supercooling would not be efficiently relieved causing the models to overpredict the spatial extent of the ice
Figure 3.13 – The difference in September sea ice thickness in metres predicted by models with and without ice shelves. (a) Hellmer’s (2004) prediction where negative values indicate an increase due to ice shelves. (b) Beckmann and Goosse’s (2003) prediction where positive values indicate an increase due to ice shelves. Note: northern latitude limit differs between the figures.

Shelves’ influence.

Studying the ocean beneath the Amery Ice Shelf (see Figure 1.1), Galton-Fenzi et al. (2012) assessed the effect of frazil ice on ice shelf mass balance by including its dynamics into the Regional Ocean Modelling System (ROMS), an ocean model widely used in the scientific community. ROMS can be summarised as a three-dimensional, time-dependent implementation of the Reynolds-averaged Navier-Stokes equation and, not surprisingly, is a computationally expensive routine even without frazil ice. Computing power limitations meant only five size classes were used (radius range of 0.05–0.5 mm), which is smaller than the number used in depth-averaged models. Any size class sensitivity is counteracted by the three-dimensional treatment of frazil ice processes. They predicted that only 30% of marine ice was formed by direct basal freezing. However, this was calculated using transfer velocities, which may not produce accurate predictions of basal freezing rates (Section 3.3.3). Nevertheless, the study produced basal accumulation estimates in good agreement with observation. For this reason and others, it is likely that frazil-inclusive ROMS studies will soon become widespread.
4.1 Overview

In late 2011 a new suite of sea ice and ocean measurements were taken in McMurdo Sound as part of Event K063 (Antarctic Sea Ice Thickness: Harbinger of Change in the Southern Ocean). The primary task of the nine scientists involved was to develop and validate geophysical methods to measure sea ice thickness. Both groups were tasked with making snow, sea ice and sub-ice platelet layer thickness measurements at a number of sites throughout the Sound, these measurements being ground validation for remote-sensing methods. The author was part of a group of four who also took concurrent oceanographic measurements, made possible by the loan of oceanographic equipment from the National Institute of Water and Atmospheric Research (NIWA). This thesis complements the largely remote-sensing-focused goal of Event K063 by contributing to the understanding of how ocean surface water interacts with the sea ice above, and hence helps explain the distribution and thickness of sea ice in McMurdo Sound.

Ice thickness measurements were made at 40 sites, the majority being part of a grid covering approximately 700 km$^2$. Oceanographic measurements were made at six of these sites and sea ice cores from four sites were analysed. All positions are shown in Figure 4.1 and the coordinates of named sites are given in Table 4.1.

4.2 Salinity and Temperature Profiles

4.2.1 Method

Three oceanographic transects were taken: two were approximately parallel to the McMurdo Ice Shelf edge (cross-sound), one for each of the spring and neap tidal regimes, and the third was in a northwest direction (long-sound). The primary motivation for the cross-sound transects was to examine the distribution of ISW along the McMurdo Ice Shelf edge. The long-sound transect, which followed the movement of the coldest water in the Sound, provides data to compare with modelling along an expected “plume path”. This was chosen using the distribution of platelet ice in cores reported in Dempsey et al. (2010). The time and date each site was visited is
Table 4.1 – Coordinates of oceanographic and sea ice core sites.

<table>
<thead>
<tr>
<th>Site Name</th>
<th>Latitude (S)</th>
<th>Longitude (E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>West</td>
<td>77° 50.0’</td>
<td>165° 20.0’</td>
</tr>
<tr>
<td>Intermediate</td>
<td>77° 50.0’</td>
<td>165° 36.0’</td>
</tr>
<tr>
<td>Central</td>
<td>77° 50.0’</td>
<td>165° 54.0’</td>
</tr>
<tr>
<td>East</td>
<td>77° 52.4’</td>
<td>166° 30.0’</td>
</tr>
<tr>
<td>Far East</td>
<td>77° 53.6’</td>
<td>166° 44.4’</td>
</tr>
<tr>
<td>Near North</td>
<td>77° 45.0’</td>
<td>165° 24.0’</td>
</tr>
<tr>
<td>Far North</td>
<td>77° 40.0’</td>
<td>165° 12.0’</td>
</tr>
</tbody>
</table>

given in Table 4.2 and sea level height relative to the mean sea level throughout the measurement period is shown in Figure 4.2.

At each oceanographic site a hole was drilled through the sea ice using a Jiffy Ice Drill coupled to a 10-inch diameter ice auger. Often a significant volume of loose crystals floated up into the freshly drilled hole and required bailing using a kitchen sieve before any equipment could be lowered through the hole. When free of obstructions, a weight on a winch was lowered to the seafloor. A marking was then put on the winch’s cord to indicate the point 20 m above the seafloor, allowing safe use of oceanographic equipment.

The principal components of the oceanographic study were the full-depth temperature and salinity profiles. At each site a Sea-Bird Electronics SBE 19plus Conductivity-
Table 4.2 – Summary of oceanographic casts. All dates are 2011 with time in NZDT (UTC+13 hours). Tidal regime is left blank if it is neither spring nor neap. Only one cast, not three, was taken at the first and second stations.

<table>
<thead>
<tr>
<th>Station No.</th>
<th>Site</th>
<th>Date and Time</th>
<th>Tidal Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>East</td>
<td>26/11 14:44</td>
<td>Spring</td>
</tr>
<tr>
<td>2</td>
<td>Central</td>
<td>26/11 17:23</td>
<td>Spring</td>
</tr>
<tr>
<td>3</td>
<td>West</td>
<td>27/11 10:12</td>
<td>Spring</td>
</tr>
<tr>
<td>4</td>
<td>Intermediate</td>
<td>27/11 14:25</td>
<td>Spring</td>
</tr>
<tr>
<td>5</td>
<td>Intermediate</td>
<td>01/12 09:57</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Far North</td>
<td>01/12 13:30</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Near North</td>
<td>01/12 16:18</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>West</td>
<td>03/12 09:26</td>
<td>Neap</td>
</tr>
<tr>
<td>9</td>
<td>Intermediate</td>
<td>03/12 12:10</td>
<td>Neap</td>
</tr>
<tr>
<td>10</td>
<td>East</td>
<td>03/12 15:19</td>
<td>Neap</td>
</tr>
<tr>
<td>11</td>
<td>Central</td>
<td>03/12 17:59</td>
<td>Neap</td>
</tr>
</tbody>
</table>

Temperature-Depth (CTD) profiler was lowered to within 20 m of the seafloor and left for five minutes to avoid any thermal-lag errors (Morison et al., 1994) and to ensure that the salinity cell was free of ice crystals. A drill-powered winch then lifted the profiler to within 10 m of the sea ice base at approximately 1 m s\(^{-1}\). Given that the profiler takes measurements at a frequency of 4 Hz, this rate of ascent provides a profile of the water column with sub-metre resolution. The accuracies (resolutions) of temperature and salinity stated by the manufacturer are 0.005°C (0.0001°C) and 0.004 (0.0004), respectively. The first upcast was immediately followed by a downcast—descending at an equivalent rate—and a second upcast. However, only an upcast was taken at the first two sites.

Initial raw data were checked on-site for any obvious errors, particularly distur-

Figure 4.2 – Sea level height in McMurdo Sound during the period in which oceanographic measurements were made. Refer to Table 4.2 for a description of stations. Tide prediction from [http://tbone.biol.sc.edu/tide](http://tbone.biol.sc.edu/tide) for a site 1 km offshore of Scott Base.
bances caused by the presence of ice crystals in the salinity cell. These data were later converted to salinity, temperature and pressure measurements using Sea-Bird Electronics software. Further processing involved removing all measurements before the start of the first and after the end of the last cast, as well as any periods in which the profiler was not moving. Data were then binned in 0.2 dbar intervals for ease of use. For the accuracy required in this thesis, pressure measured in decibars (dbar) can be considered equal to depth in metres.

Interpretation of the new data described hereafter will be biased toward the surface layer of the water column, as this is the water that either interacts with or affects the sea ice above it.

4.2.2 Measurement Variability

Before presentation of the new data, it is important to estimate the variability between measurements. Three case studies are considered: comparison between individual casts in a set of three, comparison between spring, neap and the tidal range in between, and variation between multiple casts taken during a 24-hour station by others.

At stations where three casts were taken, all but one show good agreement between the two upcasts and the downcast. The exception occurred at station 10 (East site). As will be shown, there is a significant discrepancy between the first upcast and the subsequent casts at a depth of approximately 50–160 m. The reason for this is unknown, but it was measured by both the temperature and conductivity sensors, which are situated near each other on the CTD profiler.

Other than the cast that was just described, the variation in temperature and salinity between successive casts was small. The range of both variables is shown in Figures 4.3a and 4.3b. For the three casts at every station, the differences between the maximum and minimum temperature or salinity measured in each depth bin were calculated and then plotted on top of each other. Although the discrepancy for station 10 (described above) stands out, most of the variation falls below 0.004 for salinity and 0.002°C for temperature. This can be seen in Figures 4.3c and 4.3d, which show the relative frequency of the range values calculated for all stations.

The variation described above results from measurement uncertainty and small-scale processes and is smaller than the variation brought about by tidal effects. With only one set of oceanographic equipment it was impossible to conduct oceanographic measurements simultaneously. Instead, the aim was to complete all stations on each transect as closely spaced in time as possible.

As described in Section 2.5, McMurdo Sound’s oceanography is influenced by both a diurnal tidal cycle and a longer spring–neap cycle. The effect of the latter cycle can be estimated by comparing all profiles measured at the Intermediate site, which are shown in Figure 4.4. Being part of both the cross-sound and long-sound transects, this site was home to three stations, instead of only one or two for all other sites. Surprisingly, the casts taken during stations at spring (station 4) and neap (station 9)
Over a diurnal cycle the water properties in McMurdo Sound are affected by the inflow/outflow of water from different sources, internal waves (Albrecht et al., 2006), and local mixing processes. A series of CTD casts over a diurnal cycle on the 26–27th of November 2007 at a site in the centre of the Sound—close to the Far North site described earlier—is shown in Robinson (2012). A well-mixed surface layer to a depth of approximately 150 m is evident in all of her casts. Over the 24-hour measurement period the mixed-layer salinity dropped monotonically by 0.01 and the mixed-layer temperature had a range of less than 15 mK. These two values are similar to each other, but different to the casts taken during the station in the intervening period (station 5), which are termed the “In Between” casts in Figure 4.4. The casts from station 5 show that the surface layer became warmer and more saline, whereas below 150 m it became cooler and less saline.
Figure 4.4 – (a) Salinity and (b) potential temperature profiles taken at the Intermediate site at three points in a spring–neap cycle. Bracketed numbers in the legend refer to station numbers given in Table 4.2. Estimates for the measurement variation expected over a diurnal cycle are described in Section 4.2.2.

Profiles of salinity and potential temperature from oceanographic stations in the two cross-sound transects are shown in Figure 4.5. A distribution of temperature across the Sound is evident. In both transects the surface layer was coldest at the Intermediate site, followed by West, then Central and finally East. This order changed slightly below 150 m, where the coldest water was then found at the West site. The salinity distribution is similar: the least saline surface water is found at the central site, the most saline at the east, but the order for the other two sites varies between neap and spring tidal regimes. A likely explanation for the salinity and temperature distributions is the contribution of glacial meltwater at each site. Because this meltwater is relatively fresher and colder, only a small admixture is needed before it makes a difference to
Figure 4.5 – (a) Salinity and (b) potential temperature profiles for cross-sound transects at spring tide and (c & d) at neap tide. Bracketed numbers in the legend refer to station numbers given in Table 4.2. The black dashed line is the pressure-dependent freezing temperature for water with a salinity of 34.6.
the CTD profiles.

The absolute temperature gives some indication of just how cold the water is, but for waters very close to their freezing point a more relevant quantity is the supercooling. To an accuracy of 6 mK, the level of in situ supercooling as a function of depth is the difference between the dashed freezing point line and the potential temperature profiles in Figure 4.5. Because of the depth dependence of the freezing point, the water at the surface usually exhibits the highest supercooling. At the West and Intermediate sites in situ supercooling was observed down to 59–63 m and 60–73 m, respectively. The in situ supercooled layer is deeper during neap tide at the Central and East sites when it reaches 50 m and 35 m, respectively, as opposed to only 30 m and 20 m during spring tide.

Like salinity and temperature, there is a distribution across the Sound in the magnitude of variation in water properties within the water column. The water column is nearly homogeneous at the West site, which is different to the significantly variable East site. The variability at the other two sites falls between these extremes. This variability is consistent with that described in Section 2.3 and is clearly displayed in the potential temperature–salinity diagrams in Figure 4.6. Consider first the properties of the water to 100 m depth at each site, shown by the darker scatter points. The cold, low-salinity, homogeneous surface water layers at the West and Intermediate sites are shown by the small cluster of points in the bottom left of both diagrams. These clusters are close together, indicating that surface water properties are similar at these sites. In contrast, the data points for the East site (and the Central site at spring tide) are spread widely on both diagrams, due to their larger variability. Data from the upper 10 m are not shown as its properties may be affected by the sub-ice platelet layer.

With the exception of the East site at neap tide, most of the water at all sites was below the surface freezing point, indicated by the data points being below the black dashed line. This demonstrates that most of the water must have interacted either directly or indirectly with a sub-surface heat sink such as an ice shelf. It is therefore ISW.

### 4.2.4 Long-Sound Transect Results

The sites on the long-sound transect were approximately 10 km apart and situated on a line following the suspected direction of ISW outflow (Lewis and Perkin, 1985; Dempsey et al., 2010). Therefore, the temperature and salinity profiles between sites should show little variation. With the exception of the temperature inversion at 150–180 m at the Far North site, the profiles (Figure 4.7) show this expected result. Small increases in the surface layer’s temperature and salinity occur with an increase in distance from the ice shelf edge. These small increases, however, lead to a decrease in the depth of supercooled water of approximately 40 m over the 20 km separation between the sites. Supercooled water was measured at the three sites to maximum
Figure 4.6 – Potential temperature–salinity diagram for the cross-sound transects during (a) spring tide and (b) neap tide. The darker scatter points indicate data from above 100 m and the lighter points are for all data below. The black dashed line is the salinity-dependent surface freezing temperature. Data from the top 10 m are not shown. Note: temperature and salinity data for the East and Central spring tide sites are from only one cast, not three as for all other sites.
Figure 4.7 – (a) Salinity and (b) potential temperature profiles for the long-sound transect. Bracketed numbers in the legend refer to station numbers given in Table 4.2. The black dashed line is the depth-dependent freezing temperature for water with a salinity of 34.6.

depths of 63 m, 45 m and 25 m.

Figure 4.8 shows the potential temperature–salinity diagram for the long-sound transect. Data for the top 100 m of the water column are again shown by darker scatter points. Consider the cluster of these points for each site. All three clusters are similarly sized and spaced approximately equally on the $T$–$S$ plane. This indicates that the surface layer at each site is homogeneous but changes with distance from the ice shelf edge, at an approximately constant rate. Below the surface layer the temperature and salinity characteristics between sites are similar, suggesting they may share a common source.

### 4.3 Ice Thickness

As mentioned in Section 4.1, the ice thickness data to be used in this section were collected by a number of scientists and their use was primarily intended for remote-sensing-based studies. The data presented here are to be published elsewhere. However, two types of ice thickness measurements that were collected are relevant to this thesis: the solid ice thickness and the sub-ice platelet layer thickness. These were both
measured a number of times at each of the 40 ice thickness sites (see Figure 4.1), using a tape measure weighted with a metal bar. After being dropped through a hole drilled in the sea ice, the bar lies horizontally and is pulled upward. The first sign of resistance on the bar indicates the bottom of the sub-ice platelet layer. If pulled firmly, the bar can be forced upward through this layer to reach the bottom of the solid sea ice.

At each site the total ice thickness was measured 10 times: two observers both measured the thickness at the four corners and centre of a 30 m wide cross. Using a second observer minimises the uncertainty associated with the sometimes-subjective task of finding the lower boundary of the sub-ice platelet layer. The solid ice thickness, which is a less troublesome measurement, was often measured by only one observer at each of the points on the cross. Subtraction of the solid ice thickness from the total thickness gives the sub-ice platelet layer thickness. Typical relative errors (one standard deviation) for the sub-ice platelet layer and solid ice thicknesses were ±5% and ±2%, respectively. The 10 sub-ice platelet layer measurements at each site commonly had a range greater than 50 cm.

Thickness contours for both types of ice thickness measurements are shown in Figure 4.9. Immediately evident in both plots is the spatial distribution in the measurements. Consider first the solid ice: it is thickest near the centre of the ice shelf edge, where it is nearly twice as thick as the thinnest sea ice measured, which is situated just north of the Erebus Glacier Tongue. In general, sea ice thickness decreases
Figure 4.9 – (a) Solid ice thickness contours and (b) sub-ice platelet layer thickness contours. Red dots indicate measurement sites from which contours were created and green shading indicates the non-contoured sea ice region.
most rapidly in a northeast direction from the centre of the Sound. Conversely, there is a much smaller change in a northwest direction. This spatial distribution is expected based on the ocean properties described in Section 2.3 and is confirmed by sea ice structure (Dempsey et al., 2010).

The sub-ice platelet layer thickness contours show a similar pattern, but with a much stronger spatial variation. Again, the thickest layer (7.5 m) was found just west of the centre of the Sound, and the thickness decreases very quickly in a northeastward direction. Under-ice platelets were either non-existent or present in relatively small amounts at the eastern sites, as shown by the white and light grey contours in Figure 4.9b.

### 4.4 Comparing Ice Thickness and Supercooling

The level of supercooling and sub-ice platelet layer thickness both display significant spatial variability within McMurdo Sound. As the former quantity influences the latter, the spatial distributions of each should be similar. Figure 4.10 shows the comparison between the thickness of the sub-ice platelet layer and the thickness of
the supercooled layer below. The latter thickness is 8–12 times the size of the former, except at the Central and Eastern sites where supercooling is significantly tidally influenced. It must be noted, however, that the thickness correlation may be specific to the time of year measurements were taken as the thickness of the sub-ice platelet layer is effectively an indication of the time-history of supercooling at a site.

Despite the uncertainties, it is clear that the thickness of the sub-ice platelet layer can be used, at least qualitatively, as a proxy for the level of supercooling below it. The benefit of this correlation is that the thickness contours shown in Figure 4.9b, which are based on measurements at numerous sites, also indicate the flow of supercooled water. These contours are similar to the surface supercooling contours in Lewis and Perkin (1985) (see Figure 2.5). Therefore, the new data presented here augment what was already known from this study, and others, by providing a higher-resolution map of the flow paths of supercooled water near the McMurdo Ice Shelf.

4.5 Sea Ice Cores

Full-depth sea ice cores were taken from five sites close to the ice shelf edge (see Figure 4.1). Of these five, four were analysed for ice crystal structure by the author at the University of Otago, while the fifth, from the Intermediate site, was not analysed. Thin sections of the 9 cm diameter sea ice cores were prepared on glass slides. These were then photographed through crossed polarisers. The primary objective, for each core, was to produce a near-continuous picture of sea ice structure using vertical thin sections. Horizontal thin sections were also made at various depths to allow comparison with other studies that use only this method. These depths often corresponded to points where breaks in the core meant only a short length of ice was left and so preparation of a vertical section would be impractical.

Figure 4.11 shows the key result from the thin-sectioning study, which is the cross-sound distribution in sea ice structure. The depths at which the ice structure changed within each core were determined from inspection of Figures 4.12–4.15, which show merged photographs of all thin sections for the West, Central, East and Far East cores. There are two techniques typically used to judge transitions between different ice structures: visual inspection of thin section photographs and measurements of $c$-axis orientations. Being quantifiable, the latter method is more reliable. However, it is still somewhat subjective in that various definitions exist within the literature for each type of ice structure. For example, Gow et al. (1998) and Jones and Hill (2001) both analysed several cores from McMurdo Sound and respectively reported that, on average, platelet ice composed 13% and 42% of their respective cores. It is possible that this factor-of-three discrepancy may be a result of different interpretation, although there is also the possibility of some interannual variability. For these reasons, comparisons between different studies must be treated cautiously.

In this study, ice structure is classified based on only visual inspection of thin
section photographs. For the presentation of results given in Figure 4.11, a broad definition is used for platelet ice. Any ice that has had its otherwise columnar structure clearly disturbed by the introduction of frazil crystals is labelled platelet ice. Equivalently, platelet ice is identified by the disappearance of long, narrow, vertical crystals, which are indicative of columnar ice. However, photographs from studies that report c-axis measurements (e.g., Leonard et al., 2006; Gough et al., 2012b) indicate that these thin, vertical crystals may actually appear disrupted up to 15 cm earlier than the defined platelet ice transition. An estimate for the accuracy of the depth of the columnar/platelet transition is 5 cm above the transition and 15 cm below. Comparison between only cores from this study suggests that the precision of the depth of the columnar/platelet transition can be estimated as ±5 cm.

Bubbly ice with fine-grained crystals is evident at the top of all cores and indicates either snow ice or granular ice. Neither of these structures are of interest for this thesis and so a distinction is unnecessary. Below this, in the Central, East and Far East cores, columnar ice can be seen to depths of 46, 67 and 95 cm, respectively. The rest of each core is composed of platelet ice. Columnar ice is not exhibited in the West core. Instead, the entire core consists of platelet ice (excluding the thin layer of granular ice).

Mahoney et al. (2011) state that a disruption to the columnar fabric occurs only if the whole water column is cooled to the surface freezing point, i.e., it is ISW.

---

**Figure 4.11** – (a) Map showing the location of the four sea ice cores analysed. (b) Comparison of sea ice structure. The accuracy of the depth of the transition between columnar and platelet ice is 5 cm above the transition and 15 cm below. The precision of this depth is ±5 cm.
Therefore, Figure 4.11 illustrates the incremental arrival of ISW at different sites within McMurdo Sound. Assuming the growth of columnar ice occurred at a rate equal to that measured in McMurdo Sound by Purdie et al. (2006) and assuming ice starts forming at the same time, the depths at which platelet ice first appears at the Central, East and Far East sites correspond to ice growth periods of 18–33, 34–52, and 60–82 days, respectively. An attempt was made to determine the date at which sea ice began to persist in McMurdo Sound in order to convert these relative times into a day of the year. However, because of cloud cover it very difficult to assess the state of the sea ice from satellite imagery. It is not possible to say more than that it appears that sea ice starts to persist near the end of March.

In the figures on the following four pages, a sharp colour change indicates the edge of a crystal, while the absolute colour of each crystal is unimportant. Often several tens of crystals are readily identifiable on each thin section. Flaws in the thin sections (e.g., missing parts, small cracks and a faint ‘T’ at the top of some photos) are artefacts of their preparation and should be ignored.
Figure 4.12 – West core. (a) Vertical thin sections. (b) Horizontal thin sections at selected depths. The core is 237 cm thick and 9 cm in diameter.
Figure 4.13 – Central core. (a) Vertical thin sections. (b) Horizontal thin sections at selected depths. The core is 214 cm thick and 9 cm in diameter.
Figure 4.14 – East core. (a) Vertical thin sections. (b) Horizontal thin sections at selected depths. The core is 211 cm thick and 9 cm in diameter.
Figure 4.15 – Far East core. (a) Vertical thin sections. (b) Horizontal thin sections at selected depths. The core is 208 cm thick and 9 cm in diameter.
Chapter 5

Modelling a Plume beneath Sea Ice

5.1 Motivation

The new data presented in Chapter 4 provide a snapshot of oceanographic conditions for a specific time of year and cover only a part of McMurdo Sound. While useful on their own, these new data can be extended by informing a numerical model that contains the relevant physical processes.

The aforementioned model is an adaption of the SJ04 model, i.e., the steady-state ISW plume model described in Section 3.10. The SJ04 model is adapted to McMurdo Sound as described below. Sea ice thermodynamics and the effects of local currents are both added and the entrainment and supercooling formulations are improved. Sea ice thermodynamics were described in Chapter 3. The other adaptions will be described in Sections 5.2–5.5. The integration of these changes into the model will be discussed in Section 5.6.

The underlying concept of the extended model is still a one-dimensional, depth-averaged, frazil-laden plume with heat and mass interactions. Oceanographic measurements taken in McMurdo Sound will be tested against the extended model’s output, providing some indication of the applicability to the McMurdo Ice Shelf and McMurdo Sound of such a one-dimensional plume model and the parameterisations used therein.

5.2 The “Ambient Current”

The ISW plume concept is most applicable when the plume’s own buoyancy is its main source of transport, as opposed to it being driven by wind, geostrophic currents or tidally-induced flows. This may be the case within an ice shelf cavity, but not within McMurdo Sound where buoyancy effects are much reduced or non-existent beneath the horizontal base of the sea ice. To include a physical description for all of the transport processes just described would require a significantly more complex model. However, the effects of forcings other than buoyancy must be included under sea ice because, as will be shown, they alter the plume’s evolution. The compromise taken here is to combine the effects of all these other contributions to the net flow into an “ambient current” on which the plume flow is superposed. Tidal effects are
incorporated elsewhere. Hence, the ambient current is the net flow in the absence of fluxes that drive sub-ice shelf circulation.

Consider a one-dimensional plume that begins below the ice shelf, and then continues beneath sea ice. Figure 5.1 shows the difference in the plume’s evolution with and without the effect of an ambient current. In both cases, interfacial drag causes the plume to decelerate when it reaches the sea ice because the buoyancy force is no longer counteracting the drag force. Mass continuity then requires that the plume thicken. Without an ambient current, this thickening continues until the plume simply stops moving. Conversely, the thickening is reduced if an ambient current in the direction of the plume is present. The plume still decelerates, but instead of losing momentum beyond the ice shelf edge, its mass is transported away. As the plume loses its initial momentum, its thickness approaches a steady state (Figure 5.1b). Ultimately, the plume will become a layer of ISW being transported at the same velocity as the ambient current.

Here, and elsewhere, the discontinuity in ice draft that occurs at the ice shelf edge will be ignored. The edge of the McMurdo Ice Shelf is approximately 20 m thick (McCrae, 1984) and likely has an effect on the plume’s evolution. However, the effect of this draft discontinuity is not known.

It is assumed that the ambient current is driven by external forces (caused by processes that are not considered in the model) that are equal and opposite to the drag force it would experience in the absence of a plume. Because the ambient current alters the velocity of the plume relative to the ice above, it affects heat and mass transfers at the ice–ocean interface. Quantitatively, this effect is easily accounted for by adding the ambient current velocity, $U_a$, to the interfacial friction velocity (Equation 3.8) and precipitation formulation (Equation 3.50) as shown below.

\[
\begin{align*}
  u_z^2 &= C_d \left( (U + U_a)^2 + U_T^2 \right) \\
  p' &= -\frac{\rho_i}{\rho_w} C_i w_i \cos(\vartheta) \left( 1 - \frac{(U + U_a)^2 + U_T^2}{U_z^2} \right) \text{He} \left( 1 - \frac{(U + U_a)^2 + U_T^2}{U_z^2} \right)
\end{align*}
\]
5.3 A Robust Supercooling Formulation

For simplicity, it is assumed that the ambient current is fixed in magnitude, present at all depths and parallel to the plume’s velocity (hence the simple addition of $U$ and $U_a$ in Equations 5.1 and 5.2). These assumptions restrict application of the model to the western side of McMurdo Sound, where there is a consistent net flow away from the ice shelf (Section 2.4.1). It would be inappropriate to apply the extended model in the eastern Sound, where flow is more variable and often causes a net transport into the ice shelf cavity (Robinson et al., 2010). Excluding plume flow, northward flow in western McMurdo Sound may result from factors such as a current setting northward after travelling westward across the Ross Ice Shelf front or a pressure gradient set up by an east–west salinity gradient across the Ross Sea continental shelf (Section 2.4.1).

The concept of an ambient current was investigated by Fong and Geyer (2002) for a scenario similar to an ISW plume: a surface-trapped river plume propagating into the ocean. They showed that the addition of an ambient current (parallel to the plume’s own primary direction of motion) greatly enhanced the freshwater transport, even when the ambient current’s velocity was significantly smaller than typical plume velocities. Further, they showed that, given sufficient time, the river plume always evolved to a steady-state horizontal width regardless of the ambient velocity—a result analogous to that described earlier for the plume’s thickness.

### 5.3 A Robust Supercooling Formulation

Supercooling is very relevant to this thesis, and so there is a need to devise a robust formulation. The supercooling formulation used in the ISW plume models has undesirable features that do not seem to represent reality. The problem with the initial formulation is that supercooling is strongly dependent on the plume’s depth, as illustrated in Figure 5.2. In the original formulation (Figure 5.2b), the presence of supercooling ($T - T_f < 0$) is determined by whether the plume’s temperature is greater or less than the freezing point at mid-depth in the plume. As a result it is possible for a change in the thickness of the plume to change it from being supercooled to being in an above-freezing state. This occurs without any change in either the plume’s thermohaline properties, or its depth within the water column.

In the new formulation (Figure 5.2c), the variable $T_{sc}$ is introduced to denote the depth-averaged supercooling. It is the integral of the temperature deficit $(T_f - T)$ over the supercooled region, divided by the plume thickness, and is negative for water below freezing. Unlike the original formulation, the depth-averaged supercooling is now negative whenever the top of the plume is below its in situ freezing temperature.

Using the notation shown in Figure 3.12, $T_{sc}$ can be written as

$$T_{sc} = \frac{1}{D} \int_{-D}^{0} (T - T_f) \text{He}(T_f - T) \, dn$$

where the freezing temperature $T_f$ is a function of depth, and hence a function of $n$. 
New depth-averaged supercooling formulation

\[ T_{sc} = \text{shaded area/plume thickness} \]

Original depth-averaged supercooling formulation

\[ T - T_f = \text{length of blue/red line} \]

Figure 5.2 – Comparison of the original and new depth-averaged supercooling formulations for three different scenarios: when the plume’s temperature is equal to, greater than and less than the freezing point at mid-depth \((D/2)\). (a) Schematic of the depth-dependent, \textit{in situ} supercooling. (b) The depth-averaged supercooling \((T - T_f)\) for the old formulation, which is represented by the length of the blue (negative) or red (positive) line. (c) In the new formulation, the depth-averaged supercooling \(T_{sc}\) is proportional to the shaded area, and is always negative. \(D_{sc}\) is the “supercooled depth”.

The Heaviside function means that only supercooled water is included in the integral.

The normal coordinate \(n\) is related to the depth coordinate by

\[ z = z_b + n \cos(\vartheta) \quad (5.4) \]

where \(z = 0\) at sea level and is positive upward.

Employing the linear freezing point relationship (see Section 3.6) and changing from the \(n\) to \(z\) coordinate, Equation 5.3 can be rewritten as

\[ T_{sc} = \frac{1}{D} \int_{z_b-D\cos(\vartheta)}^{z_b} \left( T - \left( a_p S + b_p T_p + c_p z \right) \right) \text{He}(T_f - T) \frac{dz}{\cos(\vartheta)} \quad (5.5) \]

The plume’s temperature and salinity do not vary with depth making it is easy to determine the point where \(T_f - T = 0\), and hence remove the need for the Heaviside function. The distance between this point and the top of the plume will be termed
5.4 Frazil Growth with Salt Diffusion

the “supercooled depth”, denoted $D_{sc}$ (see the bottom right of Figure 5.2 for a visual interpretation).

\[
T - \left( a_{fp} S + b_{fp} + c_{fp} (z_b - D_{sc} \cos(\vartheta)) \right) = 0 \tag{5.6}
\]

\[
D_{sc} = \frac{\left( a_{fp} S + b_{fp} + c_{fp} z_b \right) - T}{c_{fp} \cos(\vartheta)} \tag{5.7}
\]

The $\cos(\vartheta)$ term is needed in Equations 5.6 and 5.7 because, like $D$, $D_{sc}$ is measured normal to the ice base. In practice, $\cos(\vartheta) \approx 1$, but the term is kept for generality. Using $D_{sc}$ to remove the Heaviside function yields $T_{sc}$ in its final form.

\[
T_{sc} = \frac{1}{D \cos(\vartheta)} \int_{z_b}^{z_b - \min(D, D_{sc})} \left( T - \left( a_{fp} S + b_{fp} + c_{fp} z \right) \right) \, dz \tag{5.8}
\]

The min function is needed in case the supercooled depth exceeds the plume’s thickness.

$T_{sc}$ provides a much better estimate of the supercooling of seawater surrounding frazil crystals than $T - T_f$. The original formulation gave values that were typically sub-millikelvin.

\section{5.4 Frazil Growth with Salt Diffusion}

The accuracy increase achieved by using $T_{sc}$ is only useful if the effect of salt diffusion is also included in the frazil growth formulation. If limited by molecular processes, the salt rejected by growing crystals diffuses away much more slowly than heat. This causes the crystals to be surrounded by water of above-average salinity, and hence a lower freezing temperature. Following Galton-Fenzi et al. (2012) and Holland and Jenkins (1999), growth is slowed by a factor $\Theta'$ to quantify salt’s rate-limiting effect, where

\[
\Theta' = 1 - \frac{a_{fp} c_w S_c \text{Nu} \kappa_T}{L \text{Sh} \kappa_S} \tag{5.9}
\]

$S_c$ is the salinity of water at the crystal edges, $\kappa_S$ is the mass diffusivity of salt in seawater ($8.0 \times 10^{-10}$ m$^2$ s$^{-1}$), and Sh is the Sherwood number, the ratio of the actual salt mass transfer normal to the crystal boundary to that by diffusion alone. Calculations using the three-equation formulation (Holland and Jenkins, 1999) show that the salinity $S_c$ differs by less than 1% from the surrounding seawater. Hereafter it will be set as a constant (34.6). All that remains to calculate $\Theta'$ is an estimate of the Sherwood number. To make this estimate, it is assumed that mass transfer from a frazil crystal is analogous to heat transfer. Therefore, the Sherwood number is formulated equivalently to the Nusselt number (Equation 3.25) with the Prandtl
Modelling a Plume beneath Sea Ice

Figure 5.3 – (a) Frazil growth scaling factor ($\Theta'$) as a function of frazil crystal radius, for two possible values of $\eta$. $\Theta'$ is the factor by which frazil growth is slowed due to salt diffusion. (b) The combined effect on frazil growth due to turbulent transfer and the salt diffusion. Discontinuities in each graph are due to the discontinuities in the formulations for $Nu$ and $Sh$.

When considering the effects of both turbulent transfer and salt diffusion, the growth of “large” crystals is increased overall, but the converse is true for “small” crystals. The transition from “small” to “large” is dependent on the Kolmogorov length scale $\eta$, which is related to the turbulent dissipation rate. Figure 5.3b shows the value of $Nu/\Theta'$, which is the factor that frazil growth is scaled by to account for both turbulent transfer and the low diffusivity of salt.

With the revised depth-averaged supercooling formulation and salt’s rate-limiting number replaced by its mass analogue, the Schmidt number.

$$Sh = \begin{cases} 
1 + 0.17m^*Sc^{1/2} & \text{if } m^* \leq Sc^{-1/2} \\
1 + 0.55m^{2/3}Sc^{1/3} & \text{if } Sc^{-1/2} < m^* \leq 1 \\
1.1 + 0.77\alpha_T^{0.035}m^{2/3}Sc^{1/3} & \text{if } m^* > 1 
\end{cases} \tag{5.10}$$

Figure 5.3a shows the value of $\Theta'$ as a function of the frazil radius. The difference in the molecular diffusivities of heat and salt is large ($\kappa_T = 175\kappa_s$). However, across most of the frazil radius range, frazil growth is only slowed by a factor of approximately two. There are two reasons for this. First, the Sherwood number is larger than the Nusselt number. This reduces the difference in the actual transport rates of heat and salt. Second, as frazil grows, the rejected salt causes an increase in the salinity gradient at the crystal edge, thereby increasing the transport rate of salt. $\Theta'$ decreases with an increase in crystal radius because heat and mass transfer become more influenced by convection and $Sh$ increases more rapidly with crystal size than $Nu$. 

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With the revised depth-averaged supercooling formulation and salt’s rate-limiting
5.5 Entrainment and Detrainment

As described in Section 3.9, the entrainment formulation used in ISW plume models is a one-way process: detrainment is not considered. This was not, however, the primary motivation for changing the formulation used in the extended model. Instead, an alternative was needed for a horizontal upper surface (e.g., constant-thickness sea ice) because the entrainment rate is zero due to the \( \sin(\vartheta) \) term in Equation 3.65. More importantly, this implies an absence of any heat and mass transfers between the plume and ambient fluid.

The new parameterisations used here come from a study by Baines (2001a), whose work was mentioned in Section 3.9 (see also Baines, 2001b). He suggests his experimental work is appropriate for modelling flows in geophysical situations, including shallow and deep plumes in stratified environments. Adapting his original notation slightly, entrainment and detrainment \( (d') \) are written as

\[
e' = E_e U \\
d' = E_d U
\]

where \( E_e \) and \( E_d \) are the entrainment and detrainment coefficients given by

\[
E_e = C_1 / \text{Ri} \\
E_d = C_1 / \text{Ri} + 0.2 M^{0.4} \sin(\vartheta) \tag{5.15}
\]

\[
C_1 = 0.001 + 10^{-5}(1.3689\vartheta^2 - 1.1970\vartheta^3) \tag{5.16}
\]

\[
\text{Ri} = \left( \frac{D \Delta \rho g}{U^2} \right) \tag{5.17}
\]

\[
M = \frac{DU}{(\Delta \rho g)^{1.2}} N_{bf}^3 \tag{5.18}
\]

\( N_{bf} \) is the buoyancy frequency, \( \vartheta \) is measured in degrees, and the above relationships are valid for \( 0 \leq \vartheta < 4^\circ \). A new symbol is used for the new entrainment coefficient \( E_e \) to save confusion with the identically named coefficient \( E_0 \) introduced in Chapter 3. It is assumed \( \cos(\vartheta) \approx 1 \) in the Richardson number definition.

The second term in Equation 5.15 is based on data with considerable scatter, especially for the values of the non-dimensionalised parameter \( M \) that are applicable to ISW plumes. The coefficient and exponent \( (0.2 \text{ and } 0.4, \text{ respectively}) \) both have uncertainties of \( \pm 25\% \). Given the very small slopes associated with sea ice, this second term is often negligible. Nevertheless, it is kept for generality. The physical basis for

\[
* \text{A subscript “bf” is used to distinguish buoyancy frequency from the variable } N \text{ introduced in Chapter 3.}
\]
the term is worthy of some discussion, however, for it seems to contradict the concept of a plume. A plume may grow (remain constant) in thickness only if entrainment exceeds (equals) detrainment.

The second term in Equation 5.15 contains both the buoyancy frequency and the basal slope. Hence, this term is nonzero for inclined flows in stratified environments. Consider first a realistic plume that is not perfectly mixed throughout its depth. The densest fluid within the plume will exist near its lower edge. When this plume reaches a position where its own density is similar to the surrounding ambient fluid, there may be a net loss of mass from the plume because some of the plume’s fluid near its lower edge will reach its own level of neutral buoyancy and spread out horizontally, i.e., detrain. This process occurs more quickly with an increase in either $N_{bf}$ or $\vartheta$, because these both increase the rate at which the plume moves through isopycnals.

Consider now the idealised plume treated in the model. The properties of this plume are perfectly homogeneous with depth. Therefore, the mass transfer through the lower edge of the plume that occurs in reality must be represented by parameterisations that consider only the depth-averaged properties of the plume and the bulk properties of the water column and basal surface. Equations 5.12–5.18 satisfy these criteria with the depth-averaged properties being the plume’s Richardson number and the bulk properties of the ambient water column being the buoyancy frequency and basal slope.

In the case of no slope, the new formulation predicts that entrainment and detrainment occur at the same rate. In the absence of heat and mass exchange at the plume’s upper boundary and frazil growth within it, the plume’s salinity and temperature can change without a net mass transfer.

5.6 The Extended Model

The model presented in this chapter predicts the evolution of a depth-averaged, frazil-laden, steady-state plume. The depth-averaged approach, while simple, will work well here because the destabilising buoyancy flux due to brine rejection leads to a nearly homogeneous surface layer, which allows rapid communication between the surface and mid-waters (McPhee, 2008; Robinson, 2012). The model begins at the ice shelf edge as a plume of supercooled water with a specified thickness, velocity, temperature, salinity, and multiple-size-class frazil ice concentration. These properties evolve as the plume travels due to heat and mass transfers with the following: sea ice at its upper edge, an ambient current (with specified thermohaline properties) at its lower edge, and frazil ice within the plume. A schematic diagram of the extended model is shown in Figure 5.4.

The conservation equations previously described in Section 3.10 must be adapted to account for all of the changes described in this chapter. However, these adaptations are relatively straightforward. Mass conservation of the mixture (Equation 3.66) can be derived by examining a mass budget across an infinitesimally wide slice of the
plume. Figure 5.5 shows mass fluxes (due to the plume moving with a velocity $U + U_a$) at each side of the thin slice of the plume. The divergence of the mass flux between positions $s$ and $s + ds$ must equal the mass added or removed by the combined effects of entrainment, detrainment, melting/freezing, and precipitation. As shown in the figure, each of these processes can be written in an equivalent form as a mass transfer rate per unit length. Denoting the bulk density of the plume’s mixture of frazil and seawater as $\rho$, mass conservation is derived as

$$\frac{\partial (D(U + U_a))}{\partial s} = \frac{\partial (D(U + U_a))}{\partial s} = \rho' - d' + m' + p'$$  \hspace{1cm} (5.21)$$

It is possible to convert to the partial differential, and thereby put the equation in an equivalent form to the original, because $U$ and $D$ are only a function of the coordinate $s$.

The key difference between Equation 5.21 and its original form (without the ambient current, Equation 3.66) is the change of $U$ to $U + U_a$. The original mass conservation equations for the water and ice fractions (Equations 3.67 and 3.68, respectively) are adapted in the same way. Detrainment of frazil is not considered because the frazil is distributed towards the upper part of the plume (Holland and
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Feltham, 2005).

\[ \frac{\partial(D(U + U_a))}{\partial s} = e' - d' + m' + f' \]

(5.22)

\[ \frac{\partial(D(U + U_a)\rho_iC_i(k))}{\partial s} = -\rho_w \left(f'(k) - p'(k)\right) \]

(5.23)

The inconsistency between Equations 5.21 and 5.22 was explained in Section 3.10.

Momentum conservation (Equation 3.62) requires a number of changes and additional terms: the drag term is larger because the layer adjacent to the ice–ocean interface now has a velocity \( U + U_a \); the increased turbulence due to tidal flow is incorporated in the same way as in Smedsrud and Jenkins (2004); the ambient current must be accounted for; and a term must added for the loss of momentum of the detrained fluid.

\[ \frac{\partial \left(D (U + U_a)\right)^2}{\partial s} = -C_d \left( (U + U_a)\sqrt{(U + U_a)^2 + U_T^2} \right) \]

\[ + C_d \left( U_a\sqrt{U_a^2 + U_T^2} \right) + D\Delta\rho g \sin(\vartheta) - d'U \]

(5.24)

The ambient current term is set such that if \( U = 0 \), then the first two terms on the right hand side cancel. The detrainment term is proportional to \( U \), which is the velocity of the plume layer relative to the ambient current.

Other than the adjustment for the ambient current, heat conservation (Equation 3.70) requires two changes: a term for the loss of the detrained fluid’s thermal energy and an adjustment for the new supercooling formulation (replacing \( T_f \) with...
5.7 Establishing Initial Conditions

\[ T + T_{sc} \).

\[
\frac{\partial (D(U + U_a)T)}{\partial s} = e'T_a - d'T + m'T_b + \left( T + T_{sc} - \frac{L}{c_w} \right) f' + \frac{K_i}{\rho_w c_w} t\frac{dT_i}{dz} \bigg|_b - m'L_i c_w (5.25)
\]

Salt conservation (Equation 3.71) also requires the addition of a detrainment term.

\[
\frac{\partial (D(U + U_a)S)}{\partial s} = e'S_a - d'S + m'S_i (5.26)
\]

To allow the model developed in this chapter to be solved in MATLAB, the variables inside the derivatives must be decoupled. However, this process gives no additional physical insight and the solutions are given in Appendix A.

5.7 Establishing Initial Conditions

5.7.1 Applying the SJ04 Model to the Ross and McMurdo Ice Shelves

The extended model described in this chapter begins at the ice shelf edge, which assumes a plume is sufficiently buoyant to reach this point. Therefore, using the plume’s final state as predicted by the SJ04 model would appear an obvious approach to obtaining initial conditions for the extended model.

The SJ04 model assumes that ISW plumes that reach the ice shelf edge have evolved from a single point source at the grounding line. However, a more recent study suggests that smaller plumes originating from multiple meltwater sources can converge downstream to form one plume with a larger volume flux (Holland et al., 2007b). Nevertheless, the SJ04 model approach was explored, but did not produce results that agreed with observation. Indeed, this approach involves the inherent and possibly unrealistic assumption that there is a single, dominant source for the ISW that enters McMurdo Sound. Numerous experiments with various input parameters failed to produce a result in which the plume remained both buoyant and frazil-laden by the time it reached the McMurdo Ice Shelf edge. In the experiments in which the plume became supercooled, all the frazil that grew was predicted to precipitate well before the ice shelf edge.

Many experiments performed here involved realistic ice shelf drafts (Timmermann et al., 2010) and RMS tidal velocities (Padman et al., 2003), which are shown in Figure 5.6, and observed thermohaline properties beneath the ice shelf (e.g., Foster, 1983; Robinson et al., 2010). In addition, simulations were also carried out using hypothetical conditions including linear ice shelves, homogeneous water columns, temperatures well above their freezing point and/or no tidal velocities.

It is not known where, if it exists, the dominant source for the ISW that enters McMurdo Sound would be, so different start points beneath the Ross Ice Shelf were tested. Initial tests involved starting at the deepest parts of the ice shelf, but the
concept suggested by MacAyeal (1985a) of a shallow plume starting in the middle of the water column (initiated by basal melt caused by an intrusion of MCDW) was also thoroughly tested. Examples of the input draft conditions used are shown in Figure 5.7. Overall, the experiments displayed the expected behaviour described by Smedsrud and Jenkins (2004), i.e., the ice shelf draft and ambient water properties have a large control of the plume’s evolution.

The numerous model runs did frequently produce the same promising result. Southeast of Ross Island, the relatively thick Ross Ice Shelf thins significantly (∼200 m) over a short distance (∼100 km) in a direction toward McMurdo Sound (see Figures 5.6a and 5.7b). This steep ice shelf draft slope increased the plume’s rate of ascent through the water column and, consequently, the plume became significantly supercooled. Unfortunately, the SJ04 model predicts that the resulting frazil ice precipitates ∼50 km before the plume reaches McMurdo Sound.

There are many possible explanations for the inability of the SJ04 model to predict a plume that reaches the ice shelf edge with a significant frazil concentration. Three likely reasons are the assumption that plumes arise from a single point source, the neglect of flows not modelled by a buoyant plume that may help transport ISW, or the inaccuracies in the parameterisations included in the model, specifically those regarding frazil ice processes.

5.7.2 A “Spin-up” Approach

The revised approach for determining initial conditions is partially reliant on the SJ04 model run for the Ross and McMurdo Ice Shelves, as it provides estimates of
5.7 Establishing Initial Conditions

Figure 5.7 – Examples of possible plume paths that were used as input for the SJ04 model. (a) A range of initial plume sources were chosen. All paths end at the McMurdo Ice Shelf Edge. Red lines indicate plumes starting at a depth greater than 500 m. Blue lines indicate plumes starting near the middle of the water column. (b) The draft against distance for the plume paths in (a), using draft data shown in Figure 5.6a. The shallow plume draft curves are offset 200 km for clarity. If frazil was generated, it was typically predicted to precipitate in the region where the draft was $-60$ to $-200$ m.

The plume’s initial depth and velocity. The initial temperature, salinity and frazil concentration, however, are found through other methods.

The plume’s initial salinity is guided by the measurements described in Section 4.2. The initial temperature is then derived from the chosen initial supercooling—specifically $D_{sc}$. The four oceanographic stations on the cross-sound transect can be considered an estimate of the thermohaline conditions at the ice shelf edge as they are each approximately 3 km from it. This distance is short in comparison to the model domain but large enough that the measurements were not influenced by processes occurring at the ice shelf edge, which are not considered in the extended model. The depth of the mixed layer is observable in the temperature and salinity profiles, but this does not necessarily correspond to the plume’s depth.

Velocity measurements were not taken at the oceanographic sites. Hence, experimentally derived estimates of the plume’s velocity at the ice shelf edge are not used. Instead, velocities suggested by the SJ04 model runs will be used. As described in Section 5.7.1, typical velocities are several centimetres per second. Corresponding depths are several tens of metres (Section 3.11). The effect of changing the magnitude of these variables will be tested in Section 6.5.

The plume’s initial frazil concentration is the only initial variable left to be determined. At the instant it passes the ice shelf edge, the plume will have a certain total frazil mass concentration—denoted $C_{i}^{tot}$—distributed over all size classes in a certain way. The thickness of the sub-ice platelet layer (Figure 4.9b) is a proxy for
Figure 5.8 – (a) The total frazil concentration during the model’s spin-up phase. Distances are negative because the spin-up phase occurs upstream of the ice shelf edge. (b) The frazil spectrum at selected distances. The distribution at the start of the spin-up phase (−69.6 km) has an equal number of crystals in each size class. Within 50 km of the ice shelf edge, the shape of the frazil spectrum reaches a steady state, and the total frazil mass concentration increases until it reaches $C_{\text{tot}}$. The frazil ice spectrum at the end of the spin-up phase provides the initial condition at the ice shelf edge.

the precipitation rate, which itself can be used to estimate $C_{\text{tot}}^i$. However, this gives no indication of the crystal size distribution.

If the plume has been supercooled for a significant distance, the frazil distribution will likely be in a pseudo-steady state: the largest crystals will not be kept in suspension and the smallest crystals will grow most efficiently (note the $r_i^{-2}$ dependence in Equation 5.11). If the initial conditions fail to match this steady state, then there will be an initial burst of change as the model adjusts. This will lead to incorrect predictions of frazil growth and precipitation, and consequently the plume’s temperature, salinity and level of supercooling.

To predict the pseudo-steady state, all initial plume properties except frazil concentration are held fixed at the beginning of the model run until the total frazil mass concentration reaches $C_{\text{tot}}^i$. This initial “spin-up” phase begins with an equal number of seed crystals in each size class and a total mass concentration of $C_{\text{tot}}^i/5$. The spin-up phase is not considered part of the model output, and hence will not be shown in the results given in Chapter 6. After this point the plume’s properties evolve in space following the conservation equations described in this chapter.

An illustration of what occurs during the spin-up phase is shown in Figure 5.8, which shows how the frazil concentration evolves under the standard conditions described in Section 5.8. At the beginning, the total concentration drops from its initial value of $C_{\text{tot}}^i/5$ because the large crystals ($r_i \geq 3.5 \text{ mm}$) quickly precipitate.
After this, the concentration increases monotonically as the crystals grow. Secondary nucleation is needed to ensure a continuous supply of small crystals. Otherwise the crystals would all become too large to be kept in suspension.

The seeding strategy used here at the start of the spin-up phase differs significantly from that used by Smedsrud and Jenkins (2004), who set the initial mass concentration of frazil in each size class to be equal, as opposed to the number concentration. Their standard conditions suggested that, at the moment the plume becomes supercooled, it contained approximately $3 \times 10^7 \text{m}^{-3}$ crystals of the smallest size class, but only $4 \text{m}^{-3}$ of the largest size.

Using an equal number concentration is an improvement to the seeding strategy, as it reduces the sensitivity to the choice of $r_i(k)$, but it does have one drawback. In many of the sensitivity tests (to be discussed in Sections 6.3–6.8), changing one or more of the input parameters meant that the plume’s frazil mass concentration was never predicted to reach $C_i^{\text{tot}}$. For example, a decrease in the drag coefficient means that many of the larger seed crystals immediately precipitate. Consequently, an insufficient number of crystals remain to allow the total concentration to recover and then increase.

In cases where this problem occurred, $n_{\text{max}}$ was increased by up to a factor of six. This change, which is applied during only the spin-up phase, increases the efficiency of secondary nucleation, and thereby increases the total number of frazil crystals within the plume. Such an adjustment may be regarded as model tuning, and hence undesirable. However, the standard value of $n_{\text{max}}$ comes from the work of Smedsrud (2002) who found that a 10-fold increase (or decrease) caused no significant changes to his model’s results.

### 5.8 Model Parameters

In this section the input parameters and parameterisations to be used for the standard model run are described. Some have already been given in Chapter 3, but are also collected here. The value of physical constants can be found in Table i (page xiii).

#### Sea Ice Properties

\[
S_i = 5.0
\]

\[
\left. \frac{dT_i}{dz} \right|_b = -10 \text{K m}^{-1}
\]

\[
C_d = 15 \times 10^{-3}
\]

\[
K_i = \frac{\rho_i}{917} \left( 2.11 - 0.011T_i + 0.09 \frac{S_i}{T_i} \right)
\]

\[
L_i = 4187 \left( 79.68 - 0.505T_i - 0.0273S_i + 4.3115 \frac{S_i}{T_i} \right)
\]
The salinity and basal temperature gradient are values typical for sea ice in McMurdo Sound. The drag coefficient is estimated from the range of values in the review by Lu et al. (2011) for sea ice in a range of conditions, choosing a value larger than the average to account for the large drag exerted by the sub-ice platelet layer (Section 2.8), which is known to build up in the supercooled conditions described below. The thermal conductivity and latent heat come from Equations 3.10 and 3.12, respectively. These two parameters are evaluated only at the sea ice base, so $T_i$ can be fixed at $-1.9^\circ$C in expressions for $K_i$ and $L_i$. Also, the density of sea ice will be approximated as that of pure ice.

**Water Column Properties**

\[
U_a = 0.03 \text{ m s}^{-1} \\
U_T = 0.04 \text{ m s}^{-1} \\
S_a = 34.7 - 2 \times 10^{-4} z \\
T_a = -1.9^\circ\text{C} \\
\varepsilon = 3 \times 10^{-8} \text{ m}^2 \text{ s}^{-3}
\]

The ambient current and RMS tidal speeds are based on typical values for western McMurdo Sound (see Sections 2.3–2.5). Salinity is based on measurements described in Section 4.2, and the temperature is equal to the surface freezing point at all depths. The turbulent dissipation rate is the value measured by Stevens et al. (2009) near Scott Base in McMurdo Sound, implying $\eta = 4 \text{ mm}$.

**Frazil Ice Properties**

\[
\bar{r}_i(k) = (0.1, 0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5, 1.7, 1.9, 2.1, 2.3, 2.5, \ldots, 2.7, 2.9, 3.1, 3.3, 3.5, 3.7, 3.9, 4.1, 4.3, 4.5, 4.7, 4.9) \text{ mm}
\]

\[
\bar{n}_{\text{max}} = 10^3 \text{ m}^{-3} \\
\alpha_r = 1/50 \\
U_c^2 = \frac{0.1(\rho_w - \rho_i)gr_w}{\rho_w C_d}
\]

All frazil properties, except the size class distribution are unchanged from that originally used by Smersdru and Jenkins (2004). The size class distribution $\bar{r}_i(k)$ was chosen for simplicity. Its range and minimum value differ from that originally used. Implications of this change are discussed in Section 6.8.3. Rise velocities (not shown, see Equation 3.47) are also unchanged. An amendment to the parameterisation of $U_c$ was suggested in Section 3.7.3, but is unnecessary here because of the large value used for $C_d$.  

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Initial Conditions

\[ U = 0.03 \text{ m s}^{-1} \]
\[ D = 50 \text{ m} \]
\[ S = 34.58 \]
\[ D_{sc} = 65 \text{ m} \]
\[ C_{i}^{tot} = 0.10 \text{ g L}^{-1} \]

The initial plume velocity is chosen to equal the ambient velocity, which suggests that plume and nonplume flows are initially each responsible for half of the net velocity. The choice of other initial conditions is explained in Section 5.7.2.
6.1 Model Validation

Measurements indicate that ISW that exits McMurdo Sound on its western side remains supercooled at least as far as 60 km from the ice shelf edge (Lewis and Perkin, 1985). Stevens et al. (2009) estimated that this supercooling could persist for ~250 km. The model developed in Chapter 5 is used here to help understand how the properties of this supercooled water evolve over such distances. The model output will also provide estimates of quantities that are difficult to measure, such as the size distribution and concentration of frazil ice.

Before any predictions are made, it is necessary to investigate the model's ability to reproduce measured conditions. The long-sound transect described in Section 4.2 provides one dataset that can be compared against model output. However, as it consists of only three sites, further data are needed. These will come from the study by Lewis and Perkin (1985), which involved a small number of measurement sites on the western side of McMurdo Sound. As shown in Figure 6.1, four of their sites lie approximately in a straight line. A further two sites lie only a short distance either side of this line; measurements from these two sites will be averaged. Therefore, in total, there are eight potential validation sites between 3 and 60 km from the ice shelf, but the first of these is used to provide initial conditions.

To test the model against measurement it is important that the input ice draft and RMS tidal speed are realistic. Although sea ice has a negligible gradient, that produced by the sub-ice platelet layer may become important. To estimate the ice draft, a slice parallel to the lines in Figure 6.1 was taken through the ice thickness contours shown in Figure 4.9. The sum of the solid ice and sub-ice platelet layer thicknesses, minus a small correction for freeboard,* produces the total ice draft against distance curve shown in Figure 6.2. Total ice thickness measurements were not taken in 1982, which is the year Lewis and Perkin (1985) took their measurements. Hence, the input ice draft that will be used in the attempt to reproduce their measured data will be assumed to be equal to that measured in 2011. The RMS tidal speed with distance from the ice shelf (also shown in Figure 6.2) was determined in an equivalent way to ice draft: a slice was taken through the contours shown in Figure 2.6.

*Freeboard is the thickness of ice floating above sea level.
**Figure 6.1** – Location of oceanographic sites providing data for model validation. Circles indicate sites on the long-sound transect described in Chapter 4 and triangles indicate a subset of sites visited by Lewis and Perkin (1985). Beside each marker is the date the site was visited, and multiple dates indicate multiple measurements. The straight lines show the fitted plume paths. Oceanographic properties for the two sites lying a significant distance from the line (shaded markers) were averaged. For the earlier study, the ice shelf and sea ice edges differed slightly to that shown here.

**Figure 6.2** – Input ice draft and RMS tidal speed used for model validation.
The upper parts of the temperature and salinity profiles shown in Figure 4.7 both exhibit little variation with depth. Hence, at each site, it is straightforward to deduce a single value for temperature and salinity that can be compared directly to the depth-averaged model. The supercooled depth $D_{sc}$ is found by subtracting the total ice thickness from the greatest depth at which in situ supercooled water was measured. A further quantity that can be used for model validation is the sub-ice platelet layer thickness, which is related to precipitation rate. The two are not directly equivalent for a number of reasons: one is a thickness, the other is a rate of change; an unknown fraction of the sub-ice platelet layer’s growth occurs after frazil has precipitated; and only one quarter of the sub-ice platelet layer is ice (Gough et al., 2012b). Overall, the sub-ice platelet layer thickness in metres should be similar, but smaller, than the precipitation rate in metres per year. For example, assume the sub-ice platelet layer (i) grew for 8 months before it was measured, (ii) has a solid ice fraction of $1/4$, and (iii) is composed of platelet crystals that reached half of their total volume due to growth while suspended in the water column. Then it is possible to roughly estimate the relationship between the sub-ice platelet layer (SIPL) thickness in metres and $p'$ in metres per year.

\[
\text{SIPL} = -\frac{8}{12} \frac{4}{1} \frac{2}{1} \frac{\rho_w}{\rho_i} p' \approx -6p' \quad (6.1)
\]

The density ratio and negative sign convert $p'$ from a seawater equivalent thickness below the ice-ocean interface to a positive ice thickness.

The average salinity and temperature of the surface layer are also easily deduced from the profiles measured by Lewis and Perkin (1985).* Their data were adjusted for the freshening trend described by Jacobs and Giulivi (2010) by reducing salinities by 0.09 and increasing temperatures by 0.09 $a_{fp}$. These changes make the 1982 data comparable to the new data without affecting any supercooling calculations. As sub-ice platelet layer thickness measurements were not made in 1982, there are no experimental data to compare against the modelled precipitation rate, and the supercooled depth calculation entails additional uncertainty.

The model output using the standard conditions described in Section 5.8, as well as the input ice draft and RMS tidal speed (Figure 6.2), is shown against measurements in Figure 6.3. The 2011 data suggest that the salinity and temperature both increase approximately linearly with distance from the ice shelf, leading to a linear decrease in the supercooled depth. Data from Lewis and Perkin (1985) suggest a similar, but slower change in temperature and supercooling. The salinity, however, exhibits no obvious trend, but this is possibly the result of the measurements being taken up to two weeks apart (Figure 6.1).

Close to the ice shelf, the model predicts a rapid temperature increase with distance, due largely to frazil growth, which slows as the supercooling decreases. The salinity increases with distance from the ice shelf edge at an approximately constant rate due

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*Data from World Ocean Database (http://www.nodc.noaa.gov/OC5/WOD/pr_wod.html).
Figure 6.3 – Comparison of the model output (solid line) with measurements of (a) $D_{sc}$, (b) salinity, (c) temperature, and (d) sub-ice platelet layer (SIPL) thickness. Input ice draft and RMS tidal speeds are shown in Figure 6.2. Circles indicate the 2011 data described in Chapter 4 and triangles are from a subset of measurements by Lewis and Perkin (1985) (see Figure 6.1) with temperature and salinity adjustments as described by Jacobs and Giulivi (2010). Grey markers indicate measurements from the Intermediate site that were not taken on the same day as the other two sites. In (a), uncertainty in $D_{sc}$ includes both the measurement uncertainty and the 3 mK uncertainty in the freezing point temperature (Millero, 1978). Estimation of uncertainties in (b) and (c) are given in Section 4.2.2 and Lewis and Perkin (1985). In (d), the quantity $-6 \times p'$ and the sub-ice platelet layer thickness share the same axis, despite being measured in different units. The factor of $-6$ comes from Equation 6.1. The sub-ice platelet layer thickness was interpolated using the contours in Figure 4.9b.
6.2 Model Behaviour

As in the model run described in Section 6.1, the plume begins at the ice shelf edge with the initial conditions described in Section 5.8. Discussion of its evolution beyond this point is the purpose of this section. To demonstrate the model’s behaviour clearly, the ice draft and RMS tidal speed will be held fixed in this section. The former will be set to zero and the latter will be set at 4 cm s\(^{-1}\). The effect of changing each of these quantities will be described in Sections 6.6.2 and 6.6.3, respectively.

6.2.1 Plume Dynamics

As shown in Figure 6.4, the plume’s dynamics are simple and predictable. Interfacial drag causes the plume to lose nearly all of its initial, buoyancy-derived momentum over the first 10 km, i.e., \(U\) goes to zero over the first 10 km. To compensate, the plume’s thickness increases. Apart from small mass fluxes through the top and bottom of the plume, mass continuity requires that \(D(U + U_a)\) stay constant. Therefore, the relative sizes of \(U(s = 0)\) and \(U_a\) determine the relative change in plume thickness. Hereafter, the velocity at the ice shelf edge \(U(s = 0)\) will be denoted \(U(0)\). In the

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Figure 6.4 – The plume’s total net velocity, \(U + U_a\), and thickness with distance from the ice shelf, for the standard model run.
standard run the ratio $U(0) : U_a$ is 1 : 1, and the plume’s thickness doubles. If the ratio were 2 : 1, the plume thickness would treble.

### 6.2.2 Thermodynamics

In comparison to its dynamics, the modelled plume’s thermodynamics are complex. Indeed, the plume’s thermodynamics depend on its dynamics, i.e., plume properties and heat and mass transfers depend on the plume’s velocity and thickness. Figure 6.5a shows the depth-averaged supercooling, which decreases with distance due to both ice growth and the increase in plume thickness. To identify the relative importance of frazil growth and interfacial growth, two new quantities are introduced: the interfacial heat flux and the frazil heat flux. The interfacial heat flux is denoted $F_i$ and is negative for a transfer of heat into the ocean. It is equal to the residual heat flux at the interface that is not accounted for by upward conduction or phase change at the ice bottom.

$$F_i = -K_i \frac{dT_i}{dz} \bigg|_b + \rho_w L_i m' = \alpha h u_s \rho_w c_w (T - T_b) \quad (6.2)$$

The second equality follows from Equation 3.15. Frazil growth causes a transfer of heat into the plume’s water fraction, but the frazil heat flux, denoted $F_f$, is defined

![Figure 6.5](image-url)
to be negative for ice growth. This allows direct comparison between $F_f$ and $F_i$.

$$F_f = \rho_w f L \quad (6.3)$$

Figure 6.5b shows the magnitudes of $F_i$ and $F_f$ as a function of distance from the ice shelf. Initially, the magnitude of the frazil heat flux is much larger (more negative) than its interfacial equivalent. The relatively large depth-averaged supercooling close to the ice shelf causes rapid frazil growth, but this does not persist. The coupled effect of a lower frazil concentration, due to precipitation, and a reduced supercooling causes the frazil heat flux to become insignificant after 20 km. Like $F_f$, the magnitude of $F_i$ is large close to the ice shelf. Again, part of this is attributed to the relatively large supercooling there. A second, equally relevant factor is the increased level of interfacial heat transfer that occurs close to the ice shelf due to the plume’s high velocity there, which raises the efficiency of turbulent heat and mass transfers (see Equations 3.5, 3.6 and 5.1).

A quantity related to the interfacial heat flux is the ice growth rate. Figure 6.5c shows the value of $-m' \rho_w / \rho_i$, which is the rate of change of ice thickness due to growth at the ice–ocean interface. The absolute growth rate is dependent on the input basal temperature gradient, and therefore not of concern. However, its change with distance is important. Near the ice shelf, where the interfacial heat flux is highest, ice is predicted to grow approximately 85 cm yr$^{-1}$ faster than ice 50 km away. Alternatively, the modelled growth rate at the Intermediate site (3 km from ice shelf) was 31 cm yr$^{-1}$ faster than that at the Far North site (23 km from the ice shelf). The measured difference in ice thickness between these sites was 18 cm, which is in good agreement considering the sea ice would have been approximately 8 months old at the time of measurement. It must be noted that, in reality, the sub-ice platelet layer interferes with the growth of “solid” ice (Leonard et al., 2006; Gough et al., 2012b), but this is not modelled.

6.2.3 Frazil Population Dynamics

The plume’s frazil concentration is highest as it passes out from underneath the ice shelf, as shown in Figure 6.6. This is expected because the initial frazil size spectrum is calculated from the plume’s initial conditions when velocity and supercooling are at their maximum. These conditions promote the growth of crystals to the larger size classes. The plume’s deceleration then causes these larger crystals to rise out of suspension. Supercooled water remains, and therefore crystals continue to grow, but precipitation dominates growth. This process can be identified in Figure 6.7, which shows the frazil ice size spectrum at selected distances from the ice shelf. Results are presented as both a mass concentration and number concentration. Near the ice shelf, most of the ice mass kept in suspension is due to crystals with a radius of $2.5 \pm 1.0$ mm, but this range decreases with distance from the ice shelf. At distances
from the ice shelf edge of greater than 15 km the range drops to $1.5 \pm 0.5$ mm. The significance of precipitation as the plume moves away from the ice shelf is indicated by the redistribution of the individual curves down and to the left.

Although sub-millimetre crystals make up only a very small part of the total mass of frazil ice, they constitute a significant fraction of the total number of crystals. Crystals in the smallest size class are being continually produced by secondary nucleation, and growth of these leads to significant numbers in the size classes above. In terms of number concentration, the peak in the crystal size spectrum remains at 1.1–1.3 mm, beyond the first few kilometres of the plume’s travel.

### 6.3 Effects of the Ambient Current

The importance of the ambient current to the plume’s dynamics was described in Section 5.2. Without the enhanced transport due to this current, the plume decelerates and rapidly becomes very thick. Because the plume’s properties are depth averaged, this thickening has consequences for all other plume properties.

To assess the effect of the ambient current on the plume’s evolution, two contrasting scenarios were tested against the standard model run, in which $U(0) = U_a$. In the first, the plume’s initial velocity was high (6 cm s$^{-1}$), but the ambient current was set to zero. In the second, the ambient current was set to 6 cm s$^{-1}$, but the plume had no initial velocity, i.e., $U(0) = 0$. These velocity combinations were chosen such that the total velocity, $U + U_a$, at the ice shelf edge was the same for each scenario, which also means all other initial conditions were unchanged.

Apart from the variation in $U(0)$ and $U_a$, model runs followed the standard conditions described in Section 5.8. The ambient current’s effect is summarised in Figure 6.8, which shows four important plume properties: $T_{sc}$, $D_{sc}$, total frazil concentration and precipitation rate. For the case of no ambient current, the model run stops after 10 km, by which time the plume’s predicted thickness is 500 m.
6.3 Effects of the Ambient Current

Figure 6.7 – The plume’s frazil ice spectrum at selected distances from the ice shelf displayed in terms of both (a) mass concentration and (b) number concentration. Coloured dots indicate the actual value in each size class. Black dots indicate the peak in each curve. A dip occurs at \( r_i = 1.1 \text{ mm} \) in the first four number concentration curves due to the discontinuity at \( r_i = 1 \text{ mm} \) in the value of \( \text{Nu}/\Theta' \) (see Figure 5.3).

6.3.1 Without the Ambient Current

Without an ambient current, the depth-averaged supercooling, \( T_{sc} \), quickly reduces to zero with distance from the ice shelf. This is due to a combination of latent heat released by ice growth and the increasing plume depth, with the latter being more significant. The supercooled depth, \( D_{sc} \), does not exhibit the same rapid decline because its value is not dependent on the plume’s thickness. Instead, \( D_{sc} \) decreases more slowly with distance than in the standard run. The reason is that very little frazil is kept in suspension after the first few kilometres. This means the plume’s temperature and salinity, and hence \( D_{sc} \), do not change significantly because the only mechanism of change is interaction at the ice–ocean interface. This interaction is weak due to the plume’s low velocity.

Relative to the standard run, the total frazil concentration and precipitation rate differ only slightly from the model run. The spike in precipitation, and hence drop in total concentration, is slightly larger because the plume undergoes a larger deceleration.

6.3.2 Ambient Current Only

The ambient-current-only scenario is significantly different to the standard model run. Close to the ice shelf, \( T_{sc} \) decreases slowly with distance because the plume is not thickening and therefore covers a certain distance more quickly. However, after 10 km,
Figure 6.8 – The plume’s evolution with different combinations of initial and ambient current velocity. (a) Depth-averaged supercooling. (b) Supercooled depth. (c) Mass concentration of frazil in suspension. (d) Precipitation rate. The solid line is the standard model run, in which $U(0) = U_a$. The black, dashed line shows the plume’s evolution without an ambient current. The black, dash-dot line shows the converse scenario: the plume’s velocity is due to only the ambient current. Grey lines indicate scenarios in which $U(0)$ is small relative to $U_a$. 

Velocities (cm s$^{-1}$)
- $U(0) = 3, U_a = 3$
- $U(0) = 6, U_a = 0$
- $U(0) = 0, U_a = 6$
- $U(0) = 1, U_a = 5$
- $U(0) = 2, U_a = 4$
its magnitude drops below that of the standard run, because the continually higher velocity maintains frazil in suspension, which efficiently relieves the supercooling. The decline in $D_{sc}$ can be explained in an similar way. Heat transfers are driven by temperature differences. Therefore, as the level of supercooling decreases, so too do the heat fluxes causing this decrease. Thus $T_{sc}$ and $D_{sc}$ decline asymptotically.

More notable than the change in the supercooling is the frazil population behaviour. The frazil concentration does not immediately decrease because there is no longer a deceleration occurring at the beginning. Instead, it increases slightly overall. More importantly, there is no longer a spike in the distribution of precipitation. This contrast in the frazil behaviour between model runs is significant enough that it warrants further investigation with similar scenarios, i.e., those in which $U(0)$ is low relative to the ambient velocity. These are described in the following section.

### 6.3.3 Other Velocity Combinations

The grey lines in Figure 6.8 show two possible combinations of $U(0)$ and $U_a$ that lie between those of the standard model and the ambient-current-only scenario. In keeping with the reasoning described at the start of Section 6.3, these further model tests are set up such that $U(0) + U_a = 6\, \text{cm s}^{-1}$. As expected, the outputs of these further two model runs lie between the standard run and the ambient-current-only run. Qualitatively, the behaviour of $T_{sc}$ and $D_{sc}$ is very similar to the standard run.

The total frazil concentration, however, reveals a new trait about the model. Small changes in the ambient velocity lead to significant changes in the volume of frazil kept in suspension. With an ambient current of $3\, \text{cm s}^{-1}$, i.e., the standard run, $0.9\, \text{mg L}^{-1}$ of frazil remains after 50 km. Increasing the ambient current to 4 and $5\, \text{cm s}^{-1}$ increases this concentration to 9.4 and 49 mg L$^{-1}$, respectively. This link between velocity and frazil concentration also explains the precipitation rate. Increasing the magnitude of $U_a$ relative to $U(0)$ reduces the size of the spike that occurs near the ice shelf edge. Because of the now smaller deceleration, fewer of the larger crystals are prone to immediate precipitation.

### 6.4 Salt-affected Frazil Growth

Improvements to the parameterisation of frazil growth were introduced in Sections 3.7.1 and 5.4 through the treatment of turbulent heat transfer and the rate-limiting effect of salt. These changes are tested against Smedsrud and Jenkins’s (2004) original formulation, which involved setting $\text{Nu} = 1$ and $\Theta' = 1$.

Under standard conditions, the Kolmogorov length scale, $\eta$, is 4 mm and the value of $\text{Nu}/\Theta'$ is less than one for the typical size range of crystals kept in suspension (Figure 5.3). This suggests that, with the improved growth formulation, the total frazil growth rate should be decreased.
Figure 6.9 – Comparison of frazil growth using two different formulations. The improved formulation (solid line) uses radius-dependent values of $\nu$ and $\Theta'$, whereas these are both set to one in the original formulation (dashed line). (a) Frazil heat flux. (b) The mean radius of all crystals kept in suspension. The dash-dot line shows the results when using the improved formulation but increasing the turbulent dissipation rate such that $\eta = 1$ mm.

Overall, the change in the growth formulation makes very little difference to most of the model output. Figure 6.9a shows the frazil heat flux for both the original and improved formulations over the first 25 km under standard conditions. The small difference between the solid and dashed curves means that other predicted variables, e.g., supercooling, also show only a small change.

Figure 6.9b shows the key difference between the growth formulations, which is the mean size of crystals in suspension. The increased growth of crystals, especially small crystals, with the original formulation leads to the predicted mean crystal radius being 0.2–0.3 mm larger. For example, the growth of crystals with sub-millimetre radius is 2–3.5 times faster relative to the improved formulation.

A third situation was tested, using the improved formulation and an increased turbulent dissipation rate such that $\eta = 1$ mm. This decrease in the Kolmogorov length scale increases the crystal growth rate by a factor between 1.4 and 2.7 depending on crystal size (see Figure 5.3b). The initial frazil heat flux is approximately doubled due to this growth increase, but the mean crystal size increases by only up to 8%.

### 6.5 Different Initial Conditions

#### 6.5.1 Initial Frazil Concentration

Changing the initial frazil mass concentration, $C_i^{\text{tot}}$, leads to predictable changes in the model output. The frazil heat flux and precipitation rate both behave simply: they increase if $C_i^{\text{tot}}$ is increased and vice versa. Further, the value of $C_i^{\text{tot}}$ has little effect on the distribution of these two quantities with distance. For example, regardless of
the initial concentration, the frazil heat flux declines to 25% of its initial value after approximately 5 km, and the predicted peak in precipitation (see e.g., Figure 6.8d) occurs at the same location.

The behaviour of most other properties follows from the frazil heat flux. When this flux is large, the depth-averaged supercooling reduces toward zero rapidly with distance from the ice shelf edge and the temperature increases quickly. Increased frazil growth causes an increase to the plume’s salinity, but its effect is small in comparison to the addition of salt from growth at the ice–ocean interface.

Halving the initial frazil mass concentration, $C_{i}^{tot}$, causes an increase of $\sim$5% in the mean frazil radius. This occurs because the plume’s supercooling, i.e., the heat sink for frazil growth, is distributed among fewer crystals. Conversely, doubling $C_{i}^{tot}$ leads to a decrease of $\sim$10% in the mean frazil radius.

6.5.2 Initial Supercooling

Under standard conditions, the plume has a supercooled depth, $D_{sc}$, of 65 m at the ice shelf edge, or equivalently $T_{sc} = -30 \text{ mK}$. This level of supercooling is based on the average supercooling measured at the Intermediate site (Section 4.2.3). At other sites on the cross-sound transect the supercooling reached depths of 15–55 m below the ice (Figure 4.10a). The frazil concentration and ambient current velocity most likely differ significantly among these sites. However, these changes are not considered here. Instead, the effect of changing the initial supercooling is considered independently.

Three different values, one larger and two smaller, than the standard value were used to test the behaviour of the model to a change in the initial supercooled depth. These depths, which are 20, 40 and 80 m, are equivalent to $T_{sc} = -3$, $-12$ and $-42 \text{ mK}$. Figure 6.10 shows how these different initial values affect the magnitude of supercooling downstream.

The key change in model output associated with a decrease in the initial supercooling is the frazil size distribution. Reducing the initial value of $D_{sc}$ to 20 m means...
the initial mean frazil radius is 0.84 mm and 50 km away from the ice shelf edge this becomes 0.53 mm. These values are 44% and 35% smaller, respectively, than suggested by the standard model conditions.

The reduction in frazil size leads to a result that is not immediately obvious. With reduced supercooling the total frazil concentration far from the ice shelf increases significantly because the smaller crystals are not prone to immediate precipitation. The total frazil concentration still drops initially, but 50 km beyond the ice shelf edge, the concentration remains high, at half of its original value. However, the remaining frazil has an insignificant thermodynamic effect because, beyond 10 km from the ice shelf edge, the magnitude of $T_{sc}$ drops below 1 mK, so very little growth occurs.

6.5.3 Initial Plume Thickness

The effect of a change in initial plume thickness was tested by both increasing and decreasing the standard initial value of 50 m by 50%, while the value of $C_{tot}^i$ was scaled such that the total frazil ice flux at the ice shelf edge was the same in all model runs.

The rates of change of the plume’s temperature and salinity are reduced if the plume is made thicker because the heat and salt added due to ice growth is distributed throughout more water. Further, a thickness increase causes a reduction in $T_{sc}$, leading to frazil ice playing a less significant role.

The precipitation rate (Equation 5.2) is not dependent on the plume thickness, with apparent dependence being due to the adjusted value of $C_{tot}^i$.

The behaviour of the frazil size distribution with a change in plume thickness is complex because the initial thickness affects both $C_{tot}^i$ and $T_{sc}$. However, the mean frazil radius is altered by less than 5% over the first 30 km and this is the region where the frazil concentration is significant. Hence, the complex behaviour is unimportant.

6.5.4 Initial Plume Velocity

Changing $U(0)$ without changing the ambient velocity has a large impact on the initial frazil size distribution, which affects the frazil heat flux and consequently the temperature. However, regardless of its value at the ice shelf edge, $U$ drops by at least 90% of this value within the first 10 km. Beyond this point, its only other significant effect is in determining the plume’s steady-state thickness. This results in changes to the rate of increase of salinity and temperature as described in Section 6.5.3.

Figure 6.11 shows how the frazil size distribution changes with a change in plume velocity at the ice shelf edge. With an increase from 1 to 5 cm s$^{-1}$, the frazil size constituting the peak in the initial mass concentration curve increases from 1.9 to 3.5 mm. After 10 km, the average crystal size remains larger for the faster plumes, but this effect is much less pronounced. It is worth reiterating that the velocities $U_a$ and $U_T$, which are held at their standard values (see Section 5.8), also alter the
precipitation rate, i.e., the frazil size distribution is dependent on more than just the magnitude of \( U \).

### 6.6 Sea Ice Conditions

#### 6.6.1 Thermohaline Properties

Within the model, the thermohaline properties of sea ice are characterised by two input parameters: sea ice salinity and the basal temperature gradient. The former parameter’s effect was investigated by changing its standard value of 5 to 0 and to 10. Similarly, the latter parameter’s effect was investigated by changing the standard value of \(-10\) K m\(^{-1}\) to 0 K m\(^{-1}\) and to \(-20\) K m\(^{-1}\).

Changing the sea ice salinity makes no difference to the model output other than a small change to the rate at which the plume’s salinity increases. An increase in sea ice salinity reduces the input of salt into the plume. The temperature gradient just above the sea ice–ocean interface has a much larger effect. The temperature gradient’s effect on the plume’s salinity was approximately eight times larger than that of the sea ice salinity. This is expected as the temperature gradient is the key quantity affecting the growth rate of the sea ice cover, and therefore the rate of salt rejection.

Other than the sea ice growth rate and plume salinity, the only other output quantity to show any real change is the interfacial heat flux. The magnitude of this flux is reduced with an increase in the magnitude of the sea ice temperature gradient because of the increased latent heat released into the plume due to the increased ice growth at the ice–ocean interface. The overall change, however, does not exceed
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There are two interesting results from the investigation of different sea ice conditions. The first has been alluded to already: the temperature gradient above the sea ice–ocean interface has a negligible effect, if any, on the plume’s supercooling. The second is that, even without a temperature gradient, the growth rate at the interface is greater than \(17\text{ cm yr}^{-1}\) over the first 100 km. This growth represents an enhancement due to supercooled water flowing past the ice–ocean interface. Figure 6.12 shows the growth enhancement over the first 250 km beyond the ice shelf edge. This was calculated by comparing the standard model output against the growth rate calculated with the same salinities and velocities, but the plume temperature set to the surface freezing point.

**6.6.2 Basal Slope**

The presence of a basal slope means that a buoyancy force will act on the plume, and hence it will undergo a smaller deceleration beyond the ice shelf edge. Solid sea ice thickness does not differ enough to provide a basal slope that will produce any real change to the model output. In contrast, the sub-ice platelet thickness does differ significantly. To quantify its effect, the standard model run was compared against two runs in which a basal slope was present. The measured sub-ice platelet layer thickness shown in Figure 6.3d provides an along-plume slope of approximately \(0.5\text{ m km}^{-1}\). This provides the first test slope. The second test slope was chosen to be half this value.

With a slope of \(0.25\text{ m km}^{-1}\), the plume’s velocity dropped from 0.30 to 0.11 m s\(^{-1}\) over the first 40 km. With the steeper basal slope, the velocity dropped to only 0.18 m s\(^{-1}\). The change in velocity behaviour has consequences for other plume properties, as described in Section 6.3. Overall, the results are very similar to those shown in Figure 6.8. The steep basal slope is much like the scenario in which \(U(0) = 1\text{ cm s}^{-1}\) and \(U_a = 5\text{ cm s}^{-1}\). Alternatively, results for the shallower slope reproduce much of the same behaviour of the scenario in which \(U(0) = 2\text{ cm s}^{-1}\) and \(U_a = 4\text{ cm s}^{-1}\).
6.6.3 Interfacial Drag

In this model, the magnitudes of the ambient and tidal currents are not affected by drag. Thus, the drag coefficient affects only the plume’s deceleration, the efficiency of turbulent transfers at the ice–ocean interface, and the size of frazil crystals kept in suspension.

Figure 6.13 shows the effect of increasing and decreasing the drag coefficient by 40% of its standard value. A higher drag coefficient allows more frazil to stay in suspension because it reduces the critical velocity above which precipitation cannot occur (Equation 3.51). Consequently, the plume’s supercooling decreases more rapidly with distance than the standard run (Figure 6.13b). In contrast, with a low drag coefficient, precipitation is rapid and the frazil concentration drops to 4% of its initial value after only 10 km (Figure 6.13a). Hence, supercooling decreases more slowly (Figure 6.13b), due to the lack of frazil heat flux.

The interfacial heat flux (Figure 6.13c) responds to the change in supercooling. In the high drag case, its magnitude is large close to the ice shelf due to the high friction velocity (Equation 5.1). However, the interfacial heat flux also decreases rapidly with distance because of the decrease in the supercooling, which results from the higher frazil concentration, and hence higher frazil heat flux, far from the ice shelf.

The mean frazil radius (Figure 6.13d) for the high drag case is larger than the standard model run at the ice shelf edge. However, after 15 km, this order switches because fewer crystals grow to a large size in the high drag case due to the decreased supercooling. With a low drag coefficient, very few crystals with a radius larger than 2 mm are kept in suspension, Consequently, the mean radius is approximately 12% smaller than the standard result, over the first 25 km. Beyond this point, the plume’s frazil concentration drops below 0.3% of its original value and the mean radius can no longer be calculated reliably.

It is worth noting here that changing the RMS tidal speed of the ambient water column has much the same effect as changing the drag coefficient. Like $C_d$, a change in $U_T$ affects the plume’s velocity, interfacial turbulent transfers and the frazil size distribution.

6.7 The Ambient Water Column

Under standard conditions, in which the basal slope is zero, changes to the ambient water column’s temperature and salinity make very little difference to the model output. This was tested by running the model with a warm ($T_a = 0^\circ\text{C}$) and very salty ($S_a = 35.5$) ambient water column. The difference in the plume’s temperature between this result and the standard model run never exceed 1 mK. Similarly, the plume’s salinity differed by less than $5 \times 10^{-5}$ and the total frazil concentration typically differed by only 0.3 mg L$^{-1}$. These results suggest that the entrainment and
Figure 6.13 – The plume’s evolution with different drag coefficients: low (9×10⁻³), standard (15×10⁻³), and high (21×10⁻³). (a) Mass concentration of frazil ice in suspension. (b) Supercooled depth. (c) Interfacial heat flux. (d) Mean radius of frazil crystals in suspension. The mean radius for the low drag case is not calculated past 25 km, by which time the plume’s frazil concentration falls below 0.3% of its initial value.
detrainment parameterisations described in Section 5.5 are negligible. An explanation for this is that the velocity driving the entrainment/detrainment is the plume’s velocity \( U \). This is only 3 cm s\(^{-1}\) at the ice shelf edge, and drops to effectively zero over the first 10 km from the ice shelf.

Without a basal slope, the density contrast, \( \Delta\rho \), between the plume and ambient water has little effect because the buoyant force is always zero. If a basal slope were present, the plume’s buoyancy forcing would increase if the ambient water column was made warmer and saltier. A discussion of how the model would behave with an increase in the temperature and salinity of the ambient water column has already been given in Section 6.6.2 because an increase in \( \Delta\rho \) has the same effect as an increase in \( \sin(\vartheta) \) (see Equations A.1 and A.2).

### 6.8 Frazil Ice Properties

The size and concentration of frazil crystals within the plume is dependent on a number of processes and quantities: growth, precipitation, secondary nucleation, crystal aspect ratio and the choice of \( r_i(k) \). The first two processes have been discussed in this chapter already. The importance of the other frazil parameters has been investigated previously by Smedsrud and Jenkins (2004). However, given the change to the supercooling formulation (Section 5.3), and therefore the rate of frazil growth, it is worthwhile reinvestigating their effects.

#### 6.8.1 Aspect Ratio

Changing the aspect ratio alters the volume of frazil kept in suspension. This has only a small effect on most output quantities. The mean radius and precipitation rate, however, are significantly affected. Under standard conditions the aspect ratio is 1/50. Increasing the thickness of crystals such that \( a_r = 1/20 \) reduces the mean radius by 25–30%. Conversely, decreasing thickness such that \( a_r = 1/100 \) increases the mean radius by 25–30%.

Under standard conditions, much of the precipitation occurs within the first five kilometres, and the same is true with a change in aspect ratio. The average precipitation rate over this length drops by 14% when the crystals are made thinner, and increases by 22% when they are made thicker.

#### 6.8.2 Secondary Nucleation

The efficiency of secondary nucleation is determined by the value \( \pi_{\text{max}} \), the upper limit of \( \pi_i \) in Equations 3.45 and 3.46. Smedsrud and Jenkins (2004) investigated the sensitivity of their model to both a 100-fold increase and a 1000-fold decrease in this value. Surprisingly, they found that the decrease made hardly any difference to the
standard model run, and the increase led to an only 50% increase in total precipitation and maximum frazil concentration.

The model results of this chapter are more sensitive to the value of $n_{\text{max}}$. Its effect is therefore tested over a smaller range, from a 10-fold decrease to a 25-fold increase. Figure 6.14 shows how the supercooled depth and mean radius change over five different values within this range. In both cases in which $n_{\text{max}}$ is increased, the mean crystal radius is significantly decreased because crystals of the smallest size are continually being produced by collisions. The smaller crystals grow more efficiently, which reduces the supercooling rapidly with distance. They are also more easily kept in suspension, further enhancing their ability to reduce the supercooling.

An important problem arose when testing a change to a low $n_{\text{max}}$. In this case the frazil concentration never reached $C_{\text{tot}}^i$ during the spin-up phase (see Section 5.7.2). Overcoming this problem required that a small flux ($\leq 6 \times 10^{-6}$ g m$^{-2}$ s$^{-1}$) of the smallest frazil crystals be continuously added during the spin-up phase to ensure the frazil concentration reached 0.1 g L$^{-1}$. Therefore, the model runs with a low $n_{\text{max}}$ may be biased. Nevertheless, the tests were undertaken to give some indication of the model’s behaviour with a decrease in $n_{\text{max}}$.

When secondary nucleation is inefficient, the frazil concentration drops to nearly zero over the first 10 km, much like in the standard model run. The lack of production of small crystals results in less frazil growth overall, and hence the level of supercooling remains high with distance from the ice shelf. For the case $n_{\text{max}} = 0.1 \times 10^3$ m$^{-3}$, Figure 6.14a shows unexpected behaviour for the mean radius beyond 15 km, in that it begins to increase with distance. By this point the frazil concentration is below
0.5% of its original value and the mean radius calculation starts to become affected by the very low number concentrations of crystals in the smaller size classes.

### 6.8.3 Frazil Size Spectrum

As stated by Holland and Feltham (2005), “any result which is found to be dependent upon \( N_{\text{ice}} \) should be regarded as potentially suspect”. They and Smedsrud and Jenkins (2004) observed that results became significantly affected only when \( N_{\text{ice}} \lesssim 10 \). Here, tests with different values of \( N_{\text{ice}} \) reproduced this expected behaviour. In this study \( N_{\text{ice}} \) is 25, and so its value is considered sufficiently large.

These authors, however, did not test the ramifications of the choice of the smallest size class. It was just shown in the previous section that the efficiency of secondary nucleation significantly alters the level of supercooling. Therefore, it is possible that a change to the value of \( r_i(k = 1) \) would also affect the results. Indeed, it would be very undesirable if the model did display sensitivity to the choice of the smallest size class, since all crystals produced by secondary nucleation are assumed to belong to this size class. Fortunately, the results do not display such sensitivity. Rather, to within the accuracy of the line width on a typical plot, the results produced by varying \( r_i(k = 1) \) from 0.01 to 0.3 mm are all the same. Because of this insensitivity, the frazil size spectrum, \( r_i(k) \) was chosen to have evenly spaced radius values. The benefit of this is that the interpretation of Figures 6.7 and 6.11 is not biased toward the spacing of \( r_i(k) \), nor is the calculation of the mean crystal radius.

### 6.9 Changing Multiple Input Parameters

The sensitivity tests undertaken so far (Sections 6.3–6.8) have involved changing either one input parameter or two related parameters, e.g., \( U(0) \) and \( U_a \) or \( \Theta' \). If two or more unrelated parameters are changed, the model’s behaviour can be anticipated by examining the effect of each parameter separately. Consider the example of increasing \( U_a \) and \( \pi_{\text{max}} \) simultaneously. Separately, either increase would raise frazil concentration and lower \( D_{sc} \), relative to the standard run. Increasing the input parameters simultaneously means frazil concentration is raised further and \( D_{sc} \) is lowered further. Conversely, increasing \( U_a \) while decreasing \( \pi_{\text{max}} \) (or vice versa) minimises the changes in frazil concentration and \( D_{sc} \) as the change in one parameter now offsets the other.

Given that there are tens of input parameters, it is impractical to verify that the behaviour described above applies to even a small percentage of all possible parameter combinations. However, each of the five selected combinations tested exhibited predictable changes in model output. This suggests that potentially complex relationships between input parameters are insignificant.

One particular combination worth noting is the initial values of \( U \) and \( D \). If these
are both increased (decreased), the changes in the plume’s properties with distance are more gradual (rapid), reflecting the larger (smaller) volume flux of ISW. However, the model’s qualitative behaviour does not change. Conversely, if $U$ and $D$ are altered such that $UD$ is kept constant, the output shows minimal change from the standard run.
7.1 Data Interpretation

7.1.1 Overview of Study Sites

Being proximate to an ice shelf, McMurdo Sound is representative of much of Antarctica’s coastline. It is therefore worthwhile assessing the Ross and McMurdo Ice Shelves’ influence on the waters of the Sound and its landfast sea ice. Water moving out from underneath the McMurdo Ice Shelf contains tell-tale signs of its prior journey. Its ice shelf–influenced temperature and salinity and suspended frazil ice leave a signature both in and beneath the sea ice (Gough et al., 2012b).

The modelling results described in Chapter 6 show that both supercooling and frazil ice concentration reduce significantly over the first 10 km beyond the ice shelf edge. Being only 3 km from the ice shelf edge, the two cross-sound transects (Section 4.2.3) allow the ice shelf’s influence to be assessed before confounding processes become significant. The four sea ice cores, 12 ice thickness sites and four oceanographic sites along the transect allow for the development of a high-resolution picture, encompassing a range of quantities, of conditions very near the ice shelf edge. Data from the three oceanographic sites on the long-sound transect complement this picture by illustrating the rate at which oceanographic changes occur with distance from the ice shelf.

7.1.2 Conditions at the Ice Shelf Edge

The east–west variation within McMurdo Sound is evident in both ice and ocean data. This variation was described throughout Chapter 4 but, with the exception of Section 4.4, there was no quantitative comparison between the various ice and ocean measurements. This comparison is presented in Figure 7.1, which summarises the ice and ocean measurements from the cross-sound transect. The columnar/platelet ice transition and the solid ice and sub-ice platelet layer thicknesses are compared against the depths of in situ and potential supercooling, with the latter depth acting as a simple indicator of ISW. The comparison between these quantities makes it clear that there is a transition from “east” to “west” between 165°30′E and 166°E. On the western side the entire water column is potentially supercooled, i.e., the water temperature is below the surface freezing point. On the eastern side more than half of
Figure 7.1 – Conditions near the ice shelf edge. (a) Sites on the cross-sound transect. Letters indicate the initial(s) of named sites (Table 4.1). The black circle labelled LP85 indicates the site where Lewis and Perkin (1985) observed their maximum surface supercooling of 45 mK. (b) Depth of columnar/platelet ice transition. In this study, this transition is defined as the first clear disruption of the columnar structure. (c) Solid ice thicknesses. (d) Sub-ice platelet layer (SIPL) thickness. (e) Depth of the in situ supercooled layer, at both spring and neap tide, measured from the base of the sub-ice platelet layer. (f) Depth of potentially supercooled water. CTD casts stopped approximately 20 m above the sea floor. It was assumed these last 20 m were potentially supercooled if the water column above was also potentially supercooled.
of columnar ice within each core can determine the period of time at the start of the ice growth season before ISW appears at the ocean surface (Section 4.5). The distribution of columnar ice (Figure 7.1b), along with the sub-ice platelet layer thickness, suggests that the lateral extent of surface ISW flow into McMurdo Sound continually expands from the west throughout the winter. This adds weight to the argument given in Section 2.4.2 that the surface current at a location in eastern McMurdo Sound switches from an inflow into the ice shelf cavity to an outflow at some point during the winter.

7.1.3 Comparison with Other Studies

There are a number of features of the observations made in this study that agree with past studies. The brief review given below suggests that the observations described in Chapter 4 can be considered typical of end-of-winter conditions.

Robinson (2012) observed a sloping oceanographic boundary similar to that shown in Figure 7.1 in an east–west transect approximately 20 km north of the ice shelf, which she attributed to the outflow of buoyant ISW “carving a corridor northward”. This oceanographic transect coincided with that undertaken by Dempsey et al. (2010) who found that columnar ice constituted approximately half of their three “eastern” cores, but was almost non-existent in their “western” cores. Other studies in eastern McMurdo Sound, but closer to the ice shelf, typically find columnar ice to depths of 1.0–1.5 m (e.g., Leonard et al. (2006) and references cited therein). The extent of columnar ice found in this study in the West and Far East cores (Figure 7.1b) are similar to these other studies. Dempsey et al. (2010) also produced a map from ice core observations in the literature showing the relative abundance of platelet ice at different locations in the Sound. Both their map and a similar map by Barry (1988) show a pattern consistent with the observed sub-ice platelet ice thickness shown in Figure 4.9b.

Lewis and Perkin’s (1985) contours of surface supercooling suggest that in situ supercooling occurs throughout McMurdo Sound, and that the maximum surface supercooling observed was 45 mK at a site west of the centre of the ice shelf (see Figure 7.1a). In this study, the maximum surface supercooling was 40–50 mK and measured at the nearby Intermediate site.

7.2 Extension of a Simple Plume Model under Sea Ice

7.2.1 Motivation for a Simple Model

In the Southern Hemisphere, the Coriolis effect deflects northward-moving water westward, unless it encounters a topographical barrier. Within and beyond McMurdo Sound, the Victoria Land coastline provides such a barrier for several hundreds of
kilometres. The flow of buoyant ISW into and out of McMurdo Sound is therefore continuously steered northward, inviting the application of an ISW plume model with only one horizontal dimension. Brine rejection beneath growing sea ice homogenises the properties of the ocean surface layer allowing a further simplification of a depth-averaged approach. Also, being ice-covered for most, if not all, of the year, the western Sound is subject to sufficiently low variation such that a steady-state treatment should be adequate.

As described in Sections 3.12–3.13, more sophisticated models exist, but for this study the optimal compromise between tractability and sophistication was to extend the model of Smedsrud and Jenkins (2004), i.e., a model that utilises the simplifications described above. The benefits of these simplifications are wide-ranging: thermodynamic effects are easily separated from dynamic effects, numerical errors are both easily identifiable and remedied, and model runs take only seconds. This computational efficiency is beneficial in that it allows numerous sensitivity tests to be undertaken, which can inform more complex models, e.g., ROMS models (see Section 3.13). As such models can take days, weeks or even months to run, it is an expensive exercise to undertake sensitivity tests for minor adjustments such as changes to basal ice properties (Section 6.6) or the parameterisation of frazil ice growth (Section 6.4).

The modelling results described in Chapter 6 will be interpreted in Section 7.3. However, before that, it is beneficial to discuss both the strengths and shortcomings of the extension of a simple model under sea ice.

7.2.2 Shortcomings of an Extension of the Plume Model

Other than the simplifications described in Section 7.2.1, the most important process that the extended model oversimplifies is that of tidal flow. Without removing the steady state restriction, tidal flow in the model does not induce a reversal of plume flow, nor can its strength vary with time in the spring–neap cycle. Instead, model tides only increase the level of turbulence. In reality, tidal flow would either boost or hinder ISW flow to a level dependent on whether the instantaneous tide is ebb or flood and spring or neap. The tide’s effect would then be akin to either increasing or decreasing the ambient current velocity (see Section 6.3).

Other than tidal processes, the key dynamic processes missing in the extended plume model are those at the ice shelf edge. Many ice shelves end with a vertical wall that is several tens or even hundreds of metres thick. A flow moving out from beneath such ice shelves, perpendicularly to the ice front, would induce eddies behind the ice wall. However, the geometry of the McMurdo Ice Shelf minimises such an occurrence. First, the change in ice thickness between the ice shelf and contiguous solid sea ice is only approximately 20 m (McCrae, 1984), which is smaller than the modelled plume thickness. Second, the introduction of frazil ice would probably smooth this transition. The area just beyond the ice shelf edge is likely to be a location of intense frazil ice
accumulation. The step change in draft that initially induces this accumulation would then smooth over time.

The extended model’s key shortcoming regarding thermodynamics is that ice–ocean interactions are not formally treated in terms of a sub-ice platelet layer. Such a layer will form in a region where frazil ice precipitates and a negative ocean heat flux is present with a magnitude greater than approximately one-third of the upward conductive heat flux (Gough et al., 2012b). Observations confirmed that this condition was satisfied for much of McMurdo Sound (Figure 4.9b), at least at the end of the 2011 winter.

In comparison to solid sea ice, the thermal and physical properties of the sub-ice platelet layer are not well studied. As a result, parameterisations of turbulent heat and mass transfer at the base of the layer that could be applied to the extended model are new and untested (Robinson, 2012). Instead, the enhanced turbulence produced by the layer’s slushiness is mimicked using a high drag coefficient, but otherwise it is assumed that turbulent transfers are no different to those beneath solid sea ice, i.e., they can be parameterised in terms of the friction velocity, a temperature/salinity difference and an experimentally derived constant.

A further simplification related to the previous problem is that solid sea ice growth and precipitation are treated independently. The extended model assumes that precipitation does not alter the growth mechanism of solid sea ice and that supercooling enhances the growth of only solid sea ice. In reality, these processes are coupled: crystals that precipitate alter the structure of the interface and can become incorporated into the solid ice, and some supercooling is relieved through growth of the sub-ice platelet layer, i.e., not only the solid sea ice. Crystals in the sub-ice platelet layer would have reached a fraction of their final size while suspended in the water column and the rest after precipitating. This fraction likely depends on time of the year and location. Further, an average value is not well known (Section 2.7) making it difficult to model solid sea ice growth and precipitation as a coupled system.

Frazil ice processes are not well understood, nor are they easy to quantify. As a result their modelling is reliant on a number of assumptions and unknowns and model output is subject to much uncertainty. The model shortcomings associated with frazil ice are discussed separately in Section 7.4.

7.2.3 Strengths of an Extension of the Plume Model

Despite the simplicity of the extended model, its output is in good agreement with observations beneath sea ice (Figure 6.3), especially the level of supercooling. Three key adaptions were applied to the SJ04 model to produce this agreement: the use of turbulent transfer parameterisations appropriate for sea ice, the inclusion of the ambient current, and the modification of the supercooling formulation (Sections 3.3, 5.2 and 5.3, respectively).

In Chapter 6, the level of supercooling was most often described in terms of the
supercooled depth $D_{sc}$. The introduction of this parameter had two major benefits. First, it provided a single quantity that is easy to visualise and compare against measurements. Second, prediction of its value was insensitive to model changes. This is because it was (i) independent of the plume thickness, (ii) independent of the frazil seeding strategy (Section 5.7.2), and (iii) subject to a negative feedback, whereby a decrease in its value results in slower frazil growth, which reduces its rate of decline.

A valuable trait of the extended model described in Chapter 6 is that it can be compared directly against both ice and ocean data. Oceanographic observations beneath ice shelves are scarce in both time and location. As a consequence, ISW plume models lack complete in situ validation of the thermohaline evolution of the plume. For example, Smedsrud and Jenkins suggested that supercooling persisted over approximately 80 km under the ice shelf after first occurring. They were able to validate this modelled length scale to some degree by comparing the predicted resulting frazil precipitation to radar-derived marine ice thickness. However, without coincident oceanographic data from several sites on their prescribed plume paths, this validation is incomplete.

The comparison between measurements from McMurdo Sound and the output of the extended model in Chapter 6 not only serves to validate the model itself, but can also inform models applied beneath ice shelves. For example, the good agreement between measurement and model of the rate of change of supercooling with distance (Figure 6.3a) indicates that the frazil and interfacial heat fluxes can be adequately represented in ISW plume models. Conversely, an ISW plume model could not have reproduced measured conditions in McMurdo Sound without the addition of an ambient current. This is primarily because of the lack of basal slope, but also suggests that it may be inappropriate to neglect nonplume flows within an ice shelf cavity. Obviously some parameters, e.g., frazil concentration and size distribution, still lack validation but there is ongoing work looking at such quantities (e.g., Leonard et al., 2010; Purdie, 2012; Gough et al., 2012b).

### 7.3 Model Interpretation

The plume’s thermohaline properties evolve due to salt and heat fluxes at three locations: the ice–ocean interface, within the plume at the edges of frazil crystals, and the bottom of the plume. Fluxes through the bottom are negligible (Section 6.7) and so will not be discussed further.

Fluxes at the ice–ocean interface are predictable: the heat flux into the plume is largely determined by the level of supercooling and declines monotonically with distance from the ice shelf, while the salt flux is primarily a function of the input basal temperature gradient in the sea ice and stays relatively constant with distance.

Heat and salt fluxes within the plume are subject to more variation, for they are dependent on a range of frazil ice parameters. Frazil ice presents a large total
surface area and consequently has the potential to induce rapid change to the plume’s
thermohaline properties. For example, typical model results indicated that the frazil
heat flux was initially four times larger than the interfacial heat flux, but after 8–15 km,
the interfacial flux became more dominant. Similarly, the typical salt flux resulting
from frazil ice growth is initially 90–170 kg m\(^{-2}\) yr\(^{-1}\), but this drops by over 80% over
the first 10 km. By comparison, the standard temperature gradient at the base of the
sea ice of 10 K m\(^{-1}\) and the large initial supercooling result in an interfacial salt flux
of 85–115 kg m\(^{-2}\) yr\(^{-1}\), but this drops by only 20% over the first 10 km

The concentration and size distribution of frazil, and hence its associated heat flux
(Equation 3.32), is primarily determined by the upper limit on the size of crystals that
can remain in suspension. Inspection of Equations 3.51 and 5.2 shows that crystals of
a certain size will precipitate more quickly with a decrease in either \((U + U_a)^2 + U_T^2\)
or \(C_d\). These are the two most dominant quantities influencing the distance that
frazil ice will travel before rising out of suspension. Also, model output is strongly
dependent on their values. For example, an increase in the ambient velocity from 3
to 5 cm s\(^{-1}\) led to a 40-fold increase in the frazil ice concentration 50 km from the
ice shelf (Section 6.3.3). Alternatively, after only 10 km from the ice shelf edge, an
eight-fold concentration difference occurred between the low and high drag (standard
\(C_d \pm 40\%\)) sensitivity tests.

Frazil population dynamics are also affected by processes controlling their growth.
Frazil’s growth rate decreases if the turbulent dissipation rate increases or the effect
of salt diffusion is included (Sections 3.7.1 and 6.4, respectively). These changes
have a smaller influence than those described above because, under most conditions,
precipitation dominates growth and the frazil concentration drops rapidly over the
first 10 km.

Frazil growth is also affected by the efficiency of secondary nucleation, which is
controlled by the parameter \(\pi_{\text{max}}\). An increase in this parameter to five times its
standard value—a large but not unrealistic increase—reduces the mean frazil radius by
0.35–0.50 mm over the first 50 km (Figure 6.14a). As a result the suspended ice is less
likely to precipitate and the concentration remains near its initial value as opposed
to dropping to effectively zero as in the standard model run. In the model, small
crystals can be produced only through secondary nucleation. Overall \(\pi_{\text{max}}\) represents
the frazil ice parameter that can induce the most significant change to model output.

In summary, the plume’s evolution is strongly dependent on parameters controlling
the nucleation and precipitation of frazil. Changes to these parameters have the
largest effect over the first 10 km, which then influences the subsequent evolution.
Parameters affecting interfacial processes have a smaller influence on model output,
but it is still important that they are quantified correctly, for they are involved in
calculation of the sea ice growth enhancement.


7.4 Frazil Ice

The previous section highlighted the importance of frazil ice in modelling the thermodynamics of the surface ocean layer. Unfortunately, it is subject to many poorly defined parameters and many processes that are not well understood. One of the largest unknowns is the process of nucleation. As alluded to in Section 3.7.2, this problem is often ignored. A hand-waving argument is given that supercooling promotes growth of dendritic crystals at the ice–ocean interface, which can detach and become suspended by turbulence. If this process is the dominant mechanism for primary nucleation then a flux of seed crystals would be expected everywhere that freezing at the base of the ice occurs. This is not the case in ISW plume models. Instead, it is assumed that the process provides a single instantaneous flux of seed crystals and, following this, new crystals can be produced only through secondary nucleation.

The problem of nucleation is especially relevant to this study, because basal freezing is prominent and hence dendritic crystals will be abundant. Modelling results would show two key qualitative changes if seed crystals were being continuously added as the plume propagated. First, supercooling would decline over a shorter distance because these seed crystals would continuously provide an efficient heat sink. Second, the total precipitation would both increase and span a longer distance. In typical results from the extended model, 90% of the total precipitation occurs within the first 10 km and 99% occurs within the first 50 km. However, Jones and Hill (2001) analysed a sea ice core taken 70 km from the ice shelf edge and found that it was composed of 54% platelet ice, suggesting that a significant frazil ice flux remains at this location. Model results would suggest such a flux is unlikely without the introduction of seed crystals along the way. Indeed, for the initial crystals to remain after 70 km, they would have to persist in suspension for 2–3 weeks (assuming modelled velocities are correct).

There is also another process that can act as a source of crystals beyond the ice shelf edge: the resuspension of previously deposited frazil. Like primary nucleation, this process is not included in the model. In this thesis, no attempt was made to either parameterise these seeding processes or improve their understanding. However, as mentioned in Section 7.3, their inclusion would increase the model’s sophistication.

The most likely reason that secondary nucleation is included in models that overlook primary nucleation and frazil resuspension is that it is possible to derive an estimate for its rate (see Section 3.7.2). This derivation, however, has a major drawback: the number of crystals produced by collisions is strongly dependent on a single quantity, $n_{\text{max}}$, which is an empirically derived value that was reported with only a vague mention of its associated uncertainty (Smedsrud, 2002).

Smedsrud and Jenkins’s (2004) modelling suggested that the majority of the suspended frazil mass beneath an ice shelf would be composed of crystals of radius 0.3–0.8 mm. In contrast, the results given in Chapter 6 suggest the equivalent quantity in McMurdo Sound is 1–2 mm. The larger sizes primarily result from changes to
the supercooling formulation (Section 5.3) and the drag coefficient. This increase in the size estimate would have been even larger were it not for the addition of the size-dependent salt diffusion parameter ($\Theta$), which has the largest rate-limiting effect for the smallest crystals.

It is worth noting that the size estimates are based on frazil crystals being circular discs of fixed aspect ratio that undergo heat transfer at only their edges. In reality, frazil crystals would exhibit a range of aspect ratios, possibly dendritic growth and some heat loss through their faces. Also, the revised supercooling formulation does not allow melting of frazil to occur. Despite these drawbacks, video observations in McMurdo Sound can provide rough validation that modelled crystal sizes are realistic. Gough et al. (2012b) reported that most crystals observed in the water column had a diameter of less than 5 mm.

### 7.5 Predicting the Effect of Supercooling

Making a prediction of the evolution of *in situ* supercooled water beneath sea ice in McMurdo Sound is one of the central goals in this thesis. As there are several tens of input parameters that can be adjusted and few constraints on their range, it is impractical to make a single prediction. Instead, the prediction is summarised in Figure 7.2a, which shows $D_{sc}$ over 250 km for 26 different sensitivity tests (Sections 6.3–6.8). The average of all model runs is shown by the thick black line. Related to the supercooling is its effect on sea ice thickness. Figure 7.2b shows the solid sea ice growth rate enhancement for all model runs, i.e., the actual ice growth rate ($-m'\rho_w/\rho_i$) subtracted from that equivalent quantity if the plume’s temperature was always at the surface freezing point. Figure 7.2c shows the increased thickness due to precipitation in terms of solid ice equivalent ($-p'\rho_w/\rho_i$). These growth enhancements are calculated assuming the supercooling at the ice shelf edge is always 65 m deep and that growth continues year-round. In reality, the growth season is typically 8–9 months long. Two sets of sensitivity tests were excluded from Figure 7.2: those in which the initial supercooling was changed and those in which a basal slope was present (because the slope acts as a generator of supercooling and thereby skews the results).

An alternative way to quantify the effect of supercooling is by calculating the oceanic heat flux $F_w$, which is a quantity that can be compared against observations. This heat flux, when negative, increases the total ice thickness by both precipitation of frazil ice in the water column and relief of supercooling at the ice–ocean interface. In the model, precipitation is treated independently of freezing at the base of sea ice (Section 7.2.2). Therefore, the oceanic heat flux can be calculated as the upward conductive heat flux minus the latent heat flux needed to produce the total ice thickness, i.e., both solid ice and precipitated frazil ice. Both $m'$ and $p'$ are negative for ice growth and solid sea ice contains salt while frazil ice is assumed to be pure.
Therefore, $F_w$ is given as

$$F_w = -K_i \left. \frac{dT_i}{dz} \right|_b + \rho_w L_i m' + \rho_w L_p' \quad (7.1)$$

Gough (2012) derived contours of the mean late-winter oceanic heat flux in McMurdo Sound by knowing (i) the difference in depth between the columnar/platelet transition and the bottom of the sub-ice platelet layer, (ii) the solid fraction of ice in the sub-ice platelet layer, and (iii) the time taken for the sub-ice platelet layer to form. A slice through these contours happens to follow the same direction as the plume paths used in this study and can therefore be compared against the model. Both the modelled and observed oceanic heat fluxes are shown in Figure 7.3. The comparison shows that the predicted spike in precipitation near the ice shelf edge causes the magnitude of the modelled oceanic heat flux to be larger than the observed magnitude. However, after 2–5 km, these magnitudes are switched.

There were two reasons for choosing 250 km as the end point of the extended model. First, this is approximately the distance between the McMurdo Ice Shelf edge...
and the Drygalski Ice Tongue. This 150–1500 m deep piece of floating ice protrudes approximately 65 km out from Victoria Land. It provides the first major obstruction to the northward ISW flow from western McMurdo Sound, but also acts as a source of glacial meltwater (Wuite et al., 2009). Second, Stevens et al. (2009) estimated that a 50 m thick layer of ISW moving at approximately 5 cm s$^{-1}$ would remain supercooled ∼250 km beyond the ice shelf edge. Their estimate, based on a turbulent vertical heat flux from the ocean underneath, is in good agreement with our estimate from the extended model (Figure 7.2a).

Figure 7.2 shows that both the surface supercooling and the sea ice growth enhancement decay over a ∼100 km length scale. At distances of 100 km and 200 km from the ice shelf, the thickness of the \textit{in situ} supercooled layer decreases to 11 ± 6 m and 4 ± 3 m, respectively. In contrast, precipitation of frazil has a ∼10 km length scale and therefore provides a very localised thickness enhancement. To put these length scales into context, it is worth comparing them to Figure 3.13, which shows Hellmer’s (2004) and Beckmann and Goosse’s (2003) modelled predictions of the influence of ice shelves on sea ice growth. Both studies suggest the Ross and Filchner-Ronne Ice Shelves have an effect over ∼1000 km. Their models do not include frazil ice, which acts to localise the ice shelf influence. However, their models include processes that cannot be captured in a one-dimensional model. One example is the stabilisation of the water column brought about by the freshwater flux and deep convection, which affects the surface heat budget. Given that the inclusion of these more complex processes counteracts the exclusion of frazil ice, the true length scale over which ice shelves can affect sea ice growth will be less than the aforementioned value of ∼1000 km value, but more than the value of ∼100 km suggested by the extended model.
Chapter 8
Conclusions

8.1 Summary

This study aimed to (i) examine the distribution of ISW at the McMurdo Ice Shelf edge and its changes throughout the year and (ii) predict the persistence of the layer of in situ supercooled water as it propagated into and beyond McMurdo Sound. The first aim was accomplished using both oceanographic measurements and several proxies for the presence of ISW, e.g., a sub-ice platelet layer. The second aim was accomplished using a one-dimensional, depth-averaged ISW plume model with many adaptations that made it suitable for extension under sea ice and into McMurdo Sound. The new oceanographic data provided both initial model conditions and, in conjunction with historical data, downstream observations for model validation. Subsequent to this validation, the model was used to predict the evolution of supercooled water as propagated along the approximately 250 km path between the McMurdo Ice Shelf and the Drygalski Ice Tongue.

8.2 Data Analysis

Oceanographic measurements were taken beneath first-year sea ice at 11 stations (six different sites) in McMurdo Sound between 26 November and 3 December 2011. These measurements, along with analysis of four first-year sea ice cores, formed the primary dataset for this thesis, which was complemented by measurements of solid ice and sub-ice platelet layer thicknesses from 40 sites throughout the Sound.

The cross-sound transect 3 km in front of the McMurdo Ice Shelf edge clearly showed a transition from “east” to “west” in the five quantities observed: the columnar/platelet ice transition, the solid and sub-ice platelet layer thicknesses and the depths of the in situ and potentially supercooled water (Figure 7.1). Comparison between the depth of the in situ supercooled layer and the sub-ice platelet layer thickness (Figure 4.10) confirmed the expected result that the two quantities are related. This relationship, which builds on work by Barry (1988) and Dempsey et al. (2010), indicates that the thickness contours in Figure 4.9b effectively provide a high-spatial-resolution picture of the flow of supercooled water, integrated over several months, throughout the ∼700 km² area over which thickness measurements were taken.
The relative amount of columnar ice observed in the upper portions of four sea ice cores provides evidence that the northward flow of ISW at the McMurdo Ice Shelf edge increases in lateral extent during winter. Further, the delayed arrival of ISW at the Far East site is consistent with winter-long studies in the vicinity (e.g. Leonard et al., 2006; Mahoney et al., 2011).

### 8.3 Numerical Modelling

A one-dimensional, depth-averaged ISW plume model (Smedsrud and Jenkins, 2004) was adapted in several ways so that it could be extended beyond the ice shelf edge and used to track the evolution of the layer of *in situ* supercooled water within and beyond McMurdo Sound. Certain aspects of the geography of McMurdo Sound region, such as the thin ice shelf edge and the Victoria Land coastline, favoured the applicability of the extension of the model by reducing the importance of processes not included in the model, such as the Coriolis effect.

The primary adaptions to the model included (i) incorporating parameterisations for turbulent heat and salt transfer beneath growing sea ice (Section 3.3.2), (ii) accounting for the effects of currents that are present in the absence of fluxes that drive sub-ice shelf circulation (Section 5.2), and (iii) deriving an alternative formulation for the depth-averaged supercooling (Section 5.3). Changes were also made to the entrainment at the plume’s lower edge, the transfer of heat and salt at the edges of frazil ice crystals, the frazil seeding strategy and the choice of the input frazil radius $r_i(k)$.

Most of the uncertainty in the model output resulted from the parameterisation of frazil ice processes. The rate of nucleation and precipitation controlled the spatial extent over which frazil ice remained in suspension. These two processes had a greater effect on the total growth of frazil ice than the parameterisation of the growth of individual crystals. RMS tidal speed and the choice of the drag coefficient both altered the precipitation rate. These two parameters, along with the efficiency of secondary nucleation (the magnitude of $\eta_{\text{max}}$), were the input parameters that, when altered, could induce the largest variation in model output. Under standard conditions, most of the frazil ice mass kept in suspension was due to crystals with radii in the range 1–2 mm. Increases in the RMS speed or drag coefficient increased this radius range, while an increase to $\eta_{\text{max}}$ reduced it. Nevertheless, the modelled crystal sizes fit with size estimations from underwater video observations (Gough et al., 2012b).

The extended model was successful in reproducing the observations of the thickness of *in situ* supercooled layer made between 3 and 60 km from the ice shelf edge (Figure 6.3a). Further, the predicted growth rate enhancement provided by this supercooled water is in good agreement with the observed difference in thickness between the Intermediate and Far North sites, which are separated by 20 km (Section 6.2.2).

At distances greater than 60 km from the ice shelf edge, the model predicted that
interactions at the ice–ocean interface were the only cause of significant change to the level of supercooling. At distances of 100 km and 200 km from the ice shelf, the average supercooled depth dropped from an initial value of 65 m to $11 \pm 6$ m and $4 \pm 3$ m, respectively (Section 7.5).

8.4 Concluding Remarks

There exists a wealth of knowledge about McMurdo Sound from past studies in the area. As a result, the oceanography of the Sound and mechanisms of sea ice formation are relatively well understood. This study has added to that knowledge by presenting a picture of conditions very near the ice shelf edge. How one part of this picture evolved with increasing distance from the ice shelf edge was studied numerically with a simple one-dimensional model, but predictions were strongly reliant on frazil ice processes. The genesis and fate of these millimetre-sized objects are not well understood but their influence on the surface ocean layer and the sea ice structure highlights the need for their further study. For example, modellers should strive for a better treatment of frazil ice seeding, while accurate in situ observations of the size and shape of frazil crystals are needed to better understand their growth and subsequent role in the formation of platelet ice and the sub-ice platelet layer.
References


Assmann, K. M. (2004), The effect of McMurdo Sound topography on water mass exchange across the Ross Ice Shelf front, Annual report of the Füchser-Ronne Ice Shelf Program (FRISP), 15.


Daly, S. F. (1984), *Frazil ice dynamics*, CRREL Monograph 84-1.


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References


Appendix A

Decoupled Conservation Equations

For their use in MATLAB, the conservations equations given in Chapter 5 need to be solved for the derivatives of $D$, $U$, $T$, $S$ and $C_i(k)$ with respect to the along-plume coordinate $s$. In the solutions below, factors of either $U + U_a$, $D$, or both have been placed on the left sides of each equation for simplicity.

As described in Section 3.10, momentum conservation (Equation 5.24) is coupled with conservation of the mixture’s mass (Equation 5.21) to form the first system of equations. Ice mass conservation also belongs to this system because ice concentration is defined in terms of the mixture’s volume. Heat and salt conservation (Equations 5.25 and 5.26) are coupled with mass conservation of the water fraction (Equation 5.22) to form the second system of equations.

All solutions below were obtained using Mathematica. Where a size class dependence is not given, a summation across all classes is implied.

\[
(U + U_a)^2 \frac{\partial D}{\partial s} = -\Delta \rho g \sin(\vartheta) D \\
- C_d U_a \sqrt{U_a^2 + U_T^2} \\
+ C_d (U + U_a) \sqrt{(U + U_a)^2 + U_T^2} \\
+ 2(e' - d' + m' + p') (U + U_a) + d' U
\]  
(A.1)

\[
D (U + U_a) \frac{\partial U}{\partial s} = \Delta \rho g \sin(\vartheta) D \\
+ C_d U_a \sqrt{U_a^2 + U_T^2} \\
- C_d (U + U_a) \sqrt{(U + U_a)^2 + U_T^2} \\
- (e' + m' + p') (U + U_a) + d' U_a
\]  
(A.2)
Decoupled Conservation Equations

\[
D(U + U_a) \frac{\partial T}{\partial s} = \left( T_{sc} - \frac{L}{c_w} \right) f' \\
- \frac{m'L_i}{c_w} + \frac{K_i}{\rho_w c_w} \frac{dT}{dz} |_b \\
+ e'(T_a - T) + m'(T_b - T) \tag{A.3}
\]

\[
D(U + U_a) \frac{\partial S}{\partial s} = e'(S_a - S) + m'(S_i - S) - f' S \tag{A.4}
\]

\[
D(U + U_a) \frac{\partial C_i(k)}{\partial s} = \frac{\rho_w}{\rho_i} (p'(k) - f'(k)) \\
- (e' - d' + m' + p') C_i(k) \tag{A.5}
\]