Debt Sustainability, Fiscal Space, and Growth

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Abstract

This paper investigates two overarching questions: how high can public debt rise before it becomes unsustainable; and, how do sustainable changes in public debt impact growth? We review fiscal performance using annual data from a panel of 23 advanced economies, over the period 1970-2007.

Firstly, as in Ghosh et al. (2013), we estimate both a static and dynamic fiscal reaction function by regressing primary balance on debt, debt squared, and debt cubed (among others). This shows that the primary balance exhibits 'fiscal fatigue'. Fiscal fatigue is where it gradually becomes harder and harder for a country to pay down rising debt, leading to an effective 'debt limit', beyond which a country cannot borrow because debt dynamics become unsustainable.

Using the fiscal reaction function results, we empirically determine debt limits for our 23 advanced economies, defined as the maximum sustainable debt level a country attain. Next, using these debt limits, we create a fiscal space data series for each country. Fiscal space is simply the difference between a country's historical debt limit and its actual debt in any period.

Finally, we investigate the relationship between fiscal space and growth. Using modern empirical methods, we find evidence of an inverse quadratic relationship between fiscal space and growth. This implies fiscal space fits into conventional debt-growth theory, whereby an economy with high fiscal space (low debt) can increase growth by sustainably going into debt and increasing government spending. Alternatively, a country with limited fiscal space (high debt), would be better off reducing debt and increasing their fiscal space. Fiscal space provides an alternative angle to assess debt and growth dynamics, by introducing the concept of sustainability. Our results hold policy implications for government decision-makers.

Keywords:
Public debt, fiscal reaction function, fiscal space, debt sustainability, debt limit, growth, common correlated effects.

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1. Introduction

"A national debt, if it is not excessive, will be to us a national blessing." Alexander Hamilton, Letter to James Duane, September 3, 1781.

Economic conventional wisdom and theories have failed to withstand the Global Financial Crisis (GFC), and more recent European Sovereign Debt crisis. The resulting effect of these two events on markets, jobs, and economic growth has been considerable but varied across countries. Interest has been rekindled in public finances, specifically across advanced economies and the role of government, with regards to government debt and sustainability.

Fiscal policy operations in advanced economies have varied greatly in recent years. There is a stark contrast; many economies focus on austerity measures while potentially needing stimulus packages. In the United States, current federal spending on goods and services have fallen to historic lows, government consumption and investment as a percentage of Gross Domestic Product (GDP) dropping to 17.6% and 16.3%, respectively, in the fourth quarter of 2015 (CIA, 2016). Fiscal policy-makers argue they are implementing austerity measures in an attempt to lower the debt load.

Despite advanced economies' central banks arguing for more fiscal stimulus (RBNZ, 2015), governments are reluctant to undertake policy changes. "There's no getting around the fact that monetary policy in the United States and many other advanced countries has been under a substantial burden and has not gotten a lot of help from fiscal policy" (Yellen, 2016). Both the UK (Treasury, 2016a) and New Zealand (Treasury, 2016b) are aiming for fiscal surpluses in their budget within the next five years. Many EU members are also undertaking austerity measures in the wake of Greece's debt troubles. Japan, on the other hand, has recently approved a record ($96.72 trillion yen) state budget (Ministry of Finance, 2016) which could pave the way for a full debate on additional stimulus spending to spur on an otherwise stagnant economy. Canada has also announced a fiscal stimulus to revive its economy in the wake of lower oil and commodity prices (Government of Canada, 2016).

Christine Lagarde, the IMF's Managing Director, has warned that the post-GFC recovery remains too slow, and too fragile, with the risk that persistent low growth may have damaging effects on the social and political fabric of many countries (Lagarde, 2016). For example, growth is expected to remain at a modest 2 percent over 2016/17 for advanced economies (IMF, 2016). To combat low growth, accommodative monetary policy may have reached its limits.[1] Fiscal policy has the ability to focus explicitly on areas of concern, for example New Zealand's 2016 Budget has an emphasis on 'innovation', 'public

[1] See Wheeler (2016) for a discussion on challenges faced by monetary policy in turbulent times.

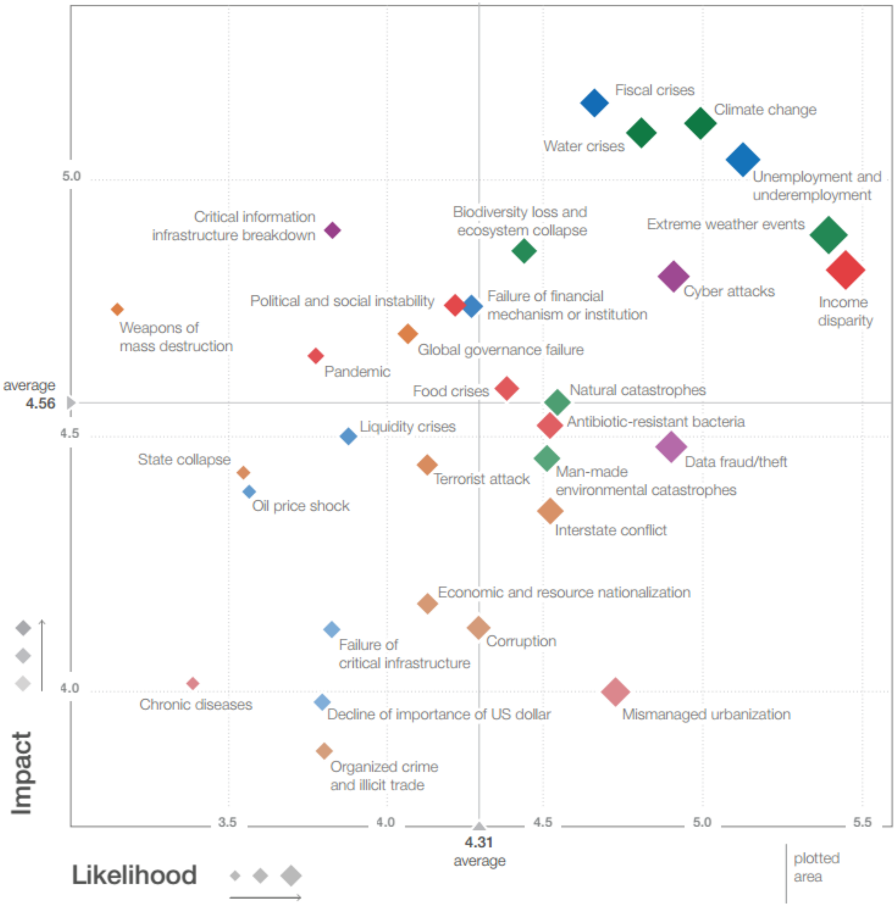
infrastructure', 'social investment', and 'health'. While fiscal policy lags[2] can also occur, spending can be immediately pumped into the economy.

Given the current precarious state of advanced economies and bleak outlook for future growth, it is arguable that fiscal support may be the best approach. But it seems like the focus for stimulating growth is increasingly centred on the direct costs that debt poses, rather than the fiscal stimulus provided. Governments tout the view that a budget surplus is a positive (English, 2016). This suggests that, from a theoretical standpoint, a government's fiscal behaviour and debt need to be subject to re-evaluation.

Government debt is seen in some economic circles as being unsustainably high in some countries; for example, Laurence Kotlikoff's (2015) testament before the US Congressional Budget Office (CBO) argues that the federal fiscal situation is much worse than the CBO estimated (Hall, 2015). Nonetheless, debt continues to rise. Rather than deleveraging their debt, major economies are currently running higher debt-to-GDP ratios than in the build-up to the great recession (circa 1929-1939), increasing global debt by $57 trillion between Q4 2007 to Q2 2012, contributing to a 17-percentage point increase in the debt-to-GDP ratio (Dobbs et al., 2016). One of the implications of this is that in many countries government bonds are seen as a risky asset, and the threat of sovereign default within one of two generations may not be negligible. The 2011 credit downgrade of the US government indicates this also. The consensus on the gravity of this problem can be documented by the 2014 Global Risks report featuring assessment by more than 700 experts and policymakers at the 2014 World Economic Forum. The report sees 'fiscal crisis' as the number 1 risk in terms of perceived financial losses, ahead of other geopolitical, societal, technological, and environmental risks (WEF, 2014). Furthermore, these fiscal crises are perceived as likely to occur in the next 10 years (see Figure 1.0).

[2] The 'inside lag' refers to getting a policy activated, while the 'outside lag' is the delayed impact of policy.

Figure 1.0: The 2014 Global Risks Landscape



Source: WEF (2014). Figure 1.1.

In modern terms, the sustainability of public finances is challenged when the government debt-to-GDP ratio reaches an excessive level. When writing about the public debt problem faced by France, Keynes (1923) highlighted the need for the French government to conduct a sustainable fiscal policy in order to satisfy its budget constraint. Keynes (1923, p.54) stated that the absence of sustainability would be evident when "the State's contractual liabilities (…) have reached an excessive proportion of the national income".

Sustainability becomes a problem when government revenues are not enough to keep on financing the costs associated with any new issuance of public debt or, again in Keynes words, when "it has become clear that the claims of the bond-holders are more than the tax payers can support" (Keynes, 1923, p. 55). At that point the government will have to take measures that restore the sustainability of fiscal policy, requiring that the State "must come in due course to some compromise between increasing taxation, and diminishing expenditure, and reducing what (…) [it] owe[s]" (Keynes, 1923, p. 59). Deleveraging debt is both difficult and rare, but a number of key issues are likely to arise when a country experiences high levels of debt. These include adverse effects on capital accumulation, productivity, and economic growth. If economic growth is indeed negatively affected by excessive debt, new approaches are sorely needed to mitigate the risks, including monitoring, management, and debt reduction mechanisms.

"Debt undoubtedly remains an essential tool for financing economic growth. But how it is created, used, monitored, and (when necessary) discharged still needs improvement" (Dobbs et al., 2015, p. 1). Conventional wisdom dictates that public debt stimulates demand and output in the short-run, but crowds out capital and reduces output in the long run (Elmendorf and Mankiw, 1998). There are possible threshold effects, such as the famous 90% debt-GDP ratio estimates by Reinhart and Rogoff (2010), non-linearities, level effects, and even growth effects of debt on GDP.

Nonetheless, the theoretical foundations for such debt and growth relationships are in need of further development. As with most of the growth literature, theory has a long way to go to catch up with empirics. Recent research (Panizza & Presbitero, 2013) demonstrates that the theoretical foundations for negative or non-linear relationships between debt and growth are rather tenuous. "There is no simple relationship between debt and growth [. . .] There are many factors that matter for a country's growth and debt performance. Moreover, there is no single threshold for debt ratios that can delineate the 'bad' from the 'good' " (IMF, 2012, Ch. 3, p. 9).

Differentiating the 'good' from the 'bad', or 'sustainable' from 'unsustainable' fiscal policies should therefore be of great interest to advanced economies. Much research today tries to characterize the relationship between debt-GDP levels and growth, without asking a key question that should concern most countries and governments: what is a sustainable level of debt? Given the current state of the world economy, this is a critical question to consider. My thesis will attempt to build towards answering this question by extending upon the current debt literature in both a theoretical and empirical context. We look to further characterize the sustainability of advanced economies' public debt levels, and also bridge the gap between the debt sustainability and growth literatures.

Theoretically, this thesis looks to investigate the relationships between debt, sustainability, and growth, incorporating Ghosh et al.'s (2013) idea of fiscal space. Our goal is to build upon Ghosh et al.'s (2013) techniques in creating a sustainable debt or 'fiscal space' measure for a panel of 23 advanced economies.[3] Empirically, we will then incorporate this sustainable

[3] 'Fiscal space' is a term prominent in current literature on debt sustainability, particularly within international institutions such as the IMF and World Bank. Fiscal Space is the flexibility of a government in its spending choices, and, more generally, to the financial well-being of a government. Heller (2005, pg.1) defined it as "room in a government's budget that allows it to provide resources for a desired purpose without jeopardizing the sustainability of its financial position or the stability of the economy." i.e. the difference between an estimated upper limit of public debt (beyond which action would have to be taken to avoid default) and actual public debt, expressed as a percentage of GDP or equivalently as the difference between the debt-limit-to-GDP percentage and the actual-debt-to-GDP percentage.

debt level into a growth equation, in order to explore the relationship between fiscal space and growth. Currently, the debt and growth literature focuses on the debt levels themselves, without taking into account what makes debt sustainable, which is where the added dynamics of Fiscal Space are promising. This paper proposes a methodology that improves our understanding of the risks surrounding debt dynamics.

The following thesis is split into a number of sections. Section 2 provides an overview of relevant literature and theory relating to governments, debt, and fiscal sustainability. Section 3 posits our Fiscal Space model theoretically, based predominately on the model presented in Ghosh et al. (2013). Section 4 presents an overview of the data used to test the theory, as well as summary statistics. Section 5 outlines the empirical strategy used to analyse the data. Section 6 reports results and interpretations. Section 7 discusses our results in relation to theory and provides policy implications.

2. Fiscal Overview

The following is split into a number of sub-sections focusing on areas of literature related to governments' fiscal sustainability and public debt. Section 2.1 expands on the notion of fiscal policy. Section 2.2 covers conventional wisdom and theory on public debt. Section 2.3 covers the government's intertemporal budget constraint (IBC). Section 2.4 builds upon the IBC and defines the ideas of sustainability. Section 2.5 extends upon the IBC and sustainability, introducing a recent method featured in the literature for assessing debt sustainability and fiscal space, used throughout this paper. Section 2.6 provides a brief overview of the prevailing debt-growth research in recent years. Section 2.7 covers current issues in the literature.

2.1. Fiscal Policy

Fiscal policy is the government's actions in collecting and spending private assets which dynamically affect aspects of the economy such as capital formation, economic growth, and intergenerational equity (Auerbach & Kotlikoff, 1987). Keynesian theory dictates that fiscal policy influences macroeconomic variables in an economy. Governments decide between adjusting spending and changing tax rates. Historically, initial views on fiscal policy were somewhat laissez-faire, i.e. less government involvement in the free market allows for better business operation and capitalism, leading to a society being better off. However, views changed post WWII to support government taking a more proactive role to regulate unemployment, business cycles, inflation, and the cost of money.

Governments therefore undertake a balancing act between stimulating the economy and imposing taxes. To stimulate the economy without imposing undue hardship on the current generation of citizens a government can issue debt via bonds, allocating the cost of public spending to future generations that derive some benefit from the spending. Bonds enable money to be raised to stimulate the economy by public spending, while simultaneously avoiding raising current taxation levels that would otherwise be needed. Historically, issuing debt has led to an economic boost in various countries; however the government is obliged to pay interest to its creditors and eventually pay back the principal in its entirety. The amount of debt issued by a government to cover fiscal spending is termed public debt, i.e. the amount of money a government owes to its creditors.

Public debt usually exists to cover shortfalls or budget deficits where a government's spending exceeds its revenue.[4] Public debt is considered a relatively safe investment and less risky than the stock market, for example. However, a number of factors can cause public debt to become riskier, such as the stability of the issuing government, exchange

[4] Public debt is classified sovereign debt if issued in a foreign currency.

rates, and valuation of returns from public spending. If a government were to default on its interest payments, or debt repayment on maturity, there would likely be serious economic outcomes.

The policy implications of paying down debt mean that government spending over time must be financed by a reduction in the consumption or leisure of at least one generation. The government's fiscal policy could let debt grow for long periods at rates faster than the growth of the economy. However, this cannot continue indefinitely. There could be unfortunate consequences under such a policy, where debt would eventually exceed national wealth, reducing the lifetime consumption of taxpayers, their savings, and having a negative effect on the capital stock (Diamond, 1965). Despite this, there are no models or real world constraints that require a government's budget to be balanced in a given year, with differences between spending and taxes simply resulting in an equal increase in the amount of outstanding government debt.

Therefore, dynamic perspectives are crucial when analysing fiscal policy. This is due to the frequency at which policy is adjusted, and because the effects are distributed over time. Static analysis can ignore this, although provides interesting insight to how things are related at a certain point in time. But dynamics offers intuition on the processes of how a situation came to be, and how it is likely to develop in the future (Plasmans et al., 2006). This perspective is also crucial in evaluating short-run benefits (e.g. tax cuts) against long-run losses (e.g. crowding out). For example, Feldstein (1974) suggests that fiscal policy is largely responsible for discouraging savings, a concern centred on the crowding out caused by government debt policies.

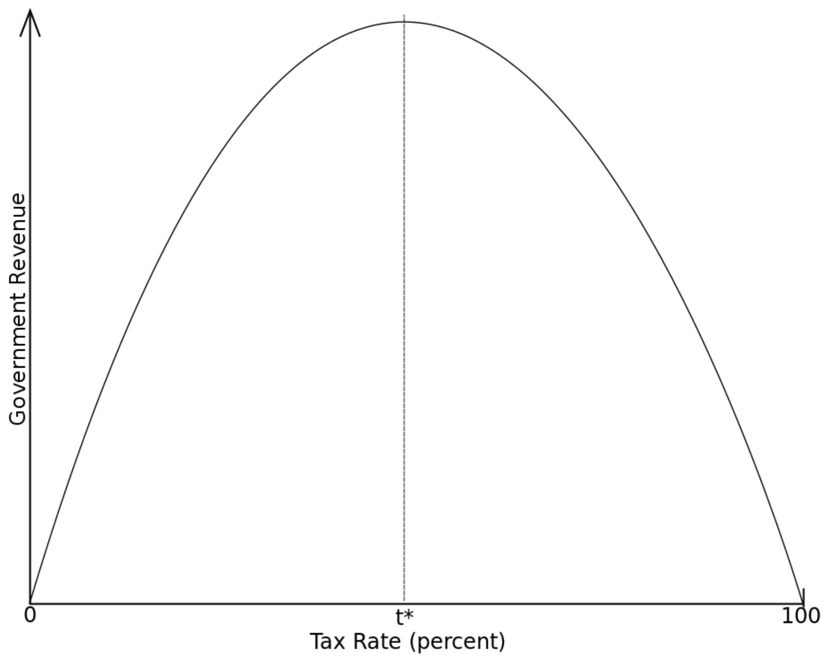
2.2. Conventional View of Public Debt

The short-run 'conventional view of public debt' is that output is determined by demand and fiscal deficits (increased spending via higher public debt) have a positive effect on disposable income, aggregate demand, and overall output (Elmendorf and Mankiw, 1999). This short-run positive effect of higher debt (or budget deficits) is likely to increase if the output gap is large or the economy is well below capacity. However, the long-run effects are more complicated.

Early empirical work (Modigliani, 1961; Diamond, 1965; Saint-Paul, 1992), finds evidence supporting this conclusion. The results suggested that a public debt increase always contributes to economic growth. Keynes (1937) argued that because of the direct link to aggregate demand, governments should spend more money in a recession in order to stimulate a recovery. Therefore, in the short-run, public debt (to fund increased fiscal stimulus) is seen as a good way for countries to boost aggregate demand. However, a government must be careful not to take on too much debt, despite the explicit short-run benefits. Barro (1974) also poses Ricardian Equivalence: an economic hypothesis asserting that consumers are forward looking, and therefore will factor in a government's budget constraint into their decision making i.e. it does not matter for the economy whether government expenditures are financed by current taxation or by issuing government bonds. If Ricardian Equivalence does not hold, an increase in private savings will not compensate for a higher government budget deficit and subsequent decrease in public savings. The extent that government spending crowds out private investment depends on how much government deficits affect interest rates. If there is excess capacity in the economy, the increased deficit and higher demand can 'crowd in' private spending.

In further examining taxes, the Laffer Curve (Laffer, 2004) is an often used. In Figure 2.0, Laffer (2004) demonstrates the compromise between total tax revenue collected by the government and actual tax rates, recognising the positive impact that lower tax rates have on incentivising work, output, and employment. Governments can cause dramatic supply-side stimulus by cutting taxes from high rates, where the economic effect can cause an actual increase in tax revenues.

Figure 2.0: Laffer Curve



Source: Laffer, A. B. (2004). Figure 1.

Most of the literature implies that debt has a negative impact on long-run growth.[5] This negative effect is generally through 'crowding out'; however the effect is quantitatively small. If high public debt increases uncertainty, or introduces expectation of future confiscation or questions of policy credibility, the negative effect could be much larger

[5] See Panizza (2013) for a summary of relevant literature.

(Cochrane, 2011). Here, higher debt could have a negative effect even in the short-run. In contrast to this theory, hysteresis can lead to a situation in which expansionary fiscal policies have positive effects on long-run growth.[6] This is essentially the debate between crowding-out and crowding-in: whether government spending complements private consumption and investment, or is a substitute for it.[7] The crowding-in effect is based on Keynesian theory, stating that in times of economic slack, deficit spending can presumably quicken the pace of economic activity. This would lead to an economic expansion, with businesses adding to their capacity to meet a greater consumer demand. In this scenario, government debt induces investment, rather than decrease it as crowding-out would predict.

However, there is yet to be consensus among economists (Alesina et al., 2015). While some suggest that now is precisely the time to apply the lessons learned during the Great Depression and that policymakers should implement expansionary fiscal policies (Krugman, 2011; Berg and Ostry, 2011; DeLong and Summers, 2012), others argue that since the high level of public sector debt has a negative effect on economic growth, fiscal consolidation is fundamental to restoring confidence and improving expectations about the future evolution of the economy (Alesina & Ardagna, 2010) and is essential to restore confidence in order to improve market expectations about the future evolution of the economy and therefore its rate of growth (Cochrane, 2011; Teles &Mussolini 2014).

For example, DeLong & Summers (2012) argue that expansionary fiscal policy is likely to be self-financing in a low-interest rate environment i.e. a new version of Say's (1803) Law.[8] There is also evidence that recessions have a permanent effect on the level of GDP (Cerra & Saxena, 2008). A key part of DeLong & Summers (2012) argument is the ability of government spending to reverse the hysteresis effects. Despite the author's dynamic view, Ramey (2012) critiques the use of a simplified model which lacks a general equilibrium and only includes low interest rates. A simple empirical investigation questions whether self-financing is indeed plausible, indicating the idea may not work in real life (Ramey, 2012).

With no consensus in the literature regarding the appropriateness or otherwise of fiscal policy, and therefore debt accumulation, on the desired outcomes of the economy, there is a clear need to re-examine questions of fiscal and debt sustainability.

[6] Hysteresis refers to a single disturbance affecting the future course of the economy, often in regards to high unemployment leading to a higher natural rate of unemployment.

[7] For example, infrastructure is not only a component of government spending, but also shows up in a firm's production function.

[8] Jean- Baptiste Say introduced the idea that aggregate production necessarily creates an equal quantity of aggregate demand i.e. supply creates its own demand. Although this law has generally been used by laissez-faire proponents, Delong & Summers (2012) have used it to argue reasons for government: "In a depressed economy, government spending creates its own financing".

2.3. Intertemporal Budget Constraint

The starting point for analyzing a government's fiscal policy is to link the deficit to its revenues, spending, and debt. In order to examine the sustainability at a basic theoretical level, the appropriate framework is a government's Intertemporal Budget Constraint (IBC). The IBC states that the initial debt level should be equal to the present value of future surpluses, i.e. a government's current and planned debt must be backed by expected future cash flows.

Fiscal sustainability centres around two key questions: Which fiscal policies are sustainable? And, what can we determine about the sustainability of particular policies encountered in practice? The majority of the literature has focused on the empirical implications of a simple ad hoc definition of sustainability: *fiscal policy is on a trajectory such that the expected present value of future primary surpluses equals the initial debt* (Bohn, 2005). Following the definitions in Bohn (2004), the path of public debt implied by an arbitrary sequence of primary surpluses and interest charges can be computed:

$$d_{t+n} = \sum_{j=0}^{n}\left(\prod_{k=0}^{n}(1+r_{t+k})\right).d_{t-1} - \sum_{j=0}^{n}\left(\prod_{k=j+1}^{n}(1+r_{t+k})\right).s_{t+j} \qquad (1)$$

where d_t is real public debt in year t, r_t denotes the appropriate version of the 'return' on debt, i.e. real interest rate, and s_t is the corresponding primary surplus. The conditions for ad-hoc sustainability are obtained in three steps and are valid in nominal, real, and GDP-ratio terms.

Firstly, a fixed value r replaces the returns, and conditional expectations are taken,

$$E_t[d_{t+n}] = (1+r)^n.d_t^* - \sum_{j=0}^{n}(1+r)^{n-j}.E_t[s_{t+j}] \qquad (2)$$

where $d_t^* = (1+r_t).d_{t-1}$ denotes debt at the start of period t, and $E_t[.]$ is the conditional expectation. Next, divide by $(1+r)^n$and rearrange to obtain,

$$d_t^* = \sum_{j=0}^{n}\frac{1}{(1+r)^j}E_t[s_{t+j}] + \frac{1}{(1+r)^n}E_t[d_{t+n}] \qquad (3)$$

Finally, it is assumed that the discounted sum converges, and the limit as n approaches infinity is taken,

$$d_t^* = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_t\left[s_{t+j}\right] + \lim_{n \to \infty} \frac{1}{(1+r)^n} E_t[d_{t+n}] \tag{4}$$

which establishes initial debt is equal to the expected present value of future primary surpluses, if and only if discounted future debt converges to zero.

Therefore,

$$d_t^* = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_t\left[s_{t+j}\right] \tag{5}$$

is the ad-hoc IBC, while,

$$\lim_{n \to \infty} \frac{1}{(1+r)^n} E_t[d_{t+n}] = 0 \tag{6}$$

is the Transversality Condition (TC).[9]

The above equations demonstrate the arbitrariness of the discount rate r, as well as the absence of an economic argument as to why potential buyers of government bonds should care about IBC's or TC's (equations (5) and (6), respectively) (Bohn, 2004). The IBC only contains economic meaning through the assertion that the TC is an accurate description of bond-holders' behaviour, for a given r-value. For individuals holding government bonds, transversality is a necessary condition and justifies the IBC as a true constraint on government policy.

The IBC was initially used by Hamilton and Flavin (1986) by applying a Dickey-Fuller unit root test to the discounted debt and surpluses series for the US for the period 1960-1984. As the mean and variance of stationary series remain constant, stationarity guarantees that debt is stable over time. Hamilton and Flavin (1986) show that both the discounted debt and surpluses series were stationary in levels implying that the series are mean reverting. Their conclusion was that the U.S. government was executing public policy subject to the IBC. However, Haug (1991) determines that the unit root tests used by Hamilton and Flavin (1986) were mis-specified by not including enough lags.

Trehan and Walsh (1991) focus on the cointegration condition linking debt to primary deficits. This condition requires an error correction mechanism that can be interpreted as a

[9] Transversality Conditions are optimality conditions often used along with Euler equations to characterise the optimal paths of dynamic economic models.

fiscal reaction function, providing a bridge towards a government's fiscal behavior.[10] Bohn (2007) provides an extension of sufficient conditions for sustainability via first-order and second-order integration and cointegration results. If the relevant debt variable is stationary after any finite number of differencing operations, the IBC is satisfied. Also, the IBC can be satisfied if revenues and with-interest spending are difference-stationary of arbitrary order, without the cointegration requirement. Bohn (2007) argues that debt will be sustainable if the primary balance reacts positively to lagged debt levels, regardless of the size of the response.[11] This response ensures that debt grows slower than the rate it is discounted. However, this is a weak sustainability criterion as it does not rule out an infinitely increasing debt-to-GDP ratio. In reality, this infinite increase seems highly implausible.

2.4. Sustainability

Debt sustainability is an important concept for advanced economies. Formally, sustainable public debt is the level of debt that allows a country to meet its current and future debt service obligations in full, without recourse to further debt relief or rescheduling, avoiding accumulation of arrears, while allowing an acceptable level of economic growth (Elkhoury, 2009; DFI, 2016). The World Bank and IMF (2001, pg. 4) hold that "a country can be said to achieve external debt sustainability if it can meet its current and future external debt service obligations in full, without recourse to debt rescheduling or the accumulation of arrears and without compromising growth". According to these two institutions, "bringing the net present value (NPV) of external public debt down to around 150 percent of the value of a country's exports or 250 percent of a country's overall revenues" would help in eliminating this "critical barrier to longer-term debt sustainability" (World Bank & IMF, 2001, pg. 4).

Debt sustainability analysis is generally conducted in the context of medium-term scenarios (IMF, 2000). Advanced economies rely predominantly on official financing where sustainability of debt is largely de-linked from the sentiments of the market, as embodied by the spread on market interest rates. Debt sustainability is a particularly blurred concept in countries to the extent that creditors base funding flows on net transfers (Powell, 2003).[12] For example, debt could suddenly become unsustainable depending on the willingness of creditors, or it could be serviced for long periods.

Debt sustainability problems can be exacerbated by many heterogeneous country characteristics including: weak public institutions and poor governance; returns on

[10] Fiscal Reaction Functions captures the reaction of fiscal policy (in terms of the government's primary balance) to public debt and macroeconomic conditions, and are regarded as a useful element in the toolkit for fiscal sustainability analysis.

[11] The primary balance refers to a government's net borrowing or lending, excluding interest payments on consolidated liabilities (OECD, 2003).

[12] Creditors provide more gross transfers to countries with higher debt servicing payments.

investment and ability to capture benefits; volatility of production and exports; and vulnerability to exogenous shocks (Kinda et al., 2016). Once the level of debt becomes unsustainably high, current investment is dissuaded and leaves little incentive for new investment. This can 'trap' countries in a downwards spiral resulting in a greater proportion of cash flow goes to servicing the debt, creating operating deficits that can only be filled by incremental debt, further adding to the debt burden. At this tipping point, the 'debt limit', dynamics become explosive and the only way out is either through forgiveness (e.g. a 'haircut' for Greece), default, or by substituting public debt for private debt (bailouts).

The notion of sustainability is not as straightforward as it may appear. Conceptually, sustainability implies judgement as to acceptable strategies for the government to satisfy its intertemporal budget constraint (Mendoza and Oviedo, 2004). Solvency is only a necessary condition for sustainability, and defining sufficient conditions involves judgement. [13] However, solvency is difficult to assess due to its forward-looking nature. There can be considerable uncertainty surrounding the primary surplus a government can generate in the future, as well as trying to accurately predict future inflation, interest rates, and productivity growth.

A common theme in the literature is to investigate past fiscal data to see if government debt follows a stationary process or to establish if there is cointegration between government revenues and government expenditures. When examining whether a fiscal policy is sustainable, the basic economic answer is that "an agent's ability to borrow is constrained by other agents' willingness to lend" (Bohn, 2008, pg.12). This hints at general equilibrium concepts, with different assumptions about lenders leading to multiple sustainable policies. The standard assumption in the literature is that lenders are infinitely-lived optimising agents (satisfying the TC), and that financial markets are complete (common pricing). Under this assumption, if the sustainability criterion such as the IBC and the government's transversality condition are violated then rational lenders would no longer buy such debt, i.e. some initial debt will not be repaid. Under standard assumptions, one cannot rule out debt policies for which there is no proper limit, or policies that have large surpluses and a negative limit. Bohn (2008) makes the further simplification that debt is non-negative, and that a limit exists.

Empirical evidence (Bohn, 1998; Bohn, 2005; Mendoza & Ostry, 2008) indicates governments tend to act responsibly with rising debt, raising the primary balance. However, this reaction has its limit. As the primary balance rises, the difficulty of raising taxes or cutting expenditures is exacerbated. Eventually, the increase in primary balance will not keep pace with rising interest payments as debt increases, eventually coming to a point where (barring extraordinary fiscal effort) debt dynamics become explosive and the

[13] Solvency is defined as the ability to meet one's financial obligations on time.

government is unable to meet its obligations. The point is the debt limit, with fiscal space being the distance between current or projected debt levels and the debt limit.

A simple proposition for ensuring sustainability, linking primary surpluses to debt, is a linear one stipulated by Bohn (1991, 1998), where the primary balance (s_t) is an increasing linear function of the initial debt-to-GDP ratio (d_{t-1} or d_t^* in the long-run):

$$s_t = \rho . d_{t-1} + \mu_t \tag{7}$$

$$s_t = \rho . d_t^* + \mu_t \tag{7}$$

for all t, where ρ>0 is a constant and μ is composed of other determinants. This suggests that sustainability can be assessed by estimating and testing a policy rule, or fiscal reaction function. Bohn (2008) focuses on a linear model, but provides the basic intuition for expanding to a more general model. The assumptions surrounding the primary balance reaction function can be weakened, for example, by allowing for a non-linear feedback rule or time-varying coefficients.

Sustainability can also only be evaluated by extrapolating forward into the indefinite future. Kraay & Nehru (2006, p.342) investigate debt sustainability and argue that "a common single debt sustainability threshold is not appropriate because it fails to recognise the role of institutions and policies that matter for the likelihood of debt distress". The authors find that there are trade-offs between debt indicators, policies, and shocks for a given probability of debt distress.[14] For example, under the joint IMF-World Bank HIPC Initiative, debt relief is calibrated to ensure that countries emerge from the process with debt-to-exports at 150 percent (present value), irrespective of (and failing to realise the importance of) other country characteristics. However, from a practical perspective, the targeted level of sustainable debt should vary across countries, along with the quality of institutions and policies.

McFadden et al. (1985) were among the first to investigate debt sustainability empirically, in the context of debt servicing difficulties. The authors construct an indicator of debt servicing difficulties, including factors such as arrears, rescheduling, and IMF support. Their findings indicate that debt sustainability is significantly predicted by the debt burden, level of per-capita income, real GDP growth, and liquidity measures (e.g. non-gold reserves). Contradicting this, the real exchange rate is found not to be significant, while country effects matter. Early literature has investigated further variables; Auten et al. (1984) focus mainly on financial variables, Berg & Sachs (1988) emphasise 'deep' structural factors such as

[14] Debt distress is the point at which debt becomes so high that default occurs. Fiscal space relates as it is the gap between this point of debt distress and the current debt level.

income inequality and trade openness, while Lloyd-Ellis et al. (1990) model the probability of debt rescheduling.

In earlier literature the standard measure of debt is usually the debt-to-export ratio, while indicators of risk or unsustainability focus on interest rates, such as the premium over benchmark interest. Empirically investigating this relationship, Cohen (1997) and Husain & Underwood (1991) find a debt-to-export threshold range of approximately 200-250 percent, beyond which the likelihood of debt default climbs dramatically. Aylward and Thorne (1998) empirically emphasise the importance of a country's repayment history and IMF-specific financial variables. Detragiache & Spilimbergo (2001) also study the importance of liquidity factors.

Unit root and cointegration tests are commonly employed to examine if time series are consistent with an intertemporal budget constraint (IBC). In the fiscal policy literature, these methods are widely used to examine the government budget constraint—the sustainability of public debts and deficits. In the international literature, the same tools are used to examine the sustainability of external debts and current account deficits. Standard empirical strategies focus on testing if the debt series is difference-stationary or if revenues and spending are suitably cointegrated. Rejections are interpreted as evidence against sustainability, usually citing Trehan and Walsh (1988, 1991), Quintos (1995), or related papers for the alleged necessity of such conditions.[15]

Bohn (2008) provides a weak sustainability criterion for debt, which does not exclude the idea of an ever-increasing debt-GDP ratio. The idea of 'fiscal fatigue' is introduced by Ghosh et al. (2013), whereby eventually the primary balance cannot keep up with interest payments as debt rises. Theoretically, there will be a debt level above which debt dynamics will be explosive and governments will necessarily default. This debt level is determined in a theoretical framework where the probability of default, the interest rate, and the debt limit are solved for simultaneously. The idea of debt sustainability is the key ingredient behind fiscal space. Fiscal space provides an alternative, fundamentals-based measure of the risk of sovereign debt default using econometric techniques to identify how nations' fiscal policies have responded historically to increases in their public debt.

2.5. Fiscal Space

Fiscal space can be broadly defined as a country's ability to engage in countercyclical policy in the case of a downturn, resulting in rising debt ratios or some depletion of existing financial assets, without jeopardising debt sustainability. Fiscal space also refers to "the

[15] Afonso (2005) provides a review of empirical work documenting widespread use of unit root and cointegration tests.

ability of government to make budgetary resources available for desired purposes" (Williams and Hay, 2005, pg. 2). With regard to developing countries, fiscal space is usually referred to as the ability to undertake "growth-enhancing investment in physical and human capital that a government can finance with borrowed funds without prejudicing the long-run sustainability of its fiscal position" (Schick, 2009, pg. 2). For operational purposes, the IMF has used an approach that focuses on the gap between the actual primary balance and the debt-stabilising primary balance (Escolano, 2010).

We define fiscal space in line with the methodology by Ghosh et al. (2013) as the difference between a debt limit (beyond which, without extraordinary measures, debt would be unsustainable) and the actual debt, with the debt limit being calculated by the reaction function of the primary balance to past debt levels. Thereby, we account for the fact that fiscal space can vary over time, for example due to interest rate changes or institutional settings such as the introduction of fiscal rules.[16]

Different authors have emphasised different aspects of the fiscal space concept by focusing on specific sectors such as health or public infrastructure (Hulbert & Vammalle, 2014) or on the broader issue of debt sustainability. Ostry et al. (2010) show the existence of a nonlinear relationship between fiscal policy and debt, such that the marginal response of the primary balance to debt is significantly weaker at high levels of debt than at more moderate levels. This finding allows them to derive a public debt limit as the debt level above which the primary balance does not keep pace with the higher effective interest payments. However, most of the attempts to derive a limit for the debt-GDP ratio have focused, so far, on advanced economies.

But what constitutes a prudent debt level is very difficult to pin down precisely in practice (Ostry et al., 2015). Saxegaard (2014) indicates this is because the prudent debt level depends on many country-specific circumstances as well as factors outside its control: debt stock, gross financing requirement, composition of debt, debt path, drivers of new borrowing, credibility of fiscal policy, long-term fiscal pressures, and risk appetite of market participants. Nonetheless, policymakers need some indicative ceiling, in particular, in situations where a rapid build-up of debt could trigger adverse market reactions.

2.6. Global Growth

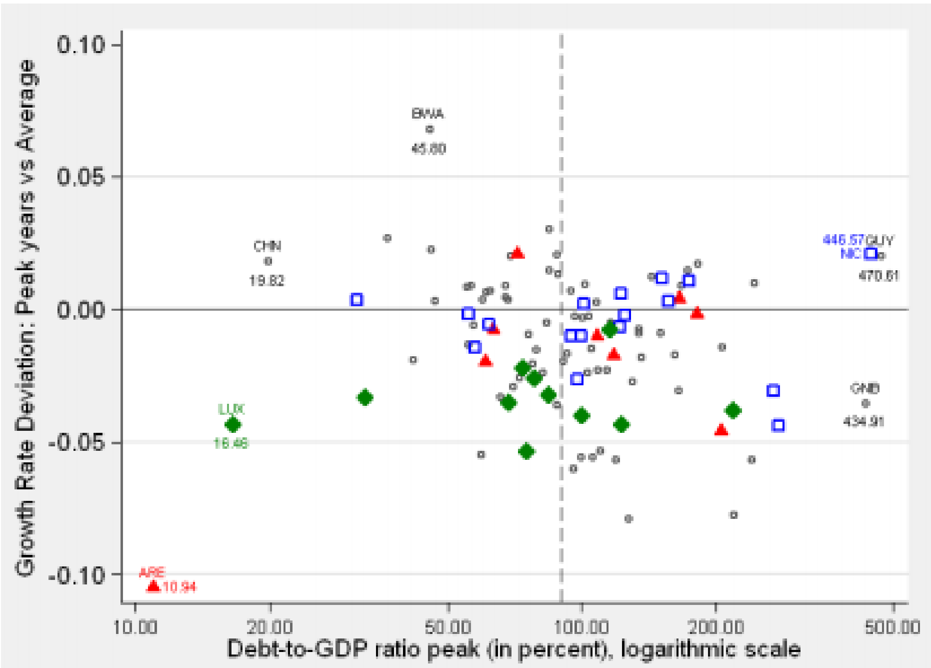
The debt-growth nexus has been increasingly motivated by a growing series of empirical papers. Reinhart and Rogoff (2010) claim that there is a particular threshold effect whereby countries that have debt-to-GDP over 90 percent experience lower growth. These findings are challenged by Herdon et al. (2013), finding that proper weighting of the data and a

[16] Please see section 3 for detailed information on fiscal space.

coding error cause the thresholds to vanish. Kumar & Woo (2010, pg. 1) find "some evidence of nonlinearity with higher levels of initial debt having a proportionately larger negative effect on subsequent growth".[17] Checherita-Westphal and Rother (2012), Eberhardt & Presbitero (2013), and Reinhart et al. (2012); also discuss the choice of debt brackets used, changes in country coverage, data frequency, econometric specification, and reverse causality going from output to debt.[18]

Eberhardt & Presbitero (2013) make a number of observations regarding the crude relationship of debt and growth. Firstly, there is a negative correlation between the maximum debt-to-GDP level and relative growth performance; however this is statistically insignificant, and only based on a correlation (see Figure 3.0 below). Secondly, there is considerable heterogeneity across countries (as demonstrated in the graph below). Lastly, and pertaining to the current debate in literature/policy, a large number of countries experienced better growth performance in their peak debt years than at any other point.

Figure 3.0: Peak Debt-to-GDP Ratio and Relative Growth.[19]



Source: Eberhardt, M. M., & Presbitero, A. (2013). Figure 2.

[17] Kumar and Woo (2010) run a growth regression at five-year frequency from 1970 to 2007 and find that a 10 percentage point increase in the initial debt-to-GDP ratio is associated with a slowdown in annual real per capita GDP growth of 0.15 percentage points per year in AEs. They test for nonlinearities by introducing three interaction terms between initial debt and dummy variables for three ranges of initial debt, 30, 60, and 90, respectively.

[18] See also Panizza and Presbitero (2013) for a survey and additional references to the literature.

[19] As per Eberhardt & Presbitero (2013) countries are arranged by their maximum debt-to-GDP ratio highlighting three years in particular: 1985 (Triangles: 9/105 countries), 1994 (Squares: 17/105), and 2009 (Diamonds: 11/105). The y-axis we plot the deviation of countries' (i) average per capita growth rate in the five years around their peak debt year (i.e. peak debt occurs in year 3) from (ii) their average per capita growth rate over the entire time horizon 1972-2009 excluding the five 'peak debt years.'

There is also the issue of causality. High debt may be caused by slow growth, or be caused by an unknown third factor, such as war or financial crisis, that both increases debt and reduces growth. Since temporary recessions naturally lead to an immediate increase in the debt ratio, this concern is particularly relevant when considering the short-term correlation between growth and debt. To partially address this issue, Reinhart and Rogoff (2010) consider prolonged periods of high debt in their analysis. Their results suggest that growth tends to be considerably lower in periods of debt overhang.

The issue of causality is also raised investigating debt and growth, with instrumental variable techniques used to properly examine the relationship. Panizza and Presbitero (2012) reject the hypothesis that high debt causes lower growth. Their chosen instrumental variable captures valuation effects, via the interaction between exchange rate volatility and foreign currency, leading to a result of no effect of debt on growth. Causality is also brought into question when looking at thresholds. It is less likely that one would observe a distinct threshold in the debt and growth relationship if low growth causes high debt. Thus, if thresholds exists, they are more likely to be driven by a causal effect of debt on growth.

Kumar and Woo (2010) instrument debt with lags, studying the impact of high public debt on subsequent growth of real per capita GDP for a panel of 38 advanced and emerging market economies (1970–2007). They apply a variety of homogeneous estimation methodologies, such as pooled OLS, fixed effects (FE) panel regression, system generalized method of moments (GMM; to account for endogeneity of growth regressors), and consider a variety of possible covariates of debt and growth. They complement their analysis by a growth accounting framework, which allows for an exploration of the channels (factor accumulation versus factor productivity) through which public debt may influence growth. Checherita-Westphal and Rother (2012) deal with simultaneous determination of public debt and growth (by using external instruments). They restrict their sample to 12 euro area countries (1970-2008), instrumenting debt-to-GDP at each point in time with the average debt-to-GDP ratio of the other 11 countries in the sample during the same time period. Checherita-Westphal and Rother (2012) find a non-linear relationship between debt and growth with a threshold ranging between 90 and 100 percent of debt to GDP levels. They use fixed-effects, two-stage least-squares (2SLS) and GMM techniques for estimation and employ a quadratic functional form for the growth-debt regression equation. They also analyze the channels through which public debt is likely to affect economic growth.

Overall, the predictions of the theoretical literature on the long-run effects of public debt on output growth are ambiguous, predicting negative as well as a positive effect under certain conditions. Even if we rely on theoretical models that predict a negative relationship between output growth and debt, we still need to estimate the magnitude of such effects empirically. The empirical evidence on the relationship between debt and growth until recently focused on the role of external debt in developing countries, and so far there have been only a few studies that include evidence on the developed economies.

2.7. Literature Issues

The majority of studies focused on debt are quite rich in empirical results, proposing a number of robustness checks. Four features distinguish the analyses in the existing literature.

1) Country coverage and data used: studies either use external/total debt, with coverage varying between the Euro area, OECD economies, developing countries, or emerging and developed countries;
2) Non-linear and threshold modelling techniques for the hypothesised relationship: a range of linear and squared debt terms in the regression, spline regressions using preconceived thresholds, endogenous threshold regressions, etc.;
3) Time horizon of the results: short-run or long-run debt-growth relationships depending on static or dynamic empirical specifications, or the use of time-averaged or annual data;
4) The identification strategy: standard instrumental variables (IV) or 2SLS estimators, or GMM estimators (Arellano & Bond, 1991).

Also, within existing panel literature, there are a number of stringent assumptions to be questioned. Firstly, empirical analysis is carried out assuming that the correlation across countries does not matter in panel regressions due to cross sectional independence. Secondly, it is assumed that all countries follow the same equilibrium path regardless of their level of economic development, industrial structure, or institutional environment. Thirdly, all countries are assumed to be subject to the same debt threshold, which is econometrically implemented by use of exogenous/endogenous debt thresholds, or by adopting a polynomial specification for debt within a pooled empirical model, giving no insights regarding heterogeneity across countries, or within countries over time.

2.7.1. Homogeneity

There are a number of reasons to assume the debt-growth relationships are heterogeneous across countries. Vulnerability to public debt depends upon current debt levels, but also on debt composition and the speed that debt is acquired. Unfortunately, existing data for the analysis of debt and growth represent a mixture of information relating to general and central government debt, plus debt in different monetary denominations. All of this implies that comparability of the debt data across countries may be compromised (Panizza & Presbitero, 2013).

The effects of public debt also depend on the reason for debt accumulation and what the debt is used for: consumed, invested, and on what economic activity. Different stocks of debt may impinge differently on economic growth. In particular, one could argue that debt could hinder GDP growth when it becomes unsustainable, affecting interest rates and triggering a financial crisis, thus affecting the level of GDP. However, the capacity to tolerate high debt is dependent on country-specific characteristics, related to past crises and the macro and institutional framework (Reinhart et al., 2003). The argument of heterogeneity in the debt-growth relationship is a simple extension, which provides greater modelling flexibility and further allows for empirical testing of the validity of this assumption via formal testing of the equality of relevant coefficients.

Reinhart & Rogoff (2010, pg. 24) suggested that "debt thresholds are importantly country-specific", while recent papers emphasise the heterogeneity of the debt-growth nexus across countries (Kourtellos, Stengos & Tan, 2011). Haque, Pesaran & Sharma (1999) provide a detailed discussion of the consequences of neglected parameter heterogeneity in the context of static and dynamic cross-country savings regressions. They particularly indicate the potential for seeming nonlinear relations as a result of misspecification, concluding that "[t]he linearity hypothesis may be rejected not because of the existence of a genuine nonlinearity…, but due to slope heterogeneity" (Haque, Pesaran & Sharma, 1999, pg.11).

2.7.2. Estimation Methods

Many debt studies consider a variety of estimation methods (e.g. Kumar & Woo, 2010). Methods include: pooled OLS, robust regressions, between estimators (BE), fixed effects (FE) panel regressions, and system GMM (SGMM) dynamic panel regressions. However each method involves a trade-off. Estimators that may seem attractive to tackle a specific econometric problem can lead to different types of bias (Reed, 2011). For example, when an omitted variable bias co-exists with measurement errors that are likely in the cross-country data, dealing with the omitted variable bias can exacerbate the measurement errors.

The nature of interactions and dependencies that generally exist over time and across individual units in the panel is complex. For instance, observations on firms, industries, regions, and countries tend to be cross-correlated as well as serially dependent. As pointed out by Breitung & Pesaran (2005), the problem of cross-sectional dependence is particularly difficult to deal with since it could arise for a variety of reasons including spatial spillover effects, common unobserved shocks, social interactions, or a combination of these factors.

2.7.3. Endogeneity and Bias

There are a number of sources of biases that can cause inconsistent estimates of the coefficients in panel growth regressions. The first is the omitted-variables bias (so-called heterogeneity bias) resulting from possible correlation between country-specific fixed effects and the regressors, affecting the consistency of pooled OLS and BE (between estimator) estimates (Wooldridge, 2009). The second is the endogeneity problem due to potential correlation between the regressors and the error term, which would affect the consistency of pooled OLS, BE and FE (Wooldridge, 2013). Specific to dynamic panels, there is a dynamic panel bias which will make FE estimates inconsistent. The third is classical measurement errors (errors in variables) in the independent variables, which affects the consistency of pooled OLS, BE, and FE estimator, although the bias tends to be exacerbated in FE and moderated in BE (Greene, 2003).

OLS estimates are sensitive to outliers, both observations with unusually large errors or influential observations with unusual values of explanatory variables (Easterly, 2005). Thus, robust regressions tend to be implemented. These are essentially iterated re-weighted least squares regression in which the outliers are dropped and the observations with large absolute residuals are down-weighted. Standard pooled estimators can also lead to heterogeneity bias in the common factor model (Pesaran & Smith, 1995). Pooling introduces data dependencies in the residual term if variables are integrated (Lee, Pesaran & Smith, 1997), a property typically assigned to macro data in the debt-growth literature. Heterogeneity misspecification enters linear combinations of integrated variables into the error term, raising the potential for spurious regression (Phillips & Sul, 2003). Existing research has found very different results when moving away from full-sample analysis in homogeneous parameter regression models and investigating sub-samples along geographic, institutional or income lines (International Monetary Fund, 2012; Kourtellos, Stengos & Tan, 2014).

The problem of reverse causality or endogeneity is especially important with debt and growth. Reverse causality is avoided in some literature (Kumar & Woo, 2010) by examining the impact of initial levels of government debt on subsequent growth. However, this does not necessarily solve the endogeneity problem: that debt and growth may be jointly determined by third variables. External instrumental variables for initial government debt are difficult to find. Endogeneity issues can be addressed by using the SGMM approach of Arellano & Bover (1995) and Blundell & Bond (1998). This method uses suitable lagged levels and lagged first differences of the repressors as their instrument. The SGMM empirical approach has gained popularity within the economic literature and is used widely in applied economic research.[20]

[20] However, this method has pitfalls – see Roodman (2006).

Cross-sectional dependence is a rapidly growing field of study in panel data analysis. Conditioning on variables specific to the cross section units alone typically does not deliver cross section error independence and it is well known that neglecting cross section dependence can lead to biased estimates and spurious inference. Chudik el al. (2011) focus on the econometric literature and consider the problem of estimating the slope coefficients of large panels, where cross section units are subject to a number of unobserved common factors that may rise with the number of countries/cross-sections. It is established that the Common Correlated Effects (CCE) estimator introduced by Pesaran (2006) remains asymptotically normal under certain conditions on the loadings of the infinite factor structure, including cases where methods relying on principal components fail.

2.7.4. Non-linearities and Model Selection

Many empirical studies on debt and growth either include squared debt terms or use spline specifications in their empirical framework to capture the heterogeneous impact of debt across different levels of indebtedness (Checherita-Westphal & Rother, 2012). However, this specification (pooled) is part of a model that assumes common parameters across countries. This poses a question: does non-linearity differ across different countries, or within countries, over time?

Eberhardt and Presbitero (2013) consider nonlinearity *across countries* by adopting a standard linear regression model, of a type that accounts for both observed and unobserved heterogeneity. Identification of the long-run and short-run coefficients on debt is achieved by use of Pesaran's (2006) CCE estimator, which accounts for the presence of unobserved heterogeneity through a simple augmentation of the regression equation.

Nonlinearity in the debt-growth relationship is also considered *at the country-level*. Caner, Grennes & Koehler-Geib (2010) argue that, provided a debt-threshold exists, this would arguably differ across countries given heterogeneity in financial market development, openness, and institutional development, amongst other causes. Kourtellos, Stengos & Tan (2011) argue that if there exist heterogeneities in the debt-growth relationship (thresholds) then there may be other nonlinearities inherent in the empirical model employed to investigate this phenomenon. They find that while there does not exist a generic threshold or tipping point beyond which debt has a detrimental effect on growth, there does exist such a threshold determined by countries' level of democracy.

The main concern for empirical analysis is the most appropriate specification with regards to the time-series properties of the data: reliable inference on a relationship involving variable series that are non-stationary involves establishing that these variables are cointegrated, and within both time-series and panel time-series econometrics a number of alternative approaches are available to test for this property. Crucially, cointegration defines a *linear* combination of variables integrated of order one (in our case) which is stationary (i.e. integrated of order zero). Difficulties for the analysis of potentially nonlinear relationships such as that between debt and growth arise given that the order of integration of the square or cube of an integrated variable is not defined within the linear integration and cointegration framework. Eberhardt and Presbitero (2013) apply methods on the order of summability and the concept of co-summability from the time-series econometric literature (Rico & Gonzalo, 2011; 2013). These methods provide pre-estimation testing as to the validity of the empirical equation incorporating country-specific nonlinearities. Their study is the first to adopt these methods in the panel context, further addressing the concerns over cross-section dependence.

Ergodicity is critical because one only observes a single realization of history from which one must infer how policy responds to disturbances and how it would respond under various contingencies.[21] This may be impossible. A non-linear rule may, for example, show no evidence of a feedback from debt to primary balances if debt happens to be below the debt limit throughout the sample; or in case of a time-varying rule, all the within-sample ρ_t values may equal zero.[22] The ergodicity problem reinforces the notion that the primary deficit-to-GDP ratio is an increasing linear function of the initial debt-to-GDP ratio, and is a sufficient but not necessary condition. A stable, positive feedback from debt to the primary balance would justify calling fiscal policy sustainable, whereas a missing or seemingly unstable feedback would be consistent with either a non-sustainable policy or with a data set insufficient for identification. The implicit stationarity and ergodicity assumptions apply analogously to unit root, cointegration, and VAR-based tests. They seem unavoidable because sustainability constraints are inherently forward looking (Blanchard, 1990).

[21] A stochastic process is said to be ergodic if its statistical properties can be deduced from a single, sufficiently long, random sample of the process. Conversely, a process that is not ergodic is a process that changes erratically at an inconsistent rate (Walters, 2000).
[22] See Equation (8).

3. Fiscal Space Theoretical Model

The following section outlines the theory behind our empirical investigation. Ghosh et al. (2013) propose a theoretical model based around the concept of fiscal space, to analyse debt sustainability.[23] In practical terms, fiscal space is typically approximated by the gap between a certain debt limit and the actual debt. The debt limit, which is critical for assessing fiscal space, is defined as the debt level beyond which the debt position could bring about macroeconomic risks.[24]

Bohn (1998, 2008) is the first to investigate fiscal space, in respect to the dynamics of US debt-to-GDP data. Bohn's (2008) fiscal space is binary, being either zero or infinite, depending on how the primary balance reacts to past public debt. Assuming a linear relationship for any amount of debt, fiscal space is infinite if the reaction of the primary balance is sufficiently strong and zero otherwise. Ostry et al. (2010) and Ghosh et al. (2013) extend Bohn's (2008) definition of fiscal space by taking into account the work of Ostry & Abiad (2005), and Mendoza & Ostry (2008), who propose inclusion of squared and cubic debt terms when calculating the response term.

Authors also calculate fiscal space based on measures of tax revenues. Aizenman & Jinjarak (2010) use years of tax revenues that are necessary to repay a country's debt. Brun (2006) calculates fiscal space as the ratio of the current level of revenues to potential tax revenues. Bi (2012) derives a dynamic Laffer curve of taxation, using a general equilibrium model, creating country-specific fiscal limits, a measure of the willingness and ability of governments to service their debts. Park (2012) also generates a Laffer curve and investigates how population aging trends affect fiscal space. However, the curve is of public revenues, and is generated using a standard neoclassical model. Fiscal space here is essentially the gap between current tax revenues and the peak of the Laffer curve (i.e. the maximum tax revenues possible).

Fiscal space can also be linked to implicit liabilities, e.g. aging costs. Marcel (2014) uses the S1 indicator[25], capturing the adjustment needed to reach a debt level of 60% of GDP by 2030. Schick (2009, 2012) focuses on a short-term definition: the financial resources available to the government for fiscal policy. These policies are growth-enhancing investment in physical and human capital, financed through borrowed funds, without compromising the long-run fiscal sustainability of the government.

This paper does not aim to contribute to the discussion about the various definitions of fiscal space, but relies on the recent approach used in Ghosh et al. (2013) to further the

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[23] See Section 2.6 for more information.

[24] Debt may not be unsustainable here, but there are high risk premiums on borrowing due to default risk, significant accumulation of arrears, or other signs of difficulty in servicing public debt.

[25] For the definition of the S1 indicator see European Commission (2012).

investigation. We base our theory and initial empirical work following the estimation settings and data selection in Ghosh et al. (2013). The following section outlines the theoretical model in regards to fiscal space and debt sustainability.

3.1. Debt Sustainability

3.1.1. Sovereign Debtor and Creditors

The government's IBC is:

$$d_{t+1} - d_t = (r_t - g)d_t - s_{t+1} \tag{9}$$

where d is one-period debt-to-GDP at the end of the period, g is the growth rate of real GDP[26], s is the primary balance (% of GDP), r is the real interest rate on debt accrued in period t and due in period $t+1$.[27] In equilibrium, the interest rate will be an increasing function of the probability of default (shown below).

Ghosh et al. (2013) make three main assumptions regarding the behaviour of sovereign debtor and creditors.

3.1.1.1. Assumption 1 – Fiscal Reaction Function with Fiscal Fatigue

The simple fiscal reaction function, in a reduced form:

$$s_{t+1} = \mu + f(d_t) + \varepsilon_{t+1} \tag{10}$$

where μ captures all systematic determinants of the primary balance other than lagged debt, $f(d_t)$ is the response of primary balance to lagged debt, and ε is an i.i.d. shock to the primary balance. Other authors that have postulated this fiscal reaction function are Bohn (2008), De Mello (2008), Burger et al. (2011), Egert (2014), and Goldey & Lavoie (2007). In this case the model deviates from Bohn's (2005) linear treatment of the fiscal reaction function, to include Ghosh et al.'s (2013) idea of fiscal fatigue.

The function $f(d)$ is assumed to be contiunuosuly differentiable and have the property that there exists a debt ratio $d^m > \overline{\varepsilon}$ such that:

$$\mu + f(d^m) - \overline{\varepsilon} \geq (r^* - g)d^m \; and \; f'(d) < r^* - g \;\; \forall d > d^m \tag{11}$$

[26] Here growth is assumed constant and exogenous, similar to Ghosh et al. (2013). These assumptions are questionable; however that is not our focus here.
[27] Another assumption is centred on the risk free interest rate, r^*. The interest rate is endogenous and greater than or equal to the risk free interest rate.

to capture the idea of fiscal fatigue. Hence at d^m, and with the worst primary balance shock, debt is non-increasing, and for any higher debt ratio the response of primary balance is lower than the growth-adjusted interest rate.

3.1.1.2. Assumption 2 – Default due to Inability to Pay

The government defaults if and only if debt exceeds the debt limit, $\overline{d}$. $\overline{d}$ is the maximum maturing debt a government can roll-over and finance the primary balance at a finite interest rate. The default rule of the sovereign thus takes the form:

$$D_t = \begin{cases} 1 \; if \; d_t > \overline{d} \\ 0 \; otherwise \end{cases} \tag{12}$$

where D is an indicator equal to one if the government defaults, and zero otherwise.[28]

3.1.1.3. Assumption 3 – Risk-neutral Creditors

Risk-neutral and atomistic creditors, who lend to the government under the following assumptions:

i) There is less than unity probability that government debt is on an explosive path (i.e. increasing without bounds regardless of all future realizations of ε).

ii) A *finite* interest rate exists that compensates risk-neutral lenders for the endogenous risk of default:

$$1 + r^* = (1 - p_{t+1})(1 + r_t) + p_{t+1}\theta(1 + r^*) \tag{13}$$

where p_{t+1} is the probability of default in the next period (when current debt matures), and θ is an assumed recovery value in the event of default.[29] If the default probability is positive but less than unity, this implies the default risk premium is a positive, increasing, and convex function of default probability.

iii) In the event that there are multiple interest rates satisfying the above condition, creditors choose the lowest interest rate.

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[28] The debt limit could vary over time with the governments' ability-to-pay. This model could be further adjusted to incorporate time varying ability-to-pay in future research, as long as μ is non-stochastic.

[29] Ghosh et al. (2013) determine the permissible range of the recovery value needs to have an upper bound to ensure that the government has the ability to pay the recovery value in the event of default. The authors assume that $0 \leq \theta \leq \overline{\theta} = 1 - \overline{\varepsilon}/(1 + r^*)d^m$. This assumption ensures that the debt limit under uncertainty will lie below the debt limit under no uncertainty, but will allows for large recovery values.

3.2. Fiscal Reaction Function

Solvency can be gauged by the behaviour of a country's primary balance. To ensure solvency, Ghosh et al. (2013) assume governments act responsibly; raising the primary balance in response to rising debt, meeting increased interest payment obligations. However, if this were to occur indefinitely, the balance would eventually exceed GDP. Beyond a certain point 'fiscal fatigue' occurs; the key idea is that it becomes difficult to keep cutting expenditure or raising taxes at higher levels of debt. At these high levels, the primary balance is still an increasing function of lagged debt, but now at a decreasing rate. Ostry et al. (2010) and Ghosh et al. (2013) therefore posit that a reasonable specification for the model, incorporating fiscal fatigue, is a cubic function of the lagged debt-to-GDP ratio.

The fiscal reaction function, based on Assumption 1, is,

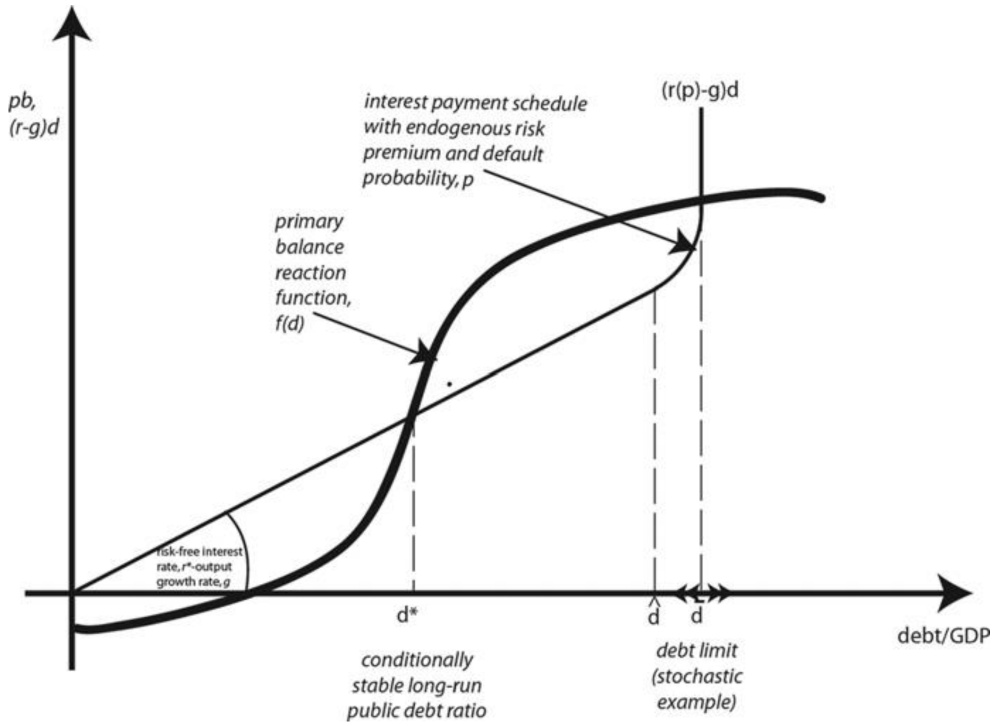
$$pb_t = f(d_{t-1}) \tag{14}$$

where pb_t is the primary balance at time t, and d_{t-1} is debt in the previous period. In the stochastic model, the primary balance is subject to explicit stochastic shocks. If shocks occur, then the market interest rate will begin to rise before the debt limit, again becoming infinite at $\overline{d}$. The function therefore becomes:

$$pb_t = \mu + f(d_{t-1}) + \varepsilon_t \tag{15}$$

with the additional default risk premium endogeneity and the market interest rate (Ghosh et al. 2013). μ captures all systematic determinants of the primary balance other than lagged debt, $f(d_{t-1})$ is the response of primary balance to lagged debt (continuous cubic function), and ε is an i.i.d. shock to the primary balance. Figure 4.0 displays a graph of this theorised relationship, emphasising the debt limit and cubic functional form.

Figure 4.0: Fiscal Reaction Function (Stochastic)



Source: Ghosh et al., (2013). Figure 3.

The model emphasizes that the market dynamics depend on the available fiscal space, not strictly on the debt level. The stochastic case has two components in the standard debt dynamics equation:

$$d_t - d_{t-1} = (\overline{r} - g)d_{t-1} - pb_t + \varepsilon_t \tag{16}$$

There are two equilibrium points at the intersections of the primary balance, pb_t, and the interest payment schedule, $(\overline{r} - g)$. The long-run equilibrium, d^*, is the conditional convergence point for debt-to-GDP, and is a dynamically stable equilibrium. Below d^*, debt increases as interest payments exceed the primary balance; above d^*, debt decreases as the primary balance exceeds interest payments. The debt limit, $\overline{d}$, provides the second, dynamically unstable, equilibrium. If debt exceeds $\overline{d}$, the primary balance is never satisfactory to meet the interest payment and debt increases further. When debt explodes, countries face market shutout thereafter. The debt limit and available fiscal space can be elevated, for example, by a decrease in the interest rate, or an increase in the output growth rate.[30] Structural reforms can improve the primary balance, shifting the $f(d_{t-1})$ schedule upwards, lowering the long-run debt level and raising the debt limit and fiscal space.

[30] Both rotate the $(r_t - g)d_{t-1}$ clockwise.

Recent literature on fiscal reaction functions aim to identify stable relationships between fiscal policy and its determinants (Mélitz, 1997; Galí and Perotti, 2003; Wyplosz, 2005). However, these exercises do not establish causality, only extracting information on key considerations that are correlated with policy decisions. Debt sustainability is one such consideration, with cyclical developments, and institutions affecting a government's incentives.

3.3. Debt Limit

Debt limits are focused on sustainability and the question of what causes debt to become unsustainable. Ghosh et al. (2013) impose a number of conditions to ensure the debt limit is an accurate reflection of this. The debt limit is a key concept characterising the largest debt at which the government can borrow at a finite interest rate. Firstly, the model's equilibrium default probability must be characterised, as well as the existence of well-defined bounds within which the debt limit is determined.[31]

There are three main heuristic properties to the debt limit. Firstly, a reduction of $\bar{d}$ can occur due to counterclockwies rotation of the $(r^* - g)d$ schedule, for example, when the economy's output growth rate decreases, or the risk free interest rate increases. Secondly, an increase in $\bar{d}$ can occur due to an upward shift of the intercept of the fiscal reaction function or the steepening of this schedule, for example, via greater fiscal adjustment or willingness to undertake adjustment. Third, $\bar{d}$ is dynamically unstable. Positive or negative shocks cause a return to the long-run debt ratio or default, respectively.

3.4. Fiscal Space and Growth

Current literature seems to predominately use public debt or fiscal deficit as variables when investigating debt-growth relationships. However these measures are inherently backwards looking. Solvency and the likelihood of default depend on a government's ability/willingness to increase revenue or cut expenditure over a potentially infinite future horizon. Therefore the forward-looking concept of fiscal space is an important next step, going beyond the current debt or deficit proxies.

Fiscal space provides an improvement over current measures such as the debt ratio. Consider two governments with the same public debt. Depending on their fiscal space, these governments may face very different environments. The response to each can also be highly non-linear – zero when there is ample fiscal space, but rising to infinity as debt

[31] See Ghosh et al. (2013) for theoretical proof of debt limit assumptions, and also for a discussion of the effects of data revisions on $\bar{d}$.

reaches the limit and fiscal space is exhausted. Markets also tend to react very late, with governments able to borrow at the risk-free rate until almost out of fiscal space.

Fiscal space (S) is essentially the difference between current debt levels, d_t, and the model-calculated debt limit, $\overline{d}$,

$$S_t = \overline{d} - d_t \tag{17}$$

which essentially indicates how much room a government has to increase fiscal policy before facing sustainability issues. We do not extend the theoretical links between debt and growth into fiscal space. Our investigation instead focuses on investigating this phenomenon empirically.[32]

Fiscal space calculations incorporate estimates of relevant interest rates on public debt (Ghosh et al., 2013). Market interest rates are a historical average (previous 10 years) of implied nominal interest rates on government debt (interest payments divided by end-period debt) relative to the growth rate of nominal GDP, and used on government debt assuming that the market rate reflects the perceived probability of default, i.e. the market rate reflects defaulting risk. These historical averages can also be replaced with IMF projections for long-term government bond yields and GDP growth. Historical interest rates are generally more favourable than projected interest rate-growth differentials, reflecting expectations of higher interest rates and slower GDP growth.

Because the relationship between fiscal space and growth has not been examined in the literature to date, we posit an initial theory extending upon the line of thinking in the debt and growth literature. At its simplest level, common theory states that high debt has a negative impact on growth, and therefore countries with high debt would be best to pay down that debt. In terms of fiscal space, our hypothesis and extension on theory would mean that countries that have little fiscal space (high debt close to their individual country debt limit) would achieve higher growth by increasing their fiscal space (paying down debt). Conversely, countries with high fiscal space (low debt relative to their individual country limit) would be able to achieve higher growth by increasing debt, having a flow-on to growth via government spending and investment.

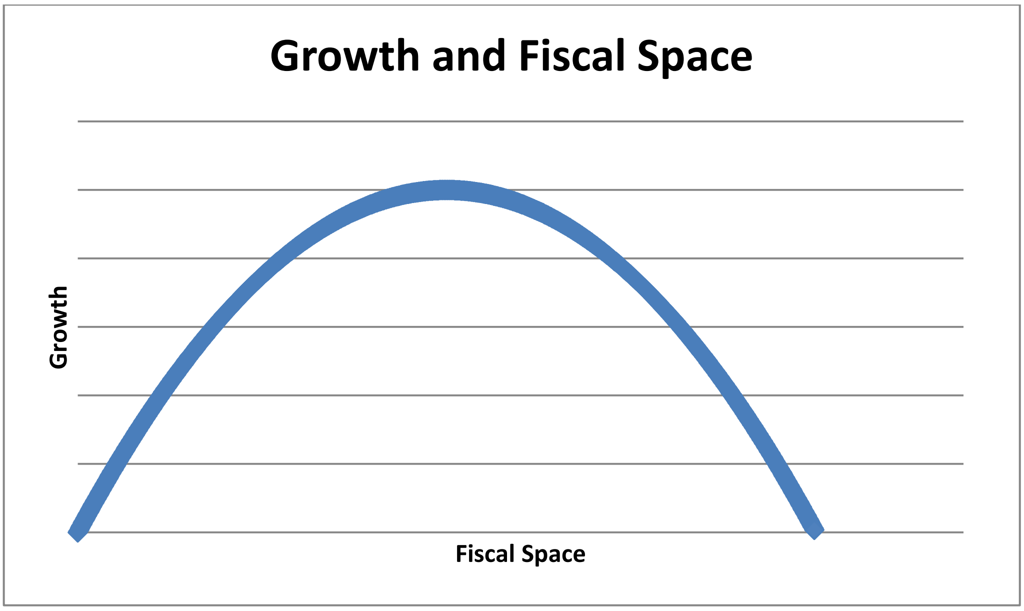
This hypothesis hints at a quadratic relationship between fiscal space and growth (see Figure 5.0). At low levels of fiscal space, countries would achieve increased growth by increasing their fiscal space to a more sustainable level. However, as countries achieve more and more fiscal space, paying down the debt and increasing fiscal space further begins to have a negative effect on growth essentially paying down money that could have been used

[32] See Section 5.3 for more details on theory, and specifics on empirical method.

sustainably to stimulate the economy. To take this one step further, we also investigate whether it matters what the government spends its money, for example the choice between investment or consumption.

We incorporate growth theory from popular literature, and control for a number of well-known growth determinants. Specifically we use a Solow (1956) and Barro (1994) model, while also incorporating modern empirical methodologies (see Section 5.3 for empirical details).

Figure 5.0: Growth and Fiscal Space Theory



Specifically, our growth theory follows work done by Moral-Benito (2010) in investigating growth determinants, while we add in fiscal space. We follow the inclusion of a range of determinants in the Solow and Barro Models including: initial GDP, investment ratio, education rates, life expectancy, population growth, trade openness, and government consumption.[33]

[33] See Section 5 onwards.

4. Data and Stylised Facts

The study uses panel data, to make use of both the time-series and cross-sectional variation in the data.[34] The countries focused on in this study are similar to Ghosh et al. (2013): a sample of 23 advanced economies, with the time span of 1969-2007. Data are available for 2007-2015; however, we have not included this period in our study to avoid the GFC period. The advanced economies include: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, Japan, Korea, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, UK, and the US. Although the data span for different variable series varies slightly across countries, the majority of the data are available for the full time span.

Focusing only on advanced economies, the theory is that they would respond to various changes or shocks in a similar manner. For example, in this context, we would assume that countries with a lower debt-to-GDP ratio would have a similar experience (in regards to their primary balance) at high debt levels, to countries that are actually observed at high debt-to-GDP levels. A large number of studies tend to use the majority of our chosen countries, plus or minus a few. For example, Reinhart & Rogoff (2010) use a sample of 20 countries, most included in our sample, with the exception of some of the advanced Asian economies such as China, Thailand, or Malaysia.

Our addition to the literature is twofold. Firstly, we extend upon Ghosh et al.'s (2013) analysis on the fiscal reaction function and debt limits. This is achieved by introducing dynamics and testing different models, inclusion of other variables, and examination of assumptions. Our second addition to the literature is investigating the relationship between fiscal space and growth. This is achieved following similar methods to the debt-growth literature; however incorporating fiscal space adds more country specific dynamics, and can also provide a gauge of how 'good' a measure fiscal space is depending on how it matches debt theory.

Section 4.1 gives an overview of our relevant fiscal data. Section 4.2 undertakes preliminary analysis focusing on stylised facts, stationarity, and cointegration.

[34] The data used in this study have been acquired from a number of sources. The fiscal data were acquired from Ghosh et al.'s (2013) dataset, and include a number of macroeconomic variables (described below) from the IMF and World Bank's World Economic Outlook (WEO) database. Further data from the WEO have been acquired for the growth regressions, based on the variables used in Moral-Benito (2010).

4.1. Fiscal Data Overview

The main variables of interest in setting up the initial fiscal reaction function are the primary balance and the debt-to-GDP ratio. The primary balance is general government primary net lending or borrowing i.e. net lending/borrowing plus net interest payable/paid (interest expense minus interest revenue), as a percentage of GDP. Debt-to-GDP is a country's gross debt as a percentage of GDP.[35] Gross debt is used because net debt is not always comparable between countries due to different definitions of debt and treatment of assets, e.g. social security. To determine the consolidated balance sheet of the general government, it also makes sense to exclude intra-government debt holdings, such as social security funds that cover pension liabilities. However this requires complex assumptions and calculations in practice. Ghosh et. al. (2013) recognize that Japan is the only advanced country with large differences between net and gross debt; however the main conclusions about fiscal space do not change.

However, "unfortunately, it is quite difficult to assess the value of many government assets and liabilities" (Taylor & Woodford, 1999, pg. 1621). Public debt is not the only liability the government holds, with other liability considerations such as pension funds and insurance liabilities (e.g. Accident Compensation Corporation in New Zealand). Governments can also hold considerable and varied assets, both financial and physical. Factors such as market values, and short vs. long-term debt issuance also add another layer of difficulty in any assessment.

For the fiscal reaction function, we include a number of control variables in similar fashion to Ghosh et al. (2013)[36]: trade openness; present and future age dependency ratio; rule of law; inflation; oil price; non-fuel commodity export prices; political stability; IMF arrangement; fiscal rules; interest rates; real GDP growth. The output gap is another key variable, calculated by taking the difference between actual and potential real government consumption spending. The government expenditure gap is also included to measure the effect of temporary fluctuations in government outlays (e.g. military spending).

In addition to these core variables, for the fiscal reaction function, further variables of interest are included that are hypothesised to influence either the primary balance or be linked to growth. To extend upon Ghosh et al.'s (2013) data have been sourced from the WEO including variables for banking crisis (ECB, 2015), population growth (Whitehead, 2006), and unemployment (Fedelino, 2009). Our growth modelling also incorporates the determinants (initial GDP, investment ratio, education rates, life expectancy, population

[35] Gross debt refers to the stock of outstanding debt. Net debt is the difference between gross debt and the financial assets a government holds.

[36] Their choice follows the literature, e.g. Roubini and Sachs (1988); Gali and Perotti (2003); Abiad and Ostry (2005); Bohn (2008); and Mendoza and Ostry (2008).

growth, trade openness, and government consumption) sourced from the WEO.[37] We include these based on popular growth theory, following Moral-Benito's (2010) choice of variable selection.

Because of limitations in data collection, with some countries having different number of observations in their time series, or having incomplete data sets, we abstract away from these issues, instead choosing to use Ghosh et al. (2013) as a base for our model, sourcing extra data from the WEO. Data are also unavailable in panel form for the market value of public debt, another reason for using gross debt, for both the time-span of data, and the ability to compare across countries. [38] Ideally, given more time or data availability, we would have incorporated net debt, or taxes. But, for all countries in our sample, our data maintain good coverage and means we can focus on extending the literature specifically down the fiscal space dimension.

4.2. Preliminary Analysis

Before investigating the empirical relationships presented in the preceding theory, it is important to undertake some preliminary analysis on the data, to determine properties and stylised facts.

4.2.1. Stylised Facts

Table 1.0 above shows summary statistics of the main variables in our data: primary balance and the debt-to-GDP ratio. According to this, our panel of countries has an average primary balance of 65 percent of GDP, indicating that the average country across our time period runs a slight budget surplus. However, in looking at the individual countries, there is clear heterogeneity in their experiences. There are countries that run low-to-negative primary balances, such as France, Ireland, Japan, Portugal, Spain, the United Kingdom, and the United States. The remaining countries tend to run a positive primary balance. Interestingly enough, it would appear that all countries have at some stage operated both a positive and negative primary balance.

[37] For detailed data description, please refer to Appendix.
[38] Countries treating or defining asset components differently, especially with regards to social security, can cause issues if net debt is used.

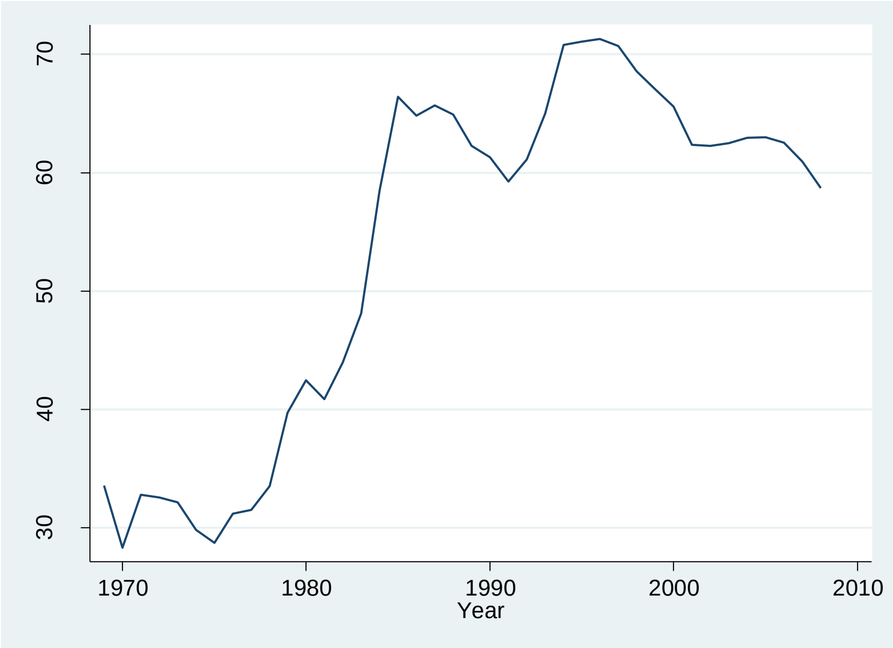
Table 1.0: Country-by-country Summary Statistics, 1970-2008

	Primary Balance			Debt-to-GDP		
country	mean	min	max	mean	min	max
Australia	0.18	-3.28	3.98	20.60	9.81	32.28
Austria	0.63	-4.20	15.81	48.50	16.73	67.62
Belgium	2.95	-7.38	6.85	100.53	59.87	136.92
Canada	0.13	-5.26	6.01	68.45	43.66	101.72
Denmark	2.26	-6.42	8.25	49.04	7.09	81.94
Finland	1.30	-8.59	7.81	26.30	6.39	57.82
France	-0.75	-3.67	1.23	45.64	20.41	66.43
Germany	0.00	-2.53	4.11	47.08	26.01	66.38
Greece	0.18	-6.82	4.65	59.04	15.65	103.72
Iceland	0.69	-3.44	6.74	38.55	23.05	58.94
Ireland	-0.98	-14.49	6.27	68.83	24.69	109.29
Israel	2.14	-7.23	14.31	120.45	79.50	262.30
Italy	0.21	-5.85	6.10	95.18	56.89	121.84
Japan	-2.03	-6.83	3.20	82.66	12.04	191.64
Korea South	1.95	-2.78	4.85	16.48	7.87	30.07
Netherlands	0.85	-2.54	5.15	57.60	37.47	78.50
New Zealand	3.23	-1.47	7.61	44.16	19.74	71.63
Norway	5.12	-4.75	16.28	45.31	28.92	68.26
Portugal	-0.58	-5.21	3.73	67.73	61.98	74.00
Spain	-0.39	-4.93	3.34	45.92	16.56	67.44
Sweden	0.96	-9.93	5.90	55.95	26.09	84.44
United Kingdom	-0.77	-6.14	3.21	42.62	31.30	49.26
United States	-0.35	-3.84	4.26	55.69	39.23	72.43
Average	0.65	-14.49	16.28	57.56	6.39	262.30

Notes: Primary Balance and Debt-to-GDP are both percentages of real GDP for a given country. Data have been sourced from WEO database, following Ghosh et al. (2013). Average value has given equal weighting to each country.

This heterogeneity across countries is also obvious when examining debt-to-GDP. There is a wide range of debt-to-GDP values across the spectrum, with South Korea having the lowest on average at 16.48, and Israel having the highest at 120.45. Given the range of debt-to-GDP being 6.39 to 262.30, it would appear that no single country has experienced debt across the entire range. Ghosh et al. (2013) account for this by arguing that because this sample focuses on advanced economies, even though they have not experienced debt at all levels, we can assume that they would all display similar behaviour over the debt ranges.

Figure 6.0: Average Debt-to-GDP of 23 Advanced Economies



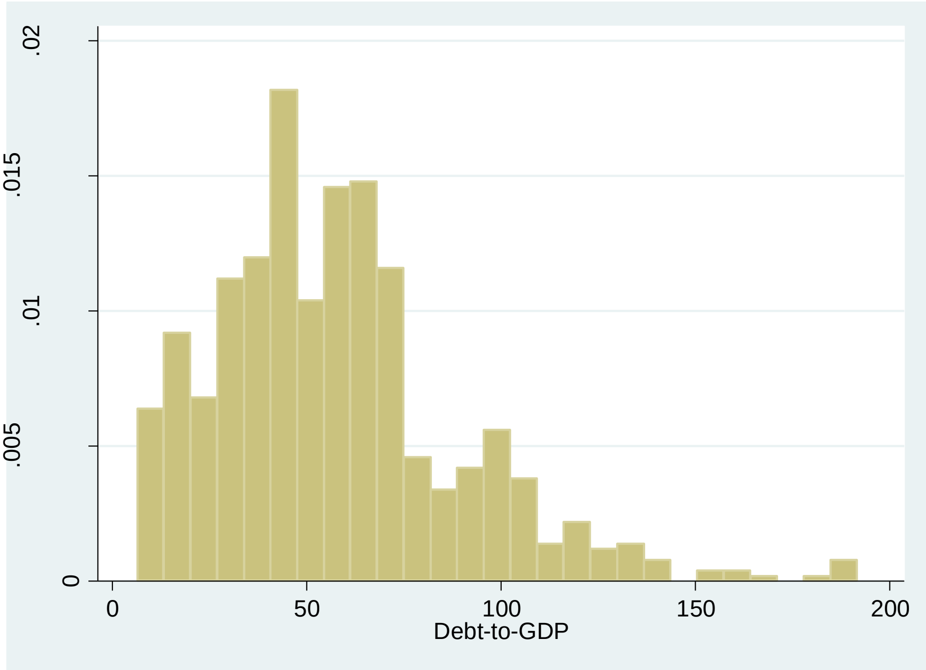
Notes: Cross-sectional average debt-to-GDP. Data have been sourced from WEO database, following Ghosh et al. (2013).

Figure 6.0 shows a plot of the mean debt-to-GDP values from our panel over time, i.e. the mean across countries in a given year (expressed as a percentage of GDP). In graphing the mean values of debt-to-GDP over time, it is clear that this series is non-stationary. Mean debt-to-GDP for our sample of advanced economies experiences a sharp rise from the mid 1970's for approximately a decade. Before this sharp increase, mean debt-to-GDP is around 30 percent. After the sharp rise, mean debt-to-GDP appears slightly more stationary, fluctuating around 65% of GDP from 1985 until 2008.

Given the sharp rise in debt-to-GDP over time, across our sampled economies, another metric to investigate would be the distribution of debt-to-GDP observations in our full sample. Figure 7.0 shows this in a density plot of debt-to-GDP observations. It is clear that the majority of debt-to-GDP observations occur below the 80 percent mark. However, the right-hand tail stretches out to a maximum of 180 percent.[39] Combining our knowledge of debt-to-GDP distributions here with Figure 6.0 would reinforce the use of panel data to draw conclusions. We have limited observations at the higher end of the spectrum, which would also appear to occur more recently in our sample given the increasing trend over time.

[39] For graphical purposes, the debt-to-GDP data displayed here has been truncated, due to very limited observations at the higher end. However, there are a small number above 200 percent of GDP, but this graph is simply used to demonstrate an approximate knowledge of debt-to-GDP distributions. See Table 1.0 for more information.

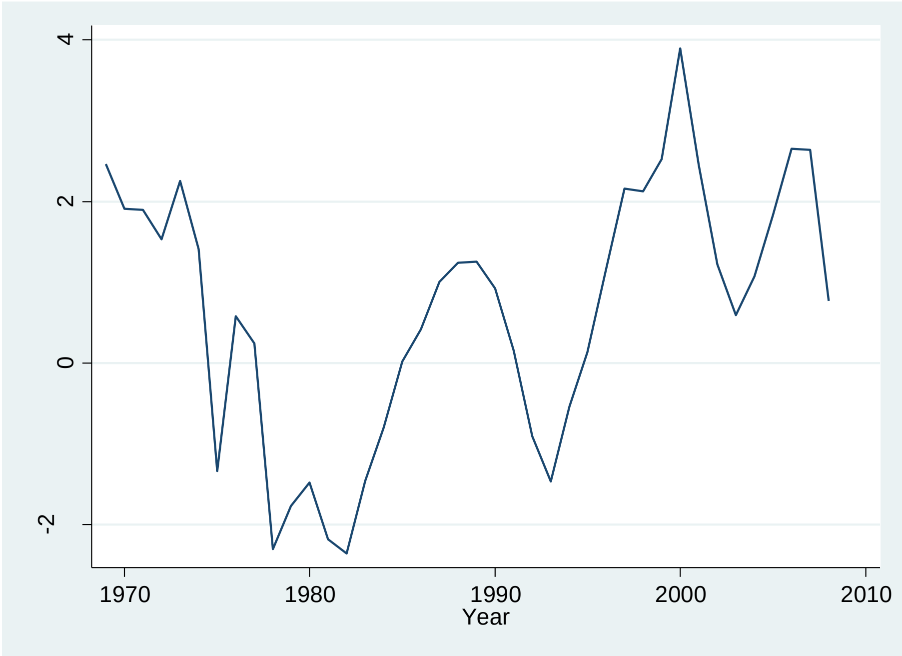
Figure 7.0: Public Debt-to-GDP distribution of 23 Advanced Economies



Notes: Distribution of debt-to-GDP data observations. Data have been sourced from WEO database, following Ghosh et al. (2013).

This increase in debt-to-GDP partially explains what we see in primary balance (Figure 8.0). Here, we also plot the average primary balance-to-GDP ratio across our sample. Around the same time that we see rising debt, the plot of average primary balance shows a decrease from 2 to -2, indicating that, on average, countries are running budget deficits and going into debt. From 1980 onwards, there appears to be an increasing 'staircase' trend where, on average, countries increase their primary balance (pay down debt), only to decrease the primary balance again. This could be due to governments generating increased revenue in response to increasing expenditure. This would imply attempts at sustainability of fiscal policy, however we can investigate this further by plotting and comparing government revenue (Figure 10.0) and government expenditure (Figure 9.0) over time.

Figure 8.0: Primary Balance of 23 Advanced Economies

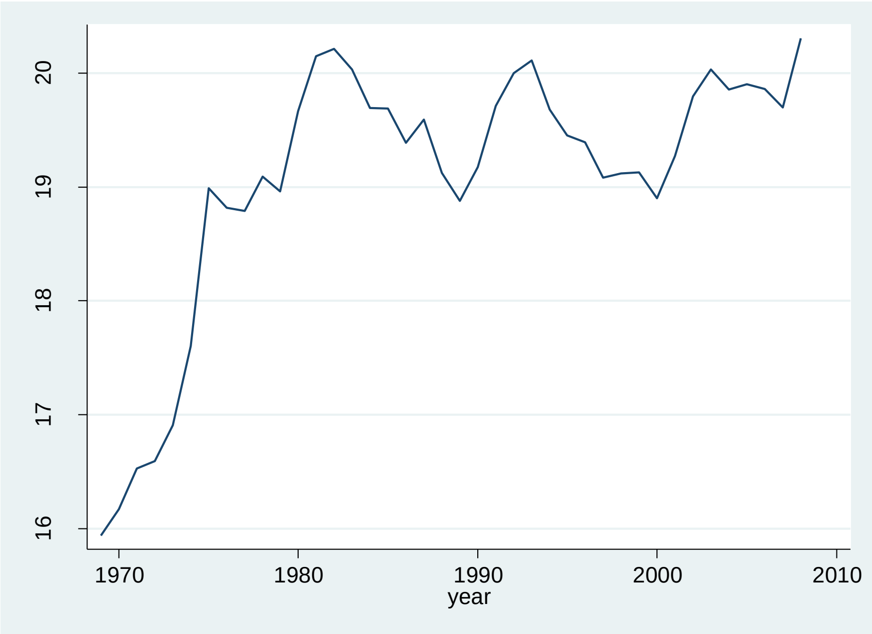


Notes: Cross-sectional average primary balance-to-GDP. Data has been source from WEO database, following Ghosh et al. (2013).

Figure 9.0 plots the mean government expenditure (as a percentage of GDP) over time for our sample. This follows a similar trend to what we see in debt-to-GDP over time. An increasing trend makes sense, given the positive correlation between government expenditure and the debt used to finance it; i.e. if expenditure increases (ceteris paribus) then debt must also increase. Government expenditure increases at a faster rate compared to debt-to-GDP. Initially, our sample has a mean government expenditure of 18 percent of GDP. However, this rises to, and fluctuates around, 19.5 percent from 1980 till 2008.

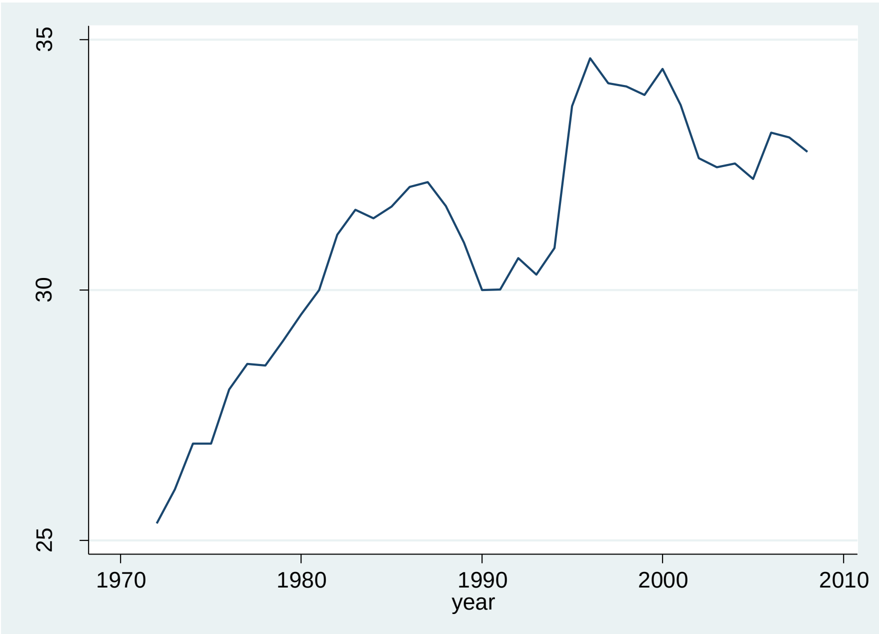
Government revenue as a percentage of GDP is plotted in Figure 10.0. Again, this plot is of the average government expenditure-to-GDP across our sample. Here we see an increasing trend, similar to that of government expenditure, but rising at a slower rate. This could indicate a larger policy lag for implementing revenue generating policies (such as increasing taxes). However, the similar increasing trends between government expenditure and government revenue, would hint at sustainability of fiscal policy (generally across the panel), where government revenue increases in response to increasing government expenditure to ensure debt-to-GDP does not perpetually increase. As a result, from 1985 till 2008, we see debt-to-GDP stabilise around 65 percent, and even start to exhibit a downwards trend from 1995 onwards.

Figure 9.0: Government Expenditure of 23 Advanced Economies



Notes: Cross-sectional average government expenditure-to-GDP. Data have been sourced from WEO database, following Ghosh et al. (2013).

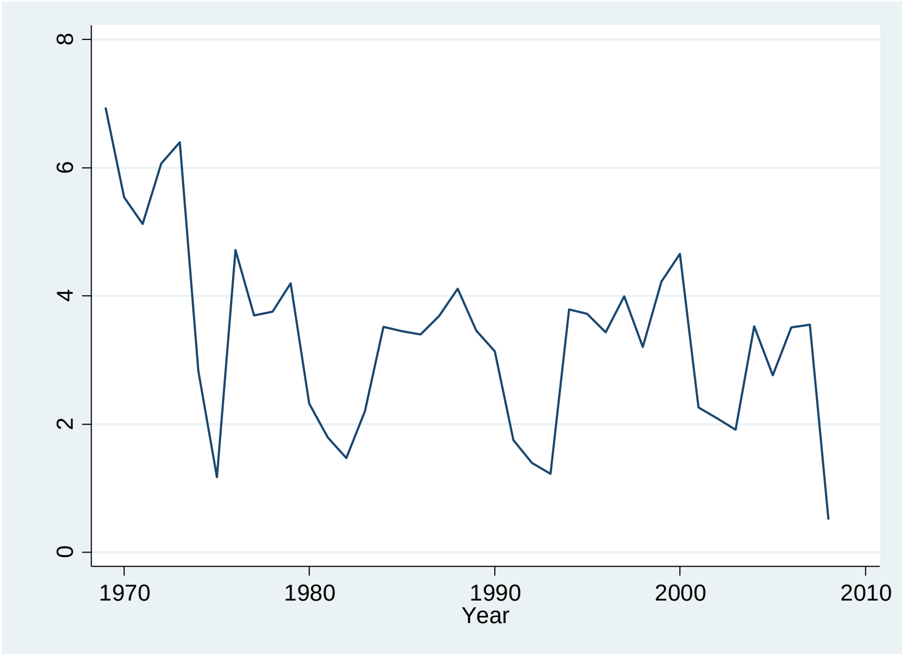
Figure 10.0: Government Revenue of 23 Advanced Economies



Notes: Cross-sectional average government revenue-to-GDP. Data have been sourced from WEO database, following Ghosh et al. (2013).

In looking at average growth of real GDP (across this group of advanced economies), there is a clear downwards trend over time (see Figure 11.0). Initially there is quite high average growth; however by the mid-1970s growth appears to have reached an average of approximately 3%. There are fluctuations around this for a number of years, then a sudden decrease in 2008 indicative of the GFC. In more recent years (not shown here) global growth has slowed further (see previous discussion for details).

Figure 11.0: Average Growth Rate of 23 Advanced Economies



Notes: Cross-sectional average GDP growth rates. Data have been sourced from WEO database, following Ghosh et al. (2013).

Our simple plots of the data above hint at the idea of fiscal sustainability. However, to delve into this further requires unit root and cointegration tests as an empirical starting point for analysis (Bohn 2007). Also, stationarity of variables is key in empirical analysis, in determining the correct specification and ensuring there are no spurious regressions (Granger & Newbold, 1974).

4.2.2. Stationarity Properties of the Data

In examining the primary balance and debt-to-GDP variables over time, it appears that they may not satisfy the criteria for stationarity, which would have implications for our empirical analysis. The key criterion for stationarity is a constant mean. Furthermore, covariance stationarity requires additionally that the covariance depends only on the time periods that the variables are apart and not on the specific time period. To analyse time-series properties of the data, a parametric approach is employed for testing of non-stationarity. Our main interest, at least for the initial fiscal reaction function, is determining how a government responds to rising debt. Therefore the main variables of interest are the primary balance and debt-to-GDP ratio.

The most common unit root test in the time-series literature is an (augmented) Dickey-Fuller test. However, our empirical analysis is done using panel techniques. While time-series unit root testing is conducted[40], it is also apparent the use of panel unit root tests are required. Using panel tests for generalised results and investigating specific countries with time-series testing can provide a useful comparison. Testing for unit roots in time-series studies is now common practice in applied research. However, testing for unit roots in panels is more recent. The development of panel unit roots can be summarised by Figure 12.0.

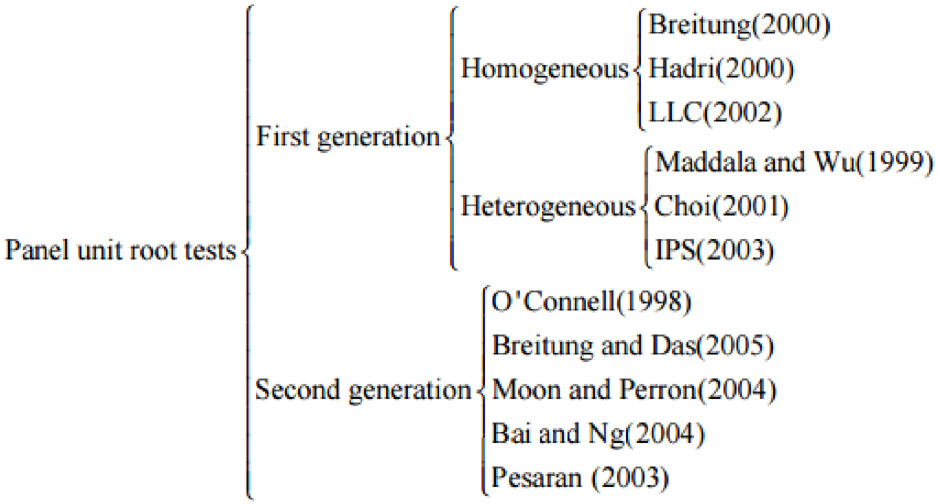
We focus on our two main variables for unit root testing: primary balance and debt-to-GDP. There are a range of factors to consider when choosing a unit root test. Firstly, we run the Dickey-Fuller GLS (see Appendix section 9.2.1.1 for results and discussion) proposed by Elliott, Rothenberg, & Stock (1992) to test for unit roots in individual country series. Then we examine second generation unit root tests (section 4.2.2.1 below). In subsequent sections, we test for and find the presence of cross-sectional dependence. Adding the cross-sectional dimension to the usual time dimension is very important in the context of non-stationary series. It is well known that unit root tests generally have low power in small sample sizes to distinguish non-stationary series from stationary series that are persistent. In order to increase the power of unit root tests, a solution is to increase the number of observations by including information relating to various individuals or countries. Thus, the use of panel data allows us to solve the low power issue of unit root tests in small samples by increasing the number of observations. As noted by Baltagi and Kao (2000, pg. 3), the econometrics of non-stationary panel data aims at combining “the best of both worlds: the method of dealing with non-stationary data from the time series and the increased data and power from the cross-section”.

The addition of the cross-section dimension, under certain assumptions, can act as repeated draws from the same distribution. Thus as the time and cross-sectional dimensions increase, panel test statistics and estimators can be derived which converge in distribution to normally distributed random variables. The cross-sectional independency hypothesis is rather restrictive and somewhat unrealistic in the majority of macroeconomic applications of unit root tests. This is an important issue since the application of tests belonging to the first generation to series that are characterised by cross-sectional dependencies leads to size distortions and low power (Banerjee, Marcellino and Osbat, 2004). Therefore, to gain a holistic understanding of the stationarity of our data we run ADF time series, Im-Pesaran-Shin (IPS), and Pesaran unit root tests. [41]

[40] See Appendix Section 9.2 for time-series unit root test results.

[41] Note: All potential combinations of lag selection, trend inclusion, and demeaning, have been investigated empirically. Conclusions regarding the presence of a unit root reinforce the results found here.

Figure 12.0: Unit Root Testing Evolution



Notes: This figure shows the classification and development of unit root testing in economic literature over time.

4.2.2.1. Pesaran's Panel Unit Root Test

The assumption of cross-sectional dependence limited to the case of common time effects on which the asymptotic results of the IPS's procedure relies (like most panel data unit root tests of "the first generation", including Levin et al. 2002) is often unrealistic and can be at odds with economic theory and empirical results (O'Connell, 1998; and Banerjee et al. 2004). Banerjee et al. (2004) shows if panel members are cross-correlated or even cross-sectionally cointegrated, all these tests experience strong size distortions and limited power.

As Breitung & Pesaran (2005) note, time series are contemporaneously correlated in many macroeconomic applications using country or regional data. Cross-sectional dependence can arise due to a variety of factors, such as omitted observed common factors, spatial spillover effects, for example via integrated financial markets, unobserved common factors, or general residual interdependence, all of which could remain even when all observed and unobserved common effects have been taken into account.

For this reason, various recent studies have proposed panel unit root tests allowing for more general forms of cross-sectional dependency, e.g. Choi (2001), Moon and Perron (2004), and Phillips and Sul (2003). We focus on Pesaran's (2007) cross-sectional augmented IPS (CIPS) test, which is a simple average of the individual CADF (cross-sectionally augmented Dickey-Fuller) tests. The data generating process (DGP) is a simple dynamic linear heterogeneous panel data model. The error term is assumed to have an unobserved one-common-factor structure accounting for cross-sectional correlation and an idiosyncratic component.

Pesaran's unit root test runs the t-test for unit roots in heterogeneous panels with cross section dependence, proposed by Pesaran (2003). Parallel to the IPS (2003) test, it is based on the mean of individual DF (or ADF) t-statistics of each unit in the panel. The null hypothesis assumes that all series are non-stationary, whereas the alternative hypothesis is that at least one of the panels is stationary. To eliminate the cross-sectional dependence, the standard DF (or ADF) regressions are augmented with the cross section averages of lagged levels and first-differences of the individual series (CADF statistics).

Expanding upon IPS, Pesaran (2007) proposes a cross-sectional augmented version of the IPS-test

$$CIPS = \frac{1}{N}\sum_{i=1}^{N} CADF_i \tag{18}$$

where $CADF_i$ is the cross-sectionally augmented Dickey-Fuller statistic for the ith cross-sectional unit.

Firstly, we run Pesaran's unit root test for debt-to-GDP. Table 2.0 shows the results. We run specifications including one to four lags, with both trend and no trend.[42] The null for this test assumes all panels are non-stationary, while the alternative hypothesis is that at least one panel in stationary. We fail to reject the null in all specifications for debt-to-GDP, indicating that debt-to-GDP has non-stationary panel properties. This result is reinforced in our time-series panel unit root test results (see Appendix section 9.2.1 for results and discussion).

<u>Table 2.0: Pesaran's Unit Root Test, Debt-to-GDP, 1970-2008</u>

Pesaran's CADF test for Debt-to-GDP

p values	1 Lag	2 Lags	3 Lags	4 lags
CIPS	0.8	0.949	0.961	0.913
CIPS & Trend	0.686	0.911	0.91	1

Notes: Table 2.0 shows the debt-to-GDP CIPS (trend and no-trend) p-value results, for different lag specifications.

Next, we run Pesaran's unit root test for primary balance, shown in Table 3.0. Again, we test specifications ranging from one to four lags, and with the inclusion of a trend. We fail to reject the null that all panels are non-stationary for primary balance. The only rejection result comes from 1 lag with no trend, which could indicate that at least one panel is stationary. According to our time-series and IPS unit root tests (see Section 9.2.1) the majority of panels display non-stationary processes; however it appears there are two

[42] One to four lags were chosen as four is the maximum number of lags we can incorporate given the span of our data.

countries that are stationary. At the panel level, this would indicate that primary balance has non-stationary properties.

Table 3.0: Pesaran's Unit Root Test, Primary Balance, 1970-2008

Pesaran's CADF test for Primary Balance

p values	1 Lag	2 Lags	3 Lags	4 lags
CIPS	0.001	0.323	0.596	0.546
CIPS & Trend	0.149	0.951	0.994	0.996

Notes: Table 3.0 shows the Primary Balance CIPS (trend and no-trend) p-value results, for different lag specifications.

Standard linear regressions are conducted under assumptions based on the Central Limit Theorem (CLT) (Rice, 2006). In the presence of non-stationarity however, the assumptions behind the CLT fail to hold. Also, economic theory often implies the existence of long-run relationships among non-stationary variables. If this occurs then cointegration techniques can be used to model these variables in regressions.

4.2.3. Cointegration Testing

After having confirmed the non-stationarity of our series of debt-to-GDP and primary balance, it is natural to test the existence of a structural long-run relationship between both series. This is the procedure we use in this section to assess fiscal sustainability on the basis of the intertemporal budget constraint.

Compared to panel unit root tests, the analysis of cointegration in panels is still at an early stage of development. So far, the focus of the panel cointegration literature has been on error-correction approaches, for example Westerlund's (2006) error-correction model approach. As is the case for panel unit root tests, panel cointegration tests are based on homogeneous and heterogeneous alternatives. The residual-based tests were developed to guard against the spurious regression problem that can arise in panels when dealing with I(1) variables. Such tests are appropriate when it is a priori known that at most there can be only one within-group cointegration in the panel. Notable contributions to this strand of the literature include Pedroni (1999, 2004), and more recently Westerlund and Edgerton (2007).

The use of panel cointegration techniques to test for the presence of long-run relationships among integrated variables with both a time-series dimension, T, and a cross sectional dimension, N, has received much attention recently. One of the most important reasons for this attention is the increased power that may be gained by accounting not only for the time-series dimension but also for the cross-sectional dimension. In spite of this, many studies fail to reject the no-cointegration null, even in cases where cointegration is strongly suggested by theory.

One explanation for this failure to reject centres on the fact that most residual-based cointegration tests, both in pure time series and in panels, require that the long-run parameters for the variables in their levels are equal to the short-run parameters for the variables in their differences. Banerjee, Dolado, and Mestre (1998) and Kremers, Ericsson, and Dolado (1992) refer to this as a common-factor restriction and show that its failure can cause a significant loss of power for residual-based cointegration tests.

As a response to this, Westerlund (2006) developed four new panel cointegration tests that are based on structural rather than residual dynamics and, therefore, do not impose any common-factor restrictions. The idea is to test the null hypothesis of no cointegration by inferring whether the error-correction term in a conditional panel error-correction model is equal to zero. The coefficient on the tests are all normally distributed and are general enough to accommodate unit-specific short-run dynamics, unit-specific trend and slope parameters, and cross-sectional dependence. Two tests are designed to test the alternative hypothesis that the panel is cointegrated as a whole, while the other two test the alternative that at least one unit is cointegrated.

However, Eberhardt (2011) states that these tests hold strong assumptions regarding the direction of causation from x to y. Also this test produces 'often quite stark results' strongly in favour of either the null in some applications, or the alternative in, unless T is large. The null here is no cointegration vs. the alternative hypothesis of cointegration.

Keeping this in mind, we investigate whether there is cointegration between primary balance and debt-to-GDP, and also government revenue and government expenditure.[43] The null for the Westerlund cointegration tests is that there is no cointegration amongst the variables. The test allows lead and lag specifications on both variables, and also the option to bootstrap p-values for all four test statistics. These are robust in the presence of common factors in the time series. Adding leads and lags to the series should eliminate feedback effects and endogeneity; however due to limits on data observation lengths we can only include up to 2 lags and 1 lead, limiting our test.

Overall the results of cointegration testing give mixed results (see Table 4.0) depending on specifications used, and even then the choice of test statistic to look at. However, this is not unusual according to Persyn & Westerlund (2008, pg. 9): "In small datasets (such as in this application with T = 32), the results may be sensitive to the specific choice of parameters such as lag and lead lengths and the kernel width."

The results below in Table 4.0 show that rejection or non-rejection of the null is specific to specification. We put more weight on the robust p-values, because bootstrapping is more

[43] See Appendix Section 9.2.2. for government revenue and government expenditure cointegration results. We find the presence of cointegration, reinforcing our conclusions for this section.

appropriate in the presence of cross-sectional dependence.[44] By adding leads and not just lags, we can allow for debt being weakly but not necessarily strictly exogenous. Also, based on the nature of governments' debt only having to be sustainable in their own country, we put more weighting on the 'group' test statistics described below. The Ga and Gt test statistics start from a weighted average of the individually estimated cross-sectional units and their t-ratios respectively. Rejection of the null should therefore be taken as evidence of cointegration of at least one of the cross-sectional units. The Pa and Pt test statistics pool information over all the cross-sectional units, so rejection of the null should therefore be taken as evidence of cointegration for the panel as a whole.

In looking at the Table 4.0 robust p-values for the group test statistics, Ga and Gt, it appears that these results reject the null of no cointegration between primary balance and GDP, concluding that there is cointegration in at least one of the cross-sectional units. Five out of 12 of the group statistics at the 5% level reject the null, and our most dynamic model with 2 lags, no trend, and a debt-to-GDP lead included also indicate support for cointegration.

Finding cointegration between primary balance and debt-to-GDP is a similar result to Trehan & Walsh (1991), who show that the TC and IBC are satisfied if this result is true. However, Bohn (2007) determines that Trehan & Walsh (1991) are examining an error-correction type specification. Error-correction has a natural economic interpretation as a reaction function describing the behaviour of the entity being studied, which fits well into the literature on fiscal behaviour (e.g. Bohn, 1998; Canzoneri et al., 1998). Bohn (2007) counters the argument that cointegration means sustainability, concluding instead that it is a necessary, but not sufficient condition. Specifically, Bohn (2007) suggests that rejections of stationary sustainability tests are invalid because in an infinite sample, any order of integration in debt-to-GDP is consistent with the TC. This would imply that the IBC may be satisfied even if particular time-series tests are not. Instead, Bohn (2007) emphasises whether a country's primary balance responds positively to debt as an indicator of sustainability. As in Bohn (1998), this depends upon the assumption that the series are stationary, or when they are non-stationary, for them to be related in a statistical sense, they must be of the same order of integration and the primary balance and government debt must be cointegrated. Additionally, another strand of the literature, as for example, Davig et al. (2011), estimate fiscal reaction functions, for the assessment of whether governments behave in a Ricardian manner.

Therefore, in the context of our results, it appears the primary balance and debt-to-GDP are cointegrated. However, to empirically investigate this idea of sustainability further we begin with a fiscal reaction function following Ghosh et al. (2013). This expands on the ideas of Bohn (2007) in the sense that the fiscal reaction function models how primary balance

[44] P-values have been bootstrapped 50 times each.

responds to increasing debt-to-GDP. Bohn's (2007) conclusion would mean that there should be a positive relationship between the two. From our preliminary testing above, we can tentatively conclude that debt-to-GDP satisfies the government's IBC; however our fiscal reaction function introduces the idea of country-specific 'debt limits', beyond which sustainability is compromised, providing a more comprehensive analysis of sustainability.

Table 4.0: Panel Cointegration Results: Primary Balance and Debt-to-GDP

Statistic	**Value**	**Z-value**	**P-value**	**Robust P-value**
		(1 1) lag, constant		
Gt	-2.283	-2.699	0.004	0
Ga	-9.941	-2.466	0.007	0
Pt	-11.882	-4.968	0	0
Pa	-10.773	-7.067	0	0
		(1 1) lag, constant, trend		
Gt	-2.351	0.09	0.536	0.46
Ga	-11.594	0.294	0.616	0.12
Pt	-13.87	-4.276	0	0
Pa	-12.89	-3.095	0.001	0
		(2 2) lag, constant		
Gt	-1.74	0.199	0.579	0.24
Ga	-8.987	-1.625	0.052	0.06
Pt	-8.468	-1.534	0.063	0.02
Pa	-8.352	-4.452	0	0
		(2 2) lag, constant, trend		
Gt	-1.813	3.265	1	0.88
Ga	-11.065	0.667	0.748	0.4
Pt	-9.684	0.51	0.695	0.24
Pa	-10.239	-1.022	0.153	0.1
		(2 2) lags, (0 1) leads, constant		
Gt	-2.222	-2.319	0.01	0
Ga	-7.842	-0.603	0.273	0
Pt	-8.356	-1.576	0.058	0.25
Pa	-8.503	-4.514	0	0
		(2 2) lags, (0 1) leads, constant, trend		
Gt	-2.538	-0.967	0.167	0.04
Ga	-8.422	2.419	0.992	0.70
Pt	-10.403	-0.827	0.204	0.02
Pa	-8.186	0.558	0.712	0.32

Note: Table 4.0 shows the 'group' and 'pooled' cointegration test results between debt-to-GDP and Primary Balance. Preliminary analysis warrants the inclusion of a constant term. From there we test a variety of lag specifications and trend inclusion.

5. Empirical Strategy

When calculating the fiscal space of a country we follow the approach described in Ghosh et al. (2013). As this paper is not primarily about the definition of fiscal space, we refrain from changing the underlying construction, instead focusing on testing robustness and assumptions regarding the model.

Firstly, we test the robustness of the empirical construction of fiscal space and the primary balance reaction function. Secondly, we investigate the link between fiscal space and growth. To our knowledge, this is the first paper to investigate such a relationship. Although this is extending upon the classic debt-growth nexus, fiscal space can further contribute to the discussion. This investigation will also further solidify whether fiscal space is a 'good' measure of debt sustainability, depending on how closely the relationship matches debt-growth theory.

5.1. Empirical Specification: Fiscal Reaction Function

Fiscal reaction functions establish how governments react to their debt burden. They specify (usually using annual data) the reaction of the primary balance-to-GDP ratio to changes in the lagged public debt-to-GDP ratio, controlling for other determinants of the primary balance. According to Bohn (1995, 2007), this represents an error correction mechanism: If the public debt-to-GDP ratio increases, rational governments attempt to arrest or reverse the rise by improving the primary balance. The rationale behind this is rooted in the government's IBC (Bohn 1998, 2007, Gali and Perotti 2003).

We first estimate a fiscal reaction function for our sample of 23 advanced economies (1969-2008) of the following form:

$$PB_{i,t} = \beta_1 D_{i,t-1} + \beta_2 D_{i,t-1}^2 + \beta_3 D_{i,t-1}^3 + \gamma X_{i,t}' + \mu_i + \epsilon_{i,t} \tag{19}$$

where the primary balance $PB_{i,t}$ of country i at time t is the dependent variable and the main focus is to estimate the reaction of this primary balance to the debt-to-GDP ratio in the previous period, $D_{i,t-1}$. To allow for the posited 'fiscal fatigue' reaction behaviour, squared and cubic debt terms are included, as well as country fixed effects μ_i. Similar to Ghosh et al. (2013) the error term $\epsilon_{i,t}$ is assumed to follow an AR(1) process.[45] Lagged debt in equation (19) would be endogenous to the extent there is persistence in the error term (ϵ), and induces a downwards bias in the estimated coefficients of the lagged independent variable. However, autocorrelation can exist for a variety of reasons. Residual

[45] We test this assumption using panel serial correlation testing methods developed by Drukker (2003), and find that the errors do appear to follow an AR(1) process.

autocorrelation can exist due to missing dynamics or model misspecification. To control for other influences on the primary balance the following set of control variables $X'_{i,t}$, in line with existing literature, is included: output gap; government expenditure gap; trade openness; inflation; age dependency; future age dependency; oil price (only for oil exporters); non-fuel commodity prices (only for non-oil commodity exports); political stability; IMF arrangement; and fiscal rules.

The lagged debt term in our fiscal reaction function could be considered a predetermined regressor. Knight et al. (1992) and Islam (1995) have also considered the predetermined nature of the lagged independent variable (in this case $D_{i,t-1}$, not $PB_{i,t-1}$) with respect to the transitory component of the error term. However, in these studies all the variables in the X vector are considered as strictly exogenous, i.e. all leads and lags of the variables are assumed to be uncorrelated with the error, ruling out the possibility of feedback. Caselli et al. (1996) and Benhabib and Spiegel (2000) allow for feedback by considering the predetermined nature of the X variables. Caselli et al. (1996) and Benhabib and Spiegel (2000) use generalised method of moments (GMM) techniques, following Holtz-Eakin et al. (1988) and Arellano & Bond (1991). GMM can generate consistent and efficient estimates of the parameters of interest, using moment restrictions, assuming the explanatory variables are predetermined. Although we acknowledge the presence of predetermined variables, we abstract away from addressing this, leaving it as a potential area for future research.

There are three sources of endogeneity bias within the fiscal reaction function. The first is the endogeneity of the output gap with respect to contemporaneous fiscal policy shocks, $\epsilon_{i,t}$. Secondly, the lagged debt level, $D_{i,t-1}$ will be correlated with the country-specific and time invariant determinants of primary balance, μ_i : countries generating lower surpluses on average—lower values of μ_i —will tend to have a higher level of public debt. This negative correlation between debt levels and the country fixed-effects would exert a downward bias on the estimated primary surplus' response to debt, if unaccounted for. We include lagged primary balance as a regressor to take account of any persistence in the primary balance behaviour due to sluggish tax and expenditure adjustment, which could help avoid this problem. As to the third source of endogeneity, to the extent that there is persistence in the idiosyncratic error term, $\epsilon_{i,t}$, the dependence of lagged debt on past surpluses will render lagged debt in equation (1) endogenous.[46]

Ideally, adequate instrumentation could address the endogeneity problems. For the first issue, the output gap needs to be instrumented with variables exogenous to the idiosyncratic primary surplus shocks. The inclusion of country fixed effects among the regressors can address the endogeneity of debt to the fixed effects term. However, if there

[46] For instance, a positive shock to the primary surplus in period $t-1$, i.e. a positive realization of ε_{t-1}, would reduce the debt stock, d_{it-1}. Thus, persistence in the idiosyncratic policy shocks (serial correlation between ε_{it-1} and ε_{it}) would result in a negative correlation between d_{it-1} and ε_{it}.

is strong serial correlation in the idiosyncratic errors this method would still be subject to the third endogeneity problem.[47] Country dummies can potentially introduce an additional bias: small sample bias of the fixed effects estimator.[48] However, all three issues could be simultaneously addressed by instrumenting both the output gap and lagged debt. But reliable instrumental variables (IV) based estimations require suitable exogenous instruments strongly correlated with the endogenous regressors. Ideal instruments are difficult to find. Moreover, IV-based regressions yield estimates with larger standard errors, and are generally not very efficient (Celasun et al., 2006).

We investigate Ghosh et al.'s (2013) specification, testing its robustness and incorporating more dynamics. We utilise a generalised least squares regression framework (see Section 6.1 for more information). Robustness is checked by the inclusion of additional variable, with a sub-sample regression testing slope homogeneity within a high debt sub-sample (Appendix section 9.2.4), and running individual time-series (Appendix section 9.2.3).

Ghosh et al.'s (2013) initial fiscal reaction function appears to be a static short-run model. Given the nature of the IBC and infinite time horizon of government's planning, it would seem appropriate to extend upon Ghosh et al. (2013) and introduce a dynamic framework for comparison. Our dynamic framework has a similar functional form as Equation (19),

$$PB_{i,t} = \beta_1 D_{i,t-1} + \beta_2 D_{i,t-1}^2 + \beta_3 D_{i,t-1}^3 + \beta_4 PB_{i,t-1} + \gamma X_{i,t}' + \mu_i + \epsilon_{i,t} \qquad (20)$$

however, the vector $X_{i,t}'$ now includes (other than the initial specification) additional lags of our three debt-to-GDP variables of interest, lagged output gap, lagged government expenditure, change in population, rule-of-law, and unemployment. Our dynamic model also includes $PB_{i,t-1}$, a lagged primary balance (dependent) variable.

Also, AR(1) errors that exist in the static model could potentially be a result of mis-specification. Due to limitations on the degrees of freedom available, and the span of time-series data available, we can only include dynamics via specific-to-general modelling (Kennedy, 2003). Following the approach described in Kennedy (2003), we begin with our static fiscal reaction function, and introduce additional lags one variable at a time, using "age-old criteria such as … significant t values on coefficients…" to determine appropriateness of the additional variable (Kennedy, 2003, pg. 83). We account for potential

[47] The inclusion of country indicator variables addresses the endogeneity of debt to η. Specifically, when country dummies are included, the mean values of the dependent and explanatory variables across all time periods for each country are obtained and the regression is performed on the variables in deviations from their country means.

[48] A large literature analyses the bias of the least squares with dummy variables estimator in dynamic models that include the lagged dependent variable as a regressor. The bias of this estimator decreases with the time dimension of the sample and the variance of the lagged dependent variable that is attributable to factors other than the disturbance terms (Kiviet, 1995, or Judson and Owen, 1999).

debt feedback through lags of debt-to-GDP, primary balance lags, as well as lags of government expenditure, output gap, oil prices, political stability, and fiscal rules. Higher order lags on the debt-to-GDP ratio are also investigated, plus leads (as currently the primary balance is regressed on lagged debt-to-GDP, so we check the contemporaneous relationship too).

Therefore, we essentially investigate two fiscal reaction function specifications: one static, and one dynamic. To gain a better understanding of how much the variation in the fiscal reaction function (an empirical estimation) affects the resulting debt limits, fiscal space, and fiscal space-growth relationship, we empirically determine fiscal space for both the static and dynamic model.[49]

5.2. Empirical Specification: Debt limit and Fiscal Space

After obtaining the estimated coefficients of both the static and dynamic fiscal reaction function, we calculate the interest rate-growth rate differentials for each of the countries and years. Following the "historical market approach" by Ghosh et al. (2013), we calculate the average of the implied nominal interest rate on government debt (share of interest expenditures to debt at end of period), $i_{i,t}$, and the average of the growth rate of nominal GDP (over the past ten years), $g_{i,t}$. To determine the debt limit, $\overline{D}_i$, for each country i, we calculate the largest root of the following equation:

$$(i_{i,t} - g_{i,t})\overline{D}_i = \beta_1 \overline{D}_i + \beta_2 \overline{D}_i^2 + \beta_3 \overline{D}_i^3 + \phi_i \tag{21}$$

where β_1, β_2 and β_3 are the coefficients estimated by the fiscal reaction function. ϕ_i is independent from the debt level, encompassing country fixed effects, other determinants, and assumes the expenditure gap and output gap are closed.

Ghosh et al. (2013) calculate two variants of fiscal space using different interest rate-growth rate differentials. Firstly, using a historical average interest rate-growth rate differential (over the past 10 years to cover a sufficiently long time span and smooth out fluctuations) creates a historical debt limit. The historical average is then replaced with IMF projections of long-term government bond yields and GDP growth (IMF, 2010). These projections are less favourable than historical interest rate-growth rate differentials, reflecting the expectations of both higher interest rates and slower GDP growth.[50]

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[49] Fiscal fatigue, or a cubic fiscal reaction function, holds for both static and dynamic models.

[50] The projected interest rate-growth rate differential is taken as the average over the next 5 years to get a medium-term perspective (such that the output gap is closed).

Finally fiscal space $S_{i,t}$ for country i at time period t is defined as the difference between the debt limit and the actual level of debt in time t. If this difference is negative then fiscal space is assumed to be zero:

$$S_{i,t} = max\left(\overline{D}_i - D_{i,t}, 0\right) \tag{22}$$

This truncation could affect the distribution of S. Our theory states that the debt limit is the maximum to which a country can borrow before facing increased restrictions and being shut out of the market. This is the reasoning behind the truncation. However, in real life, countries may get shut out of the market more gradually. This is hinted at in the deterministic fiscal space model; however it assumes the debt limit still applies. If negative fiscal space actually existed, one potential interpretation is that a country could become increasingly constrained. For following through to the relationship with growth, we use truncation and assume negative fiscal space cannot exist. Also in Ghosh et al. (2013) there are a few countries (e.g. Italy and Japan) for which the model is unable to calculate a debt limit, indicating the debt ratio does not converge to a finite level. For these countries, if $\overline{D}_i$ does not exist, then $S_{i,t}$ is assumed to be zero.

Given our two specifications (static and dynamic) for the fiscal reaction function, we undertake this process for each specification, resulting in four potential debt limits and four corresponding fiscal space series (debt limit fixed over time, but debt levels, and therefore fiscal space, vary): Static (Historical), Static (Projected), Dynamic (Historical), Dynamic (Projected). These four different measures will allow for comparison and robustness checks.

5.3. Growth and Fiscal Space

After empirically determining our fiscal space estimates, we now look to tie in this emerging strand of debt sustainability literature with the debt-growth literature, following growth-debt work by Moral-Benito (2010). Our goal here is to determine what relationship growth has with fiscal space (in terms of added dynamics over the classic debt-growth relationship), and to see whether fiscal space is an appropriate measure for debt sustainability, given the relationship we find.

The current debt-growth literature focuses on determination of empirical 'thresholds' – debt-to-GDP levels at which increasing debt starts to negatively impact growth, for example the well-known Reinhart & Rogoff (2010) '90%' threshold. Our prior calculations with the fiscal reaction function abstracted from growth, and focused solely on determining a 'debt limit', the point beyond which debt is unsustainable, not a threshold interpretation such as Reinhart & Rogoff (2010). Subsequently, we can implement growth regressions to determine if fiscal space has the empirical influence suggested by theory.

In terms of the growth literature, we follow Moral-Benito's (2010) methodology for fitting growth models, and incorporate our own estimates of fiscal space. Neoclassical growth models, such as the Solow-Swan model or running Barro regressions, provide a framework frequently used as a basis for empirical research.[51] In running the growth regressions, we utilise modern empirical methodology (see Section 6.3 for details).

It is the norm to assume that aggregate output in an economy follows a Cobb-Douglas production function. Although there is considerable work in the growth literature, the main empirical formulation focuses on two key themes: the identification of growth determinants, and the question of convergence.

Empirically, our growth model is:

$$\Delta y_{it} = \beta_1 S_{i,t} + \beta_2 S_{i,t}^2 + \psi x_{i,t} + \eta_i + \varsigma_t + \upsilon_{i,t} \tag{23}$$

where y_{it} is the logarithm of real GDP per capita for country i in period t, $x_{i,t}$ is a $k \times 1$ vector of growth determinants, η_i is a country specific fixed effect, ς_t represents a set of time dummies, and $\upsilon_{i,t}$ is the random disturbance term. We also include the initial level of GDP per capita in our neoclassical growth models because of interest in the convergence properties of growth.

In the empirical version of our Solow model, the vector $x_{i,t}$ includes proxies similar to Moral-Benito (2010): the population growth rate, the rate of technological progress, the rate of depreciation of capital, and the savings rate. The savings rate is proxied for by the ratio of real domestic investment to GDP.

Barro regressions extend the Solow model further, and are based on a wide variety of specifications given by different variables included in the vector $x_{i,t}$. The basic specification of a Barro regression is still the same as above but includes additional control and environmental variables, e.g. the average number of years of secondary education, and the logarithm of life expectancy. These variables proxy for human capital, as in Mankiw, Romer, & Weil's (1990) growth framework in the forms of knowledge and health. Among the control variables, we include the ratio of government consumption to GDP (G/GDP) as in Barro and Lee (1994).

[51] Barro regressions are aptly named extensions on the Solow-Swan model which control for other determinants of growth, allowing for predictable heterogeneity in the steady-state, beginning with Barro's (1989) paper.

5.4. Model Limitations and Issues

Previous literature on debt sustainability empirics includes Roubini and Sachs (1989), Gali and Perotti (2003), Ostry & Abiad (2005), Bohn (2005), and Mendoza and Ostry (2008). Based on this work, Ghosh et al. (2013) add a set of control variables when relating primary balances to lagged debt. This includes the output gap, government expenditure gap, fuel and non-fuel commodity prices, trade openness, three-year average inflation, a political stability index, a fiscal rules index, IMF support, and current/projected age-dependency ratios. Economic issues that could stem from the dependence of lagged debt on past values of the primary balance include persistence in the error term or serial correlation. To account for these issues, fixed effects are included in the empirical specification, along with an AR(1) error term to account for bias. This is 'textbook' autocorrelation in the errors, rendering OLS regressions non-efficient (but still unbiased), and biased standard errors. However, if this autocorrelation is due to omitted variables, then OLS estimates will be biased. Fiscal space estimates are based on the projected values of the set of structural variables included in the primary balance reaction function. Therefore changing these values shifts the predicted fiscal reaction function, affecting the resulting debt limit and fiscal space estimates. Structural changes can be quantified, to assess the impact on fiscal sustainability, and also to identify which structural improvements can shift a country's debt dynamics back to a sustainable path. The authors use an example of increasing institutional capacity (proxied by a political stability index) to the maximum value, and find that the available fiscal space increases for almost all economies.

In terms of limitations, our estimates do not take into account liquidity/rollover risks. Theoretically, the considerations only encompass a one-period model. Longer-term debt and/or stochastic endogenous growth are also not considered in the paper, but mentioned as an avenue for future research. Our sample period (1969-2007) also does not take into account post-GFC data. There is debate in literature whether secular stagnation has occurred and the world has moved to a new norm of lower growth (Summers, 2013). If this is true, any relationships drawn in this paper may have changed after the GFC with lower levels of growth, especially in regards to our growth and fiscal space estimates. Future studies may be able to provide a comparison to pre-and-post GFC debt relationships.

Our fiscal reaction function regressions have the issue of imposing slope homogeneity on counties in their primary balance reaction, enforced through the panel estimation methods. With a limited sample of countries, some with higher debt levels unobserved, the assumption is that all countries behave similarly in terms of 'fiscal fatigue'. This may make sense when examining all advanced economies, with supposedly similar characteristics. There are no countries where the primary balance can increase indefinitely, if for no other reason than that the primary surplus is unable to exceed GDP. This issue is discussed below, and in Appendix Section 9.2.4.

One of their main assumptions in the initial formation of the model (Ghosh et al., 2013) is that the growth rate of real GDP (g) is exogenous and constant in the following budget constraint:

$$d_{t+1} - d_t = (r_t - g)d_t - s_{t+1} \tag{24}$$

where d is the current-period debt-to-GDP ratio at the end of the period, g is the growth rate of real GDP (assumed constant and exogenous), s is the primary balance (% of GDP), r is the real interest rate on debt accrued in period t and due in period $t+1$. This assumption of exogenous and constant growth is dubious given the wealth of research showing a link between debt and growth. Another assumption made at this initial stage is centred on the risk free interest rate, r^*. The interest rate is endogenous and greater than or equal to the risk free interest rate. r^* is assumed to be exogenous, despite being an increasing function of the probability of default in equilibrium (a self-fulfilling prophecy).

In estimating the primary balance reaction function, an obvious source of bias arises from the endogeneity of the output gap, $ygap_{it}$, to contemporaneous policy shocks, ε_{it}. Specifically, fiscal contractions are often associated with a reduction in the output gap, and if not addressed, this negative correlation would tend to weaken the estimated impact of the output gap on the surplus.

Two other sources of endogeneity stem from the dependence of lagged debt on the past values of the primary balance. First, given that public debt represents the accumulation of primary deficits, the country-specific, time-invariant factors determining primary surplus generation capacity, η_i, will necessarily be negatively correlated with debt levels, d_{it-1} , giving rise to a second source of endogeneity. The third source of endogeneity exists only if there is persistence in the policy shock process, ε_{it}. If idiosyncratic policy shocks are correlated over time, the lagged debt term, d_{it-1}, which is endogenous to the fiscal policy shock in period $t-1$, will also be correlated with the period-t shock, ε_{it}. Like the endogeneity of debt to the country fixed-effects, this source of endogeneity would exert a downward bias on the least squares estimate of the coefficient on lagged debt, since positive realizations of the idiosyncratic shocks would tend to reduce the stock of public debt.

Although not directly generated regressors, our fiscal space estimates are created from the abstract concept of a debt limit, which does rely on the estimation of the underlying fiscal reaction function. Here, fiscal space is the gap between current debt-to-GDP and some maximum sustainable level. The estimates of fiscal space here essentially capture fluctuations of the debt-to-GDP ratio below this debt limit. So while not strictly generated, the components of the debt limit are, so we will describe the issues surrounding generated regressor bias. Pagan (1984) determines that there are three issues surrounding generated regressors: consistency, efficiency, and if valid inferences can be made with standard errors. The conclusion is that most estimates of coefficients on generated variables are consistent, and efficient, but questions are raised regarding standard errors.[52] Overall, we make the assumption that our growth regressions do not suffer from generated regressor bias. Pagan's (1984) specific definition of generated regressor is one which is extracted from another regression as a predictor or a residual. Because our fiscal reaction function is used only to find the debt limit (a 'peg-in-the-sand'), regardless of the underlying specifications, the absolute changes in the debt-to-GDP ratio will remain the same (the only difference being the maximum sustainable level we compare it to).

[52] See Murphy & Topel (1985), and Hoffman (1987) for further discussion of generated regressors.

6. Results

6.1. Fiscal Reaction Function

To examine the robustness of the fiscal reaction function's posited cubic model, we study two variations of the model. Firstly, we run a static model similar to Ghosh et al.'s (2013) function. Next, given the long-run nature of debt, we examine a range of dynamic models which introduce new variables, lags, and leads. The aim of these regressions is to examine whether the features found in Ghosh et al. (2013), the cubic reaction function and auto-correlated errors, are indeed appropriate and not caused by omitted dynamics.

6.1.1. Static Reaction Function

The classical linear regression model is,

$$Y = X\beta + u \tag{25}$$
$$\hat{\beta} = (X'X)^{-1}X'Y \tag{26}$$

where Y is the independent variable, X is a known non-singular matrix, u is an error term, and β is a vector of unknown regressions coefficients. The following assumptions are standard in the literature: linear in parameters, zero mean and normally distributed error [$E(u|X) = 0$, u~N]; no multicollinearity; no heteroskedasticity [$V(u|X) = \sigma^2 I_n$]. $\hat{\beta}$ are the coefficient estimates, and are best linear unbiased under the assumptions

Now suppose we relax the assumption regarding heteroskedasticity and autocorrelation, so $V(u|X) = \sigma^2\Omega_n$, where Ω is a symmetric, positive definite $n \times n$ matrix. $\hat{\beta}$ will still be linear and unbiased, but the Gauss-Markov Theorem does not hold anymore.[53]

It can be shown that, using an estimate $\widehat{\Omega}$ for Ω, and transforming the model, the resulting feasible GLS coefficient estimator in equation (26) becomes,

$$\hat{\beta}^* = \left(X'\widehat{\Omega}^{-1}X\right)^{-1}X'\widehat{\Omega}^{-1}Y \tag{27}$$

which is an asymptotically unbiased, efficient, and consistent maximum-likelihood estimator.

The feasible GLS regression estimation results for the fiscal reaction function relating primary balances to lagged debt (allowing for a cubic function to capture the two inflexion points in the curvature of the response) and various control variables, as well as country-

[53] See Aitken (1935) for further information on Gauss-Markov and GLS transformations.

specific fixed effects, are presented in Table 5.0. Our estimation method is feasible GLS due to the presence of AR(1) errors.[54] Feasible GLS estimation allows for AR(1) autocorrelation within panels (with a common AR coefficient) and heteroskedasticity across panels. "The estimated variance matrix is obtained by substituting the estimator $\hat{\Sigma}$ for Σ where … the residuals used in estimating Σ are first obtained from [an] OLS regression" (Stata, 2017, pg. 10).[55]

Estimates for our sample period (1970–2007) are reported, where each regression includes the sample of 23 economies. Following Ghosh et al. (2013), we include a range of variables such as the output gap to control for the effect of business cycles; the government expenditure gap to measure the effect of temporary fluctuations in government outlays (such as military spending), fuel and non-fuel commodity prices; trade openness; and the average inflation rate in the previous three years to examine the possible effects of inflation (such as bracket-creep effects, or greater fiscal effort to counter the effects of higher interest rates accompanying higher inflation) on the fiscal balance.[56] We extend their fiscal reaction function in Table 6.0 by introducing dynamics in the form of further lags of: debt-to-GDP, primary balance, output-gap, government expenditure; also variables including population growth, rule-of-law, and unemployment.

The institutional variables consist of a political stability index—which is a composite measure of the institutional capacity as well as government stability in a country—with higher values indicating lower risk; a fiscal rules index, which indicates if a country has any type of fiscal rule (balanced budget, expenditure, revenue, or debt) in a given year; and a dummy variable for an IMF program arrangement as a proxy for international influence on fiscal behaviour. We also take into account the demographic structure, and include the current and projected age dependency ratios.

The results in Table 5.0 indicate that the cubic functional form, which captures the increasing but slowing response of the primary balance to lagged debt-to-GDP, holds true and is statistically significant. The coefficient on lagged debt is not statistically significant (with the exception of specification (1)); however lagged debt squared is significant and positive, while lagged debt cubed is significant and negative. This would indicate support for Ghosh et al.'s (2013) findings of a cubic reaction function, as well as supporting the ideas of Mendoza & Ostry (2008) who find that sustainability is less assured in advanced economies when public debt is high than when it is moderate.

[54] Ghosh et al. (2013) use Prais-Winsten estimation. We run both Prais-Winsten and Stata's default feasible GLS on our fiscal reaction function for comparison, and find similar results using both methods.

[55] See Stata's (2017) xtgls manual for further information.

[56] See Roubini and Sachs (1988), Gali and Perotti (2003), Ostry and Abiad (2005), Bohn (2008), and Mendoza and Ostry (2008) for further information regarding fiscal variables of interest.

Table 5.0: Static Fiscal Reaction Function

Estimation Results for Static Fiscal Reaction Function

Sample Specification	(1)	(2)	(3)	(4)
$Debt_{t-1}$	-0.15498***	-0.00888	-0.08639	-0.05761
	(0.042)	(0.021)	(0.070)	(0.051)
$Debt\_Square_{t-1}$	0.00280***	0.00071**	0.00170*	0.00149**
	(0.001)	(0.000)	(0.001)	(0.001)
$Debt\_Cubic_{t-1}$	-0.00001***	-0.00000***	-0.00001**	-0.00001**
	(0.000)	(0.000)	(0.000)	(0.000)
Output Gap	0.48236***	0.30908***	0.44077***	0.32485***
	(0.033)	(0.026)	(0.053)	(0.050)
Government Expenditure Gap	-0.19904***	-0.28193***	-0.18259***	-0.17023***
	(0.035)	(0.028)	(0.047)	(0.036)
Trade Openness			0.14612***	0.12504***
			(0.054)	(0.039)
Age Dependency			-0.07166	-0.03403
			(0.101)	(0.056)
Future Age Dependency			-0.01539	-0.02147
			(0.067)	(0.045)
Inflation			4.62005**	5.39125***
			(2.008)	(1.626)
Oil Prices			9.52879***	9.46482***
			(3.244)	(2.672)
Non-fuel Commodity Prices			3.00490	4.42576
			(8.362)	(5.909)
Political Stability			0.06776**	0.07430***
			(0.030)	(0.022)
IMF Arrangement			-1.14205	-0.75855
			(0.999)	(0.921)
Fiscal Rules			0.29999	0.57209**
			(0.347)	(0.247)
Investment				0.16196***
				(0.053)
Banking Crisis				-0.50470**
				(0.220)
Primary $Balance_{t-1}$		0.65879***		
		(0.022)		
Constant	3.45750***	0.65484	-5.08287	-11.33952***
	(0.961)	(0.503)	(4.227)	(3.557)
Observations	642	642	496	491

Robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.
Note: Dependent variable is primary balance to GDP ratio (percent); all model specifications are estimated using feasible GLS and include country fixed-effects. Estimation allows for AR(1) autocorrelation within panels (with common AR coefficient) and heteroskedasticity across panels.

The estimated coefficients on the other variables also follow intuition from previous literature. The primary balance responds positively to the output gap indicating its cyclical nature (Tereanu et al., 2014). The government expenditure gap captures temporary increases in government outlays, which affect the primary balance negatively. More open economies tend to exhibit better fiscal performance, as do countries with higher political stability, and fuel exporters when oil prices rise. Countries with higher inflation also have a larger primary balance, potentially due to inflation eroding the real value of debt. Also in times of banking crisis, countries exhibit worse fiscal performance.

In regards to our posited cubic fiscal reaction function, the results in Table 5.0 show robust support over a variety of empirical specifications. Regression (1) includes only the set of main structural variables of interest: fiscal space, fiscal space squared, fiscal space cubed, output gap and government expenditure gap. Regression (2) includes an added dynamic term for feedback in the form of lagged primary balance as a regressor. This term is not included in the subsequent two regressions as we consider it a dynamic term, and is included in our dynamic specifications. It was included here as a robustness check, to see if simply allowing for feedback changed the proposed cubic specification. Its inclusion and significance does not change our findings, but warrants inclusion of a feedback term in our dynamic models. Regressions (3) and (4) add a richer set of primary balance determinants, with (3) being Ghosh et al.'s (2013) specification, and (4) our own adding government investment and a dummy variable for banking crisis within a country. The results are the same regardless of variable choice, with lagged debt squared having a positive significant coefficient, and lagged debt cubed having a negative significant coefficient. This would support the theory of 'fiscal fatigue'.

The empirical model includes fixed effects for individual countries to capture unobserved differences in primary balance. However, these results are based on panel estimates of the primary balance reaction to lagged debt. In line with the panel literature, this raises questions about slope homogeneity. The issue is even more complex here because the fiscal 'fatigue' or cubic reaction function is posited to hold over the full range of debt-to-GDP, whereas individual countries may only be observed over a portion of that range. Therefore, this relies on the assumption that if the debt of countries that are not observed at high debt levels were to increase to said high levels, they would behave similarly in terms of fiscal fatigue to countries that are observed at higher levels. While this is a conjecture, we support the assumption by estimating the fiscal reaction function for a sub-sample of high debt countries and low debt countries, respectively.[57] Our results in Table 19.0 are based on a regression limited to a sub-sample of only countries with high debt observations (specification (1)), and only countries with low debt observations (specification (2)). We find the cubic reaction function holds for countries at these high levels of debt, suggesting that

[57] See Appendix Section 9.2.4 for results and discussion on slope homogeneity.

our assumption would be reasonable if advanced economies do indeed have similar behaviours with respect to debt. We also find that low debt countries primary balance reacts positively to rising debt, reinforcing part of our theory, but unable to paint the picture of fiscal fatigue that the high debt countries show.

We also run regression specification (4) for individual time series. The results are presented in Appendix Section 9.2.3, Tables 15.0 to 18.0.[58] Our country findings for individual time series are mixed, which could simply be a result of limited individual-country observations for time series. However, despite this, the majority of individual regressions either indicate a positive debt-squared term with negative debt-cube term (varying significance), or an overall negative debt-squared. We leave the time-series avenue of fiscal reaction functions for future research.

6.1.2. Dynamic Reaction Function

Table 6.0 shows our dynamic specifications, extending upon the work in Ghosh et al (2013). Overall, we find support for the posited cubic reaction function (with 'fiscal fatigue'). We have extended the static model (Table 6.0 specification (1), which reproduces the results for the model in Table 5.0, specification (4)) in a number of ways. Firstly in specification (2), we include further lags of the three relevant debt variables, as well as lags of both output gap and government expenditure gap. This is done due to the long-run nature of a government's IBC and decision making. In specification (3) we add additional determinants which theory suggests could also influence the primary balance. These variables include population growth (Whitehead, 2006), rule-of-law index (Carnot, 2014), unemployment (Philip & Janssen, 2002), and the lagged primary balance (allowing for feedback effects, see section 6.1.1). In specification (4) we add another set of lags for our three relevant debt variables.[59]

Over all specifications, the coefficient on $debt_{t-1}$ remains statistically insignificant in specification (1), (3), and (4). The coefficient on $debt_{t-1}$ squared is significant and positive over all specifications. The coefficient on $debt_{t-1}$ cubed is significant and negative over the specifications. The results in Table 6.0 indicate that the cubic functional form, which captures the increasing but slowing response of the primary balance to lagged debt-to-GDP, holds true and is statistically significant and robust to specification choice and added dynamics.

[58] Please note the tables have been condensed as the time-series regressions are used simply for robustness checking.

[59] No further lags were added. Given our process was specific-to-general, and the last set of lags is insignificant, we stopped here.

Table 6.0: Dynamic Fiscal Reaction Function

Estimation Result for Dynamic Fiscal Reaction Function				
Specification	(1)	(2)	(3)	(4)
Debt$_{t-1}$	-0.05761	-0.3260***	-0.14659	-0.15052
	(0.051)	(0.084)	(0.111)	(0.116)
Debt_Square$_{t-1}$	0.00149**	0.00350***	0.00274**	0.00259*
	(0.001)	(0.001)	(0.001)	(0.001)
Debt_Cubic$_{t-1}$	-0.00001**	-0.00001***	-0.000011**	-0.000009*
	(0.000)	(0.000)	(0.000)	(0.000)
Output Gap	0.32485***	0.33316***	0.31123***	0.31706***
	(0.050)	(0.052)	(0.057)	(0.056)
Government Expenditure Gap	-0.17023***	-0.30456***	-0.3254***	-0.32090***
	(0.036)	(0.040)	(0.047)	(0.046)
Trade Openness	0.12504***	0.07715**	0.07911**	0.06001
	(0.039)	(0.038)	(0.039)	(0.038)
Age Dependency	-0.03403	-0.00119	-0.03970	-0.01736
	(0.056)	(0.053)	(0.074)	(0.073)
Future Age Dependency	-0.02147	-0.08768**	-0.08202**	-0.07785**
	(0.045)	(0.040)	(0.038)	(0.038)
Inflation	5.39125***	-2.46652	-5.48083	-5.87744
	(1.626)	(2.485)	(6.962)	(7.151)
Oil Prices	9.46482***	9.48210***	2.70534	1.95104
	(2.672)	(2.000)	(2.041)	(2.013)
Non-fuel Commodity Prices	4.42576	-3.51983	4.62230*	4.29043*
	(5.909)	(2.819)	(2.518)	(2.451)
Political Stability	0.07430***	0.04150*	0.00946	0.01653
	(0.022)	(0.023)	(0.026)	(0.026)
IMF Arrangement	-0.75855	-1.34412	0.02157	0.37533
	(0.921)	(0.896)	(0.715)	(0.711)
Fiscal Rules	0.57209**	0.65432***	0.35094	0.49889*
	(0.247)	(0.242)	(0.259)	(0.257)
Investment	0.16196***	0.10294**	0.13483***	0.12318**
	(0.053)	(0.052)	(0.051)	(0.050)
Banking Crisis	-0.50470**	-0.47820**	-0.46746	-0.45367
	(0.220)	(0.192)	(0.378)	(0.364)
Debt$_{t-2}$		0.29094***	0.22208*	0.24401
		(0.081)	(0.124)	(0.196)
Debt_square$_{t-2}$		-0.00225**	-0.00352**	-0.00228
		(0.001)	(0.002)	(0.002)
Debt_cube$_{t-2}$		0.00001***	0.000015**	0.000007

		(0.000)	(0.000)	(0.000)
Output Gap$_{t-1}$		0.04181	-0.2209***	-0.21130***
		(0.046)	(0.053)	(0.055)
Government Expenditure Gap$_{t-1}$		-0.00998	0.00434	0.00019
		(0.035)	(0.049)	(0.048)
Population Growth			0.000001**	0.000001**
			(0.000)	(0.000)
Rule of Law			-0.06962	-0.10615
			(0.752)	(0.747)
Unemployment			0.07978	0.05100
			(0.065)	(0.065)
Primary Balance$_{t-1}$			0.51339***	0.54730***
			(0.048)	(0.049)
Debt$_{t-3}$				0.00534
				(0.137)
Debt_square$_{t-3}$				-0.00142
				(0.002)
Debt_cube$_{t-3}$				0.00001
				(0.000)
Constant	-11.3395***	-3.32837	-3.35148	-4.11038
	(3.557)	(3.454)	(5.173)	(5.186)
Observations	491	295	273	272

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.
Note: Dependent variable is primary balance to GDP ratio (percent); all model specifications are estimated using feasible GLS and include country fixed-effects. Estimation allows for AR(1) autocorrelation within panels (with common AR coefficient) and heteroskedasticity across panels.

The majority of results for the original determinants are in line with our static model (Table 5.0). The primary balance still responds positively to the output gap. More open countries display better fiscal performance, as do countries with higher investment. Results for oil exporters when oil prices rise, countries with higher political stability, and banking crisis, are also similar to the static specification, but become insignificant as more dynamics are introduced. This loss of significance could be due to a number of reasons, such as correlation with the additional explanatory variables, or that the additional variables have a much stronger effect on the primary balance (however, this is just conjecture).

Although the main conclusion is that the cubic fiscal reaction function still holds, there are some interesting differences to the static model. Firstly, the effect of lagged output gap o the primary balance is negative and significant in specifications (3) and (4). This means that an increase in the output gap in the previous period affect the primary balance negatively. This is the opposite of the coefficient on the contemporaneous output gap across all specifications. This negative coefficient seems counter-intuitive, but could be plausible if the stimulating effects of government spending on demand, consumption, and investment, as well as increased tax revenue, outweighed the direct negative effect of spending. Because of the dynamic nature of the model, this may be what is witnessed over a long-run period.

Secondly, the effect of future age dependency is negative and statistically significant in specifications (2) to (4). A high dependency ratio can cause serious problems for a country if a large proportion of a government's expenditure is on health, social security and education, which are most used by the youngest and the oldest in a population. The fewer people of working age, the fewer the people who can support schools, retirement pensions, disability pensions and other assistances to the youngest and oldest members of a population, often considered the most vulnerable members of society. Combined with an ageing population in the future, there will be more people claiming social security and pensions, so this negative effect on fiscal performance could also be plausible.

Regarding our additional explanatory variables included in the regression, we find population growth affects the primary balance positively. If the population growth rate increases, this means more people in the economy, and therefore it is easier for governments to collect tax revenue etc. as a means of funding spending. Rule of law and unemployment also appear to have statistically insignificant effects. In this case, because all countries are advanced, and the regressions account for fixed effects, if these countries have innate similar characteristics such as similar rule-of-law status, then this would already be accounted for.

When adding in additional lags of debt-to-GDP, debt-to-GDP squared, and debt-to-GDP cubed to account for feedback effects, we get an interesting result – the coefficients on the additional lags or the opposite to their initial counterparts. However, when adding further lags, the coefficient signs on the twice lags variables appear the same as the initial variables. This could potentially indicate a partial adjustment phenomenon where debt has mean-reverting behaviour. Feedback effects are also allowed through the inclusion of lagged primary balance as a lagged dependent variable. We see that a higher primary balance in the previous period results in a higher primary balance in the current period, showing there is potentially some persistence.

However, questions could also be raised regarding the long-run relevance of this fiscal fatigue relationship. Despite adding additional lags of fiscal space, the short-run relationship appears to hold. To find the corresponding long-run relationship, we can transform the model in the following manner,

$$PB_{i,t} = \beta_1 D_{i,t-1} + \beta_2 D_{i,t-1}^2 + \beta_3 D_{i,t-1}^3 + \gamma X_{i,t}' + \mu_i + \epsilon_{i,t} \tag{28}$$

$$\begin{aligned} PB_{i,t} = {} & \beta_1 D_{i,t-1} + \beta_2 D_{i,t-1}^2 + \beta_3 D_{i,t-1}^3 + \theta_1 D_{i,t-2} + \theta_2 D_{i,t-2}^2 + \theta_3 D_{i,t-2}^3 \\ & + \delta PB_{i,t-1} + \cdots \end{aligned} \tag{29}$$

$$PB_t(1-\delta L) = (\beta_1 L + \theta_1 L^2)D_t + (\beta_2 L + \theta_2 L^2)D_t^2 + (\beta_3 L + \theta_3 L^2)D_t^3 + \cdots \tag{30}$$

$$Long\ Run: PB(1-\delta) = (\beta_1 + \theta_1)D + (\beta_2 + \theta_2)D^2 + (\beta_3 + \theta_3)D^3 + \cdots \tag{31}$$

$$Long\ Run: PB = \frac{(\beta_1 + \theta_1)}{(1-\delta)}D + \frac{(\beta_2 + \theta_2)}{(1-\delta)}D^2 + \frac{(\beta_3 + \theta_3)}{(1-\delta)}D^3 + \cdots \tag{32}$$

where β's are the coefficients on the initial debt-to-GDP lags (e.g. β_1 is associated with Debt_{t-1} in our regressions), θ's are the coefficients on the second lags of debt-to-GDP, and δ is the coefficient on lagged primary balance in our regression above. The third lags of debt-to-GDP are insignificant, so we focus on the specification with two lags. According to our model in Table 6.0 specification (3), we generate the following long-run equation,

$$\begin{aligned} PB = {} & \frac{(-0.14659 + 0.22208)}{(1-0.51339)}D + \frac{(0.00274 + -0.00352)}{(1-0.51339)}D^2 \\ & + \frac{(-0.000011 + 0.000015)}{(1-0.51339)}D^3 + \cdots \end{aligned}$$

$$PB = (0.1551)D + (-0.00016)D^2 + (0.0000082)D^3 + \cdots$$

According to our equation above, the long-run relationship between primary balance and debt-to-GDP may not display the cubic fiscal fatigue behaviour that our short run dynamic model does.[60] Combining the coefficients in our dynamic model and transforming into long-run primary balance (albeit back-of-the-envelope) , we find a positive coefficients on debt-to-GDP and negative coefficient on debt-to-GDP squared, while the coefficient on debt-to-GDP cubed is slightly positive. This would indicate that over the long-run primary balance does increase in response to rising debt-to-GDP (Bohn, 2007), but then subsequently decreases, displaying a negative quadratic relationship. However, our calculations here are back-of-the-envelope, and investigating a true long-run relationship would require further analysis, specifically with regards to lag specification sensitivity and the effect of auto-regressive corrections via GLS.

Overall, we find the cubic fiscal reaction function holds, regardless of inclusion of dynamic

[60] We used specification (3) here due to the insignificance of respective debt_{t-3}'s lags introduced in specification (4)

or other determinants of primary balance, at least in the short-run. The long-run equilibrium would require further analysis, but given our calculations are simple, we place weighting strictly on the results from our regressions. From this, following the methodology outlined previously, we incorporate both the dynamic and static fiscal reaction function coefficient estimates into an equation with the respective country interest rate-growth rate differential, to calculate the debt limit. Then the estimate of fiscal space is simply the gap between the debt limit and debt-to-GDP in a given year.

6.2. Debt Limit and Fiscal Space

Table 7.0 reports the respective model-implied debt limits for each country in our sample. The static debt limits are calculated by using the country's fitted static fiscal reaction function obtained above (reported in Table 5.0, specification (4)) and both the historical and projected market interest rates. Dynamic estimates are also presented, using our dynamic specification (Table 6.0, specification (4)). We have also included Ghosh et al.'s (2013) results for robustness comparison.

Table 7.0: Advanced Country Debt Limits

	Ghosh et al. (2013)		Static		Dynamic	
	(1)	(2)	(3)	(4)	(5)	(6)
	Historical	Projected	Historical	Projected	Historical	Projected
AUS	211.7	204.9	221.6	215.7	197.6	191.4
AUT	191.1	193.5	201.9	203.8	-	-
BEL	193.0	174.2	203.4	188.8	-	-
CAN	191.2	178.0	203.9	194.2	155.6	-
DNK	203.6	203.6	213.5	213.5	182.2	182.2
ESP	213.8	166.2	222.4	185.4	200.1	-
FIN	195.3	199.1	207.8	210.9	166.0	173.2
FRA	181.7	189.0	198.9	204.2	-	171.1
GER	185.7	197.1	198.1	207.4	-	-
GRC	201.5	-	212.9	-	191.3	-
IRL	230.3	189.3	236.1	199.5	208.2	-
ISL	198.3	207.9	231.8	238.2	213.4	218.6
ISR	175.2	200.4	193.5	212.4	165.6	191.2
ITA	166.8	-	189.1	-	-	-
JPN	-	-	184.4	180.5	-	-
KOR	227.3	231.4	231.3	235.3	205.2	209.0
NLD	201.6	201.6	210.3	210.3	-	-
NOR	245.3	244.9	250.3	250.0	338.2	338.1
NZL	210.6	200.1	240.8	233.7	221.6	215.9
POR	188.6	145.1	201.6	177.6	166.0	-
SWE	183.0	187.2	199.6	202.6	-	159.9
UK	180.7	201.0	197.3	212.8	158.7	187.1
USA	203.6	193.3	214.5	206.1	195.0	186.4

Note: Table 7.0 displays estimates of country debt limits. Columns 1 and 2 show Ghosh et al.'s (2013) debt limit estimates for comparison. Historical debt limits are calculated using the historical average interest rate-growth rate differential. Projected estimate are calculated using the IMF's forecast interest rate-growth rate differential. Columns 3 and 4 show estimates using our static fiscal reaction function. Columns 5 and 6 show estimates using our dynamic fiscal reaction function. '-' indicates the model is unable to calculate a debt limit.

For our sample there appears to be significant cross-country variation in the implied debt limits, which is a reflection of country-specific fixed effects and differences in interest rate-growth rate differentials. The results indicate that debt limits range from 158 percent of GDP to 250 percent of GDP depending on the country, whether the static or dynamic reaction function is used, and whether historical or projected interest rates are used. Therefore we have a variety of specifications to compare to Ghosh et al.'s (2013) as a benchmark (columns 1 and 2).

Firstly, we will focus on the static model (columns 3 and 4). In a few cases (Greece and Italy), depending on the interest rate used, no estimate of the debt limit is reported. This is because given these countries' estimated fiscal reaction function and projected interest rate-growth rate differential, debt-to-GDP would not be expected to converge to a finite steady-state, and therefore there is no maximum debt level below which convergence occurs. Therefore, for Greece and Italy, the debt limit does not exist when using the projected interest rates (lower than the historical). As such, financing difficulties that a number of these countries (e.g. Greece) have been facing should not be surprising because, based on their past fiscal behaviour but combined with current interest rate-growth rate differentials, their debt-to-GDP would not be expected to converge.

Our debt limit estimates appear to be more lenient then Ghosh et al (2013), having larger point estimates using both historical and projected interest rates. However, they appear to follow a similar trend in regards to which countries have a higher or lower point estimate. For example, countries with higher debt limits include Australia, Spain, Ireland, Iceland, South Korea, Norway, and New Zealand, which are the same as Ghosh et al. (2013) bar the addition of Iceland. Countries with relatively lower debt limits are also similar to Ghosh et al. (2013), for example, Austria, Belgium Canada, France, Germany, Israel, Italy, Japan, Portugal, Sweden, and the United Kingdom. When moving from historical to projected interest rates, our point estimates also change in a similar manner. Most countries tend to have a lower debt limit when using projected interest rates. There are a few outliers to this trend though, with Denmark and the Netherlands experiencing no change. Finland, France, Germany, Iceland, Israel, South Korea, Sweden, and the United Kingdom increase their debt limits. This trend is even evident in the dynamic model (for countries that have both historical and projected point estimates), where Denmark has no change, and Finland, Iceland, Israel, South Korea, and the United Kingdom all experience an increase in the debt limit.

Our dynamic model suffers from the inability to estimate the debt limit for a number of countries. Because we base our empirical method on Ghosh et al.'s (2013) we are able to directly compare results. However, this means we are also subject to the model's weaknesses. As outlined above (Section 5.2) the model is unable to estimate a number of country debt limits, especially so when using our dynamic model. This result can occur for two reasons. This can occur because, given the country's estimated fiscal reaction function and interest rate-growth rate differential, public debt is not expected to converge to a finite steady-state. Therefore, there is no maximum level of debt below which convergence occurs (i.e. no debt limit). However, because the cubic fiscal form holds for both the static and dynamic models, we will place more emphasis on the static model, with the dynamic results being used as a robustness check.

The debt limit estimates are based on the projected values of the set of structural variables included in the fiscal reaction function. Any changes in these would shift the predicted function and affect the debt limit. For our study this means that the dynamic model is not given as much weight as the static for drawing conclusions on empirics, nor is our long-run simple calculation used. The dynamic model's estimates on lagged debt, lagged debt squared, and lagged debt cubed, are larger, slightly larger, and approximately the same, respectively. Because the coefficients are larger for lagged debt and lagged debt square, this may mean that the function exhibits a prolonged positive rise before the 'fatigue' of the primary balance sets in, leading to our inability to estimate debt limits for countries such as Austria, Belgium, Italy, Japan, and the Netherlands. Where countries have only one estimate (either historical or projected) these appear to be in line with our static model. The dynamic model appears to be generally more pessimistic about debt limits for the countries that it can estimate, with lower point estimates across the board relative to our static model. A number of countries move closer to Ghosh et al.'s (2013) estimates compared to our more lenient static model, including Iceland, Israel, and New Zealand. For the countries with both historical and projected estimates, they move (either increase or decrease) from historical to projected with similar variation to the static model.

Although our dynamic model is limited and is only being used for comparison purposes, this does highlight a useful feature of the approach for further research. By changing the underlying estimation it would allow us to quantify the impact of any structural change on fiscal sustainability, or in terms of policy, identify what structural improvements can put the countries' debt dynamics on a sustainable path. However, we abstract away from this and keep the dynamic model as a robustness tool for Section 6.3. Plus, regarding our long-run calculations, treating the $t-1$ debt lags in isolation follows the method of Ghosh et al. (2013) but may be questionable. Further investigation is required to introduce debt interactions in our underlying model, to further investigate the cubic pattern.

Because we essentially have two different estimates of the debt limit (historical and projected), we will have two different measures of fiscal space. We make the assumption that the debt limits do not change over time (e.g. a country's historical debt limit has been *the* debt limit from 1969 to 2008), and create a fiscal space data set for our sample of 23 countries, incorporating their country-specific debt limits, and respective debt-to-GDP levels. This assumption seems plausible if we hold our underlying theory to be long-term, and have not had any shocks to, for example, interest rates and growth rates. In reality, however, the debt limit may change over time, specifically in times of high growth or financial crisis. Incorporating a moving debt limit into our model requires further work beyond the scope of this investigation.

Following Ghosh et al.'s (2013) methodology and our previously outlined theory, we truncate fiscal space by imposing a minimum of zero. However, negative estimates do occur in our dynamic model, which poses the question – what if we allowed negative fiscal space and relaxed the assumption that countries get shut out of the money market beyond their debt limit? Again, this is an avenue for further research. Using our fiscal space data set, we now examine the fiscal space-growth nexus.

6.3. Fiscal Space and Growth

Our main contribution to the literature is examining the relationship between fiscal space and growth. To our knowledge, we are the first in the literature to do so. This makes the contribution two-fold. Firstly, we are investigating a new avenue with public debt, which could have policy implications for advising governments in regards to how sustainable their policies may be, and what this could mean for countries' growth. Secondly, because fiscal space is growing in popularity in the literature, investigating a fiscal space-growth relationship allows us to determine whether fiscal space is a useful metric for assessing government spending, in the context of comparing our results to debt-growth theory.

Again, we focus on the static fiscal space estimates, with the dynamic estimates used as a robustness check.[61] We run a number of different empirical specifications to examine the relationship between fiscal space and growth. Our specifications are determined by our variable of interest – fiscal space, as well as prominent empirical growth literature, and incorporates modern empirical techniques (Moral-Benito, 2010, Chudik et al., 2013; Neal, 2015). Estimates for our sample period (1970-2007) are reported. Again, although more recent data is available, for consistency we use the same sample period as our fiscal reaction functions.

Use of panel data in estimating common relationships across countries is particularly appropriate because it allows the identification of country-specific effects that control for missing or unobserved variables. However, micro-economists have generally been more avid users of panel data, and, thus, existing panel data techniques have been devised and tested with the typical dimensions of a microeconomic panel dataset (i.e. a large number of cross-sectional units, a small number of time-series observations) in mind.

Panel data have two main attractions: the ability to control for time-invariant factors, as well as the ability to determine the direction of a causal relationship. Controlling for

[61] See appendix Section 9.3 onwards

unobservable can be accomplished with fixed effects, while the most popular causal approach is to examine a cross-lagged panel model, which achieves causal estimates based on the directional influence variables have on each other over time.[62] However, attempting to combine the two can lead to serious estimation problems.[63] The most popular estimation method has been GMM, which relies on lagged variables as instruments, or other Maximum Likelihood methods.

In the static case, if the coefficients differ randomly, all four procedures (pooling, aggregating, averaging group estimates, and cross-section regression) give unbiased estimates of coefficient means. "In the dynamic case, when the coefficients differ across groups, pooling and aggregating give inconsistent and potentially highly misleading estimates of the coefficients, though the cross-section can provide consistent estimates of the long-run effects" (Pesaran & Smith, 1995, pg.1). Therefore, to ensure that our results above (see Tables 8.0 and 9.0) are robust and accurate, we adopt relevant recently developed empirical techniques to estimate our growth model.

Unobserved common factors in the panel can lead to correlation in the errors across panel units, as well as correlation between the errors and regressors themselves. If left uncorrected, there is the potential to cause severe bias and inconsistency in the coefficients (Gormley et al. 2013). A number of panel time series estimators have been proposed that are robust to both cross-sectional dependence as well as slope heterogeneity. Pesaran (2006) introduced Common Correlated Effects (CCE) estimation, which approximates the projection space of the unobserved factors with cross-sectional averages of the variables, and accounts for slope heterogeneity by using mean group instead of pooled regression.[64]

The individual regressions take the form of:

$$y_{it} = \beta_i x_{i,t} + \delta_{xi}\overline{x}_t + \delta_{yi}\overline{y}_t + e_{it} \tag{33}$$

where y_{it} is the dependent variable for country i and time t, $x_{i,t}$ includes the regressors for country i and time t, β_i is the coefficient on the regressors, e_{it} is an error term, $\overline{x}_t = N^{-1}\sum_1^N x_t$ is the cross-sectional average of the regressor(s), and $\overline{y}_t = N^{-1}\sum_1^N y_t$ is the cross-sectional average of the dependent variable.

However, the standard CCE method has been shown to have bias in the slope coefficient of a lagged dependent variable in small samples and is therefore unsuitable in models, such as growth regressions, with a lagged dependent variables (Everaert & De Groote, 2016). The CCE model was then extended to a Dynamic CCE model (DCCE) allow for models with weakly

[62] See Kenny (2005) for information on cross-lagged panel design.
[63] See Wooldridge (2010) or Baltagi (2008) for a review of dynamic panel data models.
[64] See Pesaran (2006); Chudik & Pesaran (2015) for a full proof of the properties of CCE.

exogenous regressors and lagged dependent variables. The DCCE estimation uses lags of the cross-sectional averages in the regression:

$$y_{it} = \phi_i y_{i,t-1} + \beta_i x_{i,t} + \sum_{p=0}^{p_T} \delta_{xip} \overline{x}_{t-p} + \sum_{p=0}^{p_T} \delta_{yip} \overline{y}_{t-p} + e_{it} \tag{34}$$

where p_T is the number of average cross-section lags included in the regression. Chudik & Pesaran (2015) show that by adding cross section means, the estimator β_i gains consistency. Table 8.0 uses this DCCE specification.

Neal (2015) extends upon CCE in panel data estimation, replacing Least Squares (OLS) in the individual specific regression with GMM, and uses lagged observations of the variable to form instrument sets. In static models, the regression becomes:

$$y_{it} = \beta_i \hat{x}_{i,t} + \delta_{xi} \overline{x}_t + \delta_{yi} \overline{y}_t + e_{it} \tag{35}$$

and with dynamic models:

$$y_{it} = \phi_i \hat{y}_{i,t-1} + \beta_i \hat{x}_{i,t} + \sum_{p=0}^{p_T} \delta_{xip} \overline{x}_{t-p} + \sum_{p=0}^{p_T} \delta_{yip} \overline{y}_{t-p} + e_{it} \tag{36}$$

where $\hat{y}_{i,t-1}$ and $\hat{x}_{i,t}$, the instrument sets, are populated with any exogenous regressors, the cross-sectional averages, and lags of the endogenous regressors and dependent variable. This “allows CCE to be robust to endogenous regressors…, and also significantly improves the small sample properties of the estimator in dynamic panel data models regardless of whether the regressors are strictly exogenous, weakly exogenous, or endogenous” (Neal, 2015, pg. 2). Table 9.0 uses Neal’s (2015) CCE with GMM estimation method. Following Neal (2015) we use the lagged observations for the instrument set; lags of both fiscal space and fiscal space squared are used. Although this follows common macro-economic literature in using lags of the independent variables as regressors (see, for example; Barro, 2001), these may be weak instruments. Neal (2015) uses lagged variables as instruments in a Monte-Carlo simulation and finds CCE-GMM to be robust; however, as GMM is sensitive to instrument choice, we are wary that CCE-GMM growth regressions may need further scrutiny and development in empirical applications.

The CCE method has been shown to be robust to cross section dependence of errors, possible unit roots in factors, and slope heterogeneity. This final set of regressions incorporates both the fiscal space and growth theories, with modern dynamic panel analysis techniques. The independent variable is Δln(GDP per capita)$_{t-1}$, following equation (23). The results are shown in Tables 8.0 and 9.0 below. Our first three specifications (Table 8.0) are a standard CCE OLS regressions. However specifications (4) to (9) are dynamic CCE, where we have an included an implicit lag of the dependent variable as per equation (34). Dynamic CCE adds lags to the dependent variable cross-sectional averages to ensure consistency, as in Chudik & Pesaran (2015). The remaining specifications in Table 9.0 are estimated using Neal's (2015) CCE GMM to account for potential endogeneity in the regressors.[65] The estimates are mean group, rather than pooled, and hence are the average of the relevant coefficients across all countries.

As outlined in Sections 4 and 5.4 respectively, all estimations are run using the sample period of 1969-2007. The motivation for this is the GFC in 2008 and the resulting change in fiscal policy. A number of countries, such as the US, Australia, UK, and those in the EU, followed Keynesian theory and enacted fiscal policy stimulus to boost their lagging economies after the GFC. This stimulus often included increased government spending and tax cuts, followed by the movement towards fiscal austerity in later years. The global change in fiscal policy may have altered any previous long-run fiscal relationships. For consistency with our fiscal reaction functions, and to abstract away from the stark changes seen post GFC, we limit our sample to between 1969-2007.[66] Future analysis could include the use of a structural break test, to determine any changes post GFC.

We also run GLS regressions (see Appendix Section 9.3) as a means of assessing the robustness of our CCE and CCE-GMM results. However because our main results use empirical techniques more suited to the nature of the data (e.g. non-stationarity, heterogeneous units, and cross-sectional dependence), we give Tables 8.0 and 9.0 more weighting.[67]

[65] Please refer to Neal (2016) for information on the Stata regression module; *xtcce*. The Stata command used to run the regressions has recently been developed by Neal (2016) to implement Pesaran's (2006) CCE, Chudik & Pesaran's (2015) DCCE, and Neal's (2015) 2SLS/GMM methods.

[66] We briefly examine regressions inclusive of 2008-2015 data, but find insignificant results.

[67] Our GLS regressions do not show support for our posited quadratic theory.

6.3.1. Dynamic CCE Growth Model

Table 8.0: Dynamic CCE Fiscal Space and Growth

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Fiscal Space	0.00925	0.00925	0.0142	0.01593**	0.02408*	0.02408*	0.02691**	0.02437**	0.03569*
	(0.016)	(0.016)	(0.014)	(0.006)	(0.014)	(0.014)	(0.013)	(0.013)	(0.017)
Fiscal Space squared	-0.00002	-0.00002	-0.00006	-0.00007**	-0.00008*	-0.00008*	-0.00013**	-0.00011**	-0.0002*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Inflation		0.00026	-	-	-	-	-	-	-
		(0.000)							
Output Gap			0.02062*	0.02086**	0.01287	0.01287	0.02540**	0.02305**	0.01857
			(0.012)	(0.010)	(0.016)	(0.016)	(0.011)	(0.011)	(0.012)
Initial GDP per capita				-0.06051***	-0.04846**	-0.04846**	-0.04359**	0.00000	0.00000
				(0.020)	(0.025)	(0.025)	(0.022)	(0.000)	(0.000)
Investment$_{t-1}$					-0.00271	0.01284	0.0237***	0.0251***	0.01755**
					(0.012)	(0.012)	(0.009)	(0.009)	(0.009)
Population Growth						2.40×10^{-6}	2.56×10^{-6}	2.17×10^{-6}	1.13×10^{-6}
						(0.000)	(0.000)	(0.000)	(0.000)
Education							0.00235	-0.00303	-0.00610
							(0.007)	(0.009)	(0.009)
Life Expectancy								-0.01263	0.00150
								(0.014)	(0.002)
Government Consumption$_{t-1}$									-0.02120
									(0.018)
Constant	1.52282	1.43407	3.38287**	4.79385***	5.79233***	5.79233***	6.40233***	7.26343***	7.85937***
	(1.407)	(1.402)	(1.521)	(1.059)	(1.306)	(1.306)	(1.722)	(1.779)	(1.988)
Observations	699	699	699	699	668	631	631	631	601

Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1 Note: All model specifications are estimated using dynamic CCE mean group estimation, which add 4 lags of cross-sectional averages to the regressions. The dependent variable is Δln(GDP per capita). The fiscal space data is the 'historical' and 'static' series, which is our fullest data set. The regressions are estimated using a sample period of 1970-2007. All specifications also includes time fixed effects. Initial GDP per capita is ln(GDP per capita)$_{t-1}$. Using Stata's XTCCE command, inflation is omitted from specification (3) onwards due to multicollinearity. Please see Appendix (Section 9.1) for further information on data.

We begin our specifications in Table 8.0 ((1) and (2)) with a simple regression, similar to Chudik et al. (2013) with fiscal space and inflation as explanatory variables. The dependent variable is Δln(GDP per capita), and Δln(GDP per capita)$_{t-1}$ is included as a regressor based on equation (31). Across the majority of our DCCE model specifications we find similar results in regards to fiscal space. These point estimates of coefficients in Table 8.0 support our hypothesised quadratic effect of fiscal space on growth. From specification (4) to (9), the coefficient on fiscal space is positive and significant. Also from specification (4) to (9) the coefficient on fiscal space squared is negative and significant.

Though we find support for our hypothesised theory in terms of the signs of the coefficients, this is only one aspect of the growth regression. To build on this, we introduce additional explanatory variables suggested by the growth literature into the DCCE framework. Firstly, we implement a Solow model framework ((3) to (5)), controlling for the 'initial' GDP per capita (Bond et al., 2001), output gap (Pesaran, 2007), and savings/investment. Next, we introduce further variables based on a Barro model framework ((6) to (8)). Here we control for life expectancy, education, population growth (Durlauf et al., 2001) as proxies for human capital. We also control for government consumption, to investigate whether it matters how a government distributes its spending.

Controlling for these reinforces our quadratic hypothesis empirically, with the positive coefficient on fiscal space and negative coefficient on fiscal space squared in the majority of specifications. The remaining control variables have differing significance depending upon specification. For example, the coefficient on initial GDP per capita is significant and negative for specification (4) onwards. This supports the basic convergence intuition – countries with lower initial GDP will grow at a fast rate and 'catch up' to countries with higher initial GDP. However, this negative coefficient becomes insignificant in (8) and (9). Investment, as a proxy for savings, has an insignificant coefficient in (5) and (6). Form models (7) to (9) investment has a positive significant coefficient indicating higher savings leads to higher growth, fitting conventional theory. Unfortunately, inflation is omitted from specification (3) onwards in our model, as Stata's XTCCE is unable to estimate the coefficient due to alleged multicollinearity between inflation and other explanatory variables. However, inflation is included (un-omitted) in our GMM CCE regressions and further basic growth and fiscal space regressions (see Appendix Section 9.3). Population growth's coefficient effectively is very (eight decimal places) slightly positive but insignificant. This could be due to growth being in per-capita terms, or if the year-on-year variation in population growth in our sample is not large. The remaining coefficients are mainly insignificant. However, we find more convincing results using Neal's (2015) GMM extension on the CCE model. The CCE GMM method could give better results for a number of reasons, for example, it is better suited towards our relatively small sample, or potential for endogenous regressors.

6.3.2. GMM CCE Growth Model

Table 9.0: GMM CCE Fiscal Space and Growth

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Fiscal Space	0.0063***	0.00664***	0.00645***	0.00656***	0.00609***	0.00606***	0.09141***	0.20470***	0.01403***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.017)	(0.018)	(0.004)
Fiscal Space squared	-0.00001*	-0.00001*	-0.00001***	-0.00001***	-0.00001***	-0.00001***	-0.00018***	-0.00061***	-0.00003**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Inflation		0.39675***	0.39596***	0.34796***	0.38490***	0.38585***	13.60466***	15.75945***	2.43243**
		(0.101)	(0.092)	(0.088)	(0.086)	(0.086)	(2.525)	(2.694)	(1.220)
Output Gap			0.02056***	0.01965***	0.01792***	0.01796***	0.09081***	-0.03442	0.04209**
			(0.002)	(0.002)	(0.002)	(0.002)	(0.030)	(0.024)	(0.017)
Initial GDP per capita				0.0000012***	0.0000012***	0.000001***	-0.00004***	-0.0000037	0.000095
				(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Investment$_{t-1}$					0.00479***	0.00472***	0.02643	0.05149***	0.01498
					(0.001)	(0.001)	(0.039)	(0.018)	(0.010)
Population Growth						5.03×10^{-9}	-5.32×10^{-8}	3.90×10^{-8}**	6.28×10^{-8}
						(0.000)	(0.000)	(0.000)	(0.000)
Education							0.02230***	0.03129***	0.00313
							(0.006)	(0.008)	(0.004)
Life Expectancy								0.00000	0.11056
								(0.000)	(0.244)
Government Consumption$_{t-1}$									-0.05384**
									(0.026)
Constant	10.594***	11.22155***	10.79249***	11.47988***	11.31106***	11.40452***	0.00000	0.00000	0.00000
	(0.752)	(0.758)	(0.806)	(0.748)	(0.737)	(0.747)	(0.000)	(0.000)	(0.000)
Observations	680	680	680	680	680	680	615	615	615

Standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$. The dependent variable is Δln(GDP per capita).All model specifications are estimated using GMM CCE, which is an extension on the dynamic CCE specifications. The regressions are estimated with lags of the regressor variables to form the instrument set. We instrument Fiscal Space, Fiscal Space squared, and Δln(GDP per capita) with their own one-period lags, following Neal (2015). The fiscal space data is the 'historical' and 'static' series, which is our fullest data set. The model also includes time fixed effects. Initial GDP per capita is ln(GDP per capita)$_{t-1}$. Please see Appendix (Section 9.1) for further information on data.

Overall, the DCCE empirical methodology shows evidence of our quadratic theory when accounting for cross-sectional dependence and slope heterogeneity. Neal's (2015) technique builds upon the CCE model to overcome a few issues (outlined above). The estimation process in Table 9.0 follows that of Table 8.0 for consistency. We begin with simplistic initial models of growth, fiscal space, and inflation. Subsequently, we introduce further growth determinants one-by-one as proposed by the Solow and Barro models: output gap, initial real GDP per-capita, investment, population growth, education rates, life expectancy. Our CCE GGM model, like our dynamic CCE model, includes government consumption as an independent variable. This is to take our results a step further and attempt to address the further question of whether it matters for growth if a government's money is directed towards investment or consumption activities.

Across all regression specifications in Table 9.0, we find a positive and significant coefficient on fiscal space, as well as a significant negative coefficient on fiscal space squared. This would indicate that fiscal space and growth are related in-line with our theorised quadratic function. The size of the two fiscal space coefficients vary; however, in general, they look to be smaller than the coefficients estimated by our DCCE regressions. Our main focus, however, is the sign on the coefficients, which do not change across specifications. However, the magnitude of the fiscal space coefficients would have implications for turning points or optimal fiscal behaviour.[68] A smaller positive and negative coefficient could mean our inverse quadratic relationship is effectively distributed over a wider range of fiscal space compared to what our DCCE model would estimate.

Again, our remaining explanatory variables in the CCE GMM models appear to give similar results to our DCCE model. Inflation has a positive, significant coefficient. This could reinforce macroeconomic theory that at least some low positive level of inflation leads to growth (the reason central banks' target low, positive, and stable inflation). The output gap has a significant positive coefficient across specifications, implying cyclicality of output and growth. This cyclicality is where a larger output gap is associated with a higher growth rate, as economies below potential are liable to grow faster (Hausmann et al., 2008). The coefficient on initial GDP per capita is initially positive and significant in (4) to (6), which would go against our DCCE result. However, (7) shows the same result as our DCCE model, with (8) and (9) leaving an insignificant coefficient on initial GDP per capita as more growth determinants are controlled for. We would expect the coefficient for GDP per capita to be negative, as in the DCCE model and growth (conditional convergence) theory. Therefore, the coefficient on initial GDP per capita is an interesting result; however, Moral-Benito (2012) finds that initial GDP is (relatively) not a robust growth determinant.

[68] See Section 6.3.4. for information on turning points. Given equation (37) is comprised of both fiscal space and fiscal space squared coefficients, the magnitude clearly affects the resulting estimate.

The coefficient on investment, as a proxy for savings, is overall positive and significant in (5) to (8). This positive coefficient implies that more investment, or more savings, is associated with larger growth. Although this is a different result to our insignificant DCCE coefficient, it now fits with conventional growth theory that more investment leads to higher GDP and therefore higher growth. Both population and life expectancy appear to have mainly insignificant coefficients when they are included; however theory warrants their inclusion in the regressions nonetheless. The coefficient on education appears to be overall significantly positive in (7) and (8); as a proxy for knowledge or human capital, this result implies that as human capital increases, growth also increases. Finally, when government consumption is introduced in (9), its coefficient appears negative and significant. This would imply that when governments spend money (potentially going into debt), it matters where the money is spent. If money goes into consumption (no long term return), then overall growth prospects are lower relative to, say, if the money is used for investment. Another interesting feature of (9) is that the significance of a number of other variables disappears when government consumption is added. The significance of government consumption could imply that the distribution of where debt is spent plays a major role in growth, besides just the levels of debt or fiscal space themselves. This could be an avenue for future research.

This quadratic result between growth and fiscal space is consistent with our theory that countries with high debt (low/no fiscal space) would achieve higher growth by increasing their fiscal space (paying down debt); however, after a certain point, increasing fiscal space any further would start to negatively impact growth. But this also holds implications for countries with large amounts of fiscal space: that they can afford to decrease their fiscal space (go into debt sustainably) leading to higher growth. Policy-wise, this would indicate that countries like Greece with low fiscal space should consider trying to pay down their debt to achieve higher economic growth, while countries like New Zealand with large amounts of 'wiggle-room' can go into more debt sustainably while increasing growth.

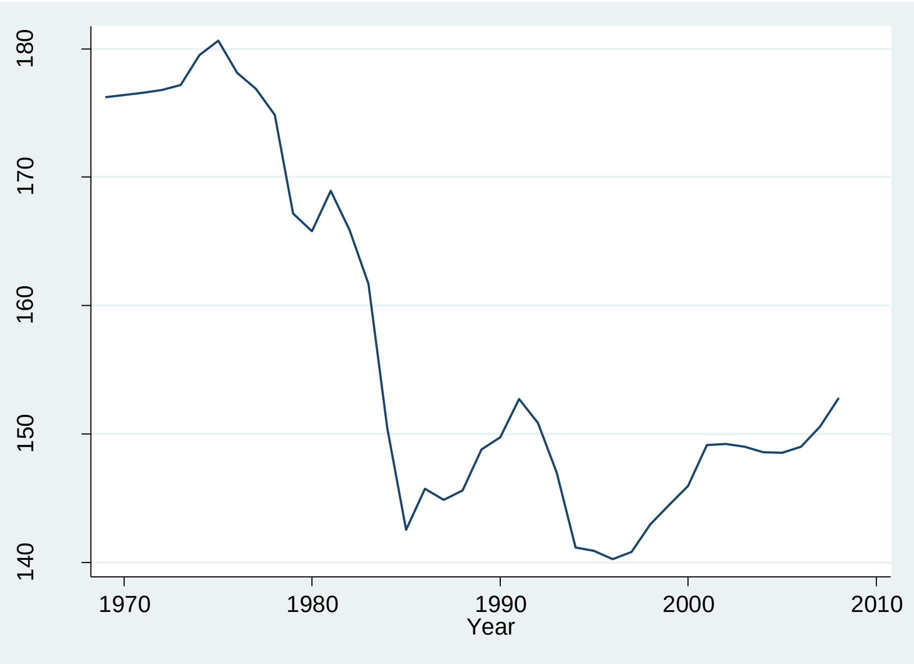
Our results in Table 9.0 indicate empirical support for our fiscal space and growth theory. A quadratic growth function with respect to fiscal space is shown by the significant positive and negative coefficient on fiscal space and fiscal space squared, respectively. This also indicates that fiscal space fits in with popular growth-debt literature, which associates a quadratic function between debt and growth, reinforcing fiscal space as a potentially 'good' measure of debt sustainability. Given the literature's focus on 'turning points' for debt and growth, we begin to bridge the gap between literature strands with a simple back-of-the-envelope calculation to show potential turning points based on our empirics (see Section 6.3.4).

6.3.3. Growth and Fiscal Space Stationarity

To ensure it is appropriate to run standard linear regressions, again we must investigate stationarity. Because the main growth components follow Moral-Benito (2010), our main focus of stationarity is our additional fiscal space variables. In a similar fashion to our preliminary cointegration testing above, we conduct a similar test between growth, fiscal space and fiscal space squared. Sims et al. (1990) determine (all be it in a time-series context) that if cointegration exists, then estimates are consistent. However, if cointegration does not exist then we are required to include first differences of fiscal space in the model.

Fiscal space appears (graphically) to be non-stationary over time (see Figure 13.0 below). This graph shows the cross-sectional panel averages for each given year. Given our non-linear fiscal space and growth results, and the appearance of being a non-stationary series, this necessitates checking cointegration to determine if we can draw conclusions regarding a long-run equilibrium relationship. Again, for consistency, we employ the use of Westerlund panel cointegration testing, where the null hypothesis is no cointegration exists.[69]

Figure 13.0: Fiscal Space-to-GDP of 23 Advanced Economies



Note: Figure 13.0 shows the average Fiscal Space, as a percentage of GDP, across our panels in a given year.

In looking at the robust p-values for the group test statistics in Table 10.0 (Ga and Gt), we find general support for rejection of the null, concluding that there is cointegration between fiscal space and growth in at least one of the cross-sectional units.

[69] See Section 4.2.3. for further information.

The presence of a cointegrating vector justifies the use of fiscal space in our regressions; however, spurious regressions are always a risk with non-stationarity. Cointegration justifies the inference of apparent empirical relationships between growth and fiscal space. Many strands of current debt-growth literature also abstract away from the risk of spurious regressions. Eberhardt & Presbitero (2013, pg. 25) state that their paper "signals a significant departure from standard empirical modelling in the literature" when they elect to take into account "well-known data properties" such as non-stationarity, that could induce biased and spurious results. Sims et al. (1990) also show that not imposing cointegration generally leads to consistent parameter estimates and valid asymptotical statistic references. In addition, if unsure about the number of cointegrating vectors, it may be preferable not to impose cointegration (Ling & Tsay, 1996). The fact that we investigate non-stationarity and cointegration for both our fiscal reaction functions, and growth and fiscal space regressions provides support for our results from the respective empirical approaches.

Table 10.0: Panel Cointegration Results: Growth, Fiscal Space & Fiscal Space Squared

Statistic	Value	Z-value	P-value	Robust P-value
		(1 1) lag		
Gt	-2.763	-6.042	0	0
Ga	-16.215	-8.708	0	0
Pt	-11.851	-5.687	0	0
Pa	-14.879	-11.575	0	0
		(1 1) lag, constant		
Gt	-2.759	-3.602	0	0
Ga	-16.621	-5.476	0	0
Pt	-11.401	-3.439	0	0
Pa	-14.786	-7.311	0	0
		(1 1) lag, constant, trend		
Gt	-3.262	-4	0	0
Ga	-19.963	-3.957	0	0
Pt	-14.772	-4.755	0	0
Pa	-18.604	-5.502	0	0
		(1 2) lag		
Gt	-2.202	-3.586	0	0
Ga	-13.748	-6.641	0	0
Pt	-11.851	-5.687	0	0.25
Pa	-14.879	-11.575	0	0
		(1 2) lag, constant		
Gt	-2.256	-1.103	0.135	0.25
Ga	-14.188	-3.699	0	0
Pt	-11.401	-3.439	0	0.5
Pa	-14.786	-7.311	0	0
		(1 2) lag, constant, trend		
Gt	-2.8	-1.48	0.069	0
Ga	-17.929	-2.686	0.004	0
Pt	-14.772	-4.755	0	0.75
Pa	-18.604	-5.502	0	0.5
		(2 2) lag		
Gt	-1.897	-2.254	0.012	0
Ga	-12.937	-5.961	0	0
Pt	-8.103	-2.867	0.002	0
Pa	-11.432	-8.352	0	0
		(2 2) lag, constant		
Gt	-1.948	0.434	0.668	0.25
Ga	-12.88	-2.743	0.003	0
Pt	-7.6	0.242	0.596	0
Pa	-11.01	-4.216	0	0
		(2 2) lag, constant, trend		
Gt	-2.467	0.333	0.63	0.5
Ga	-16.523	-1.807	0.035	0.25
Pt	-10.99	-0.562	0.287	0
Pa	-15.01	-3.066	0.001	0

Note: Table 10.0 shows the 'group' and 'pooled' cointegration test results between growth and fiscal space. We test a variety of lag specifications and trend inclusion.

6.3.4. Back-of-the-Envelope Turning Point

A popular empirical test of quadratic theories involves estimating an equation with a polynomial of the variable that is central to the non-linear relationship, then assessing a 'turning point'. If the estimated turning point of the equation is well within the range of the data, then this is usually taken as evidence that the true relationship is non-monotonic. If the estimated turning point is either at the lower or upper end or outside the range of the data, then there is little evidence that the true relationship is non-monotonic, and a monotonic non-linear function other than a polynomial might be more appropriate. For the subsequent calculations we use our CCE GMM (Table 9.0, specification (8)) results.[70]

Our empirical model above is,

$$\Delta y_{it} = \beta_1 S_{i,t} + \beta_2 S_{i,t}^2 + \psi x_{i,t} + \eta_i + \varsigma_t + \upsilon_{i,t} \tag{37}$$

$$\Delta y_{it} = (0.20407) S_{i,t} + (-0.00061) S_{i,t}^2 + \psi x_{i,t} + \eta_i + \varsigma_t + \upsilon_{i,t} \tag{38}$$

The formula for finding a turning point is,

$$S_i^* = \frac{-\beta_1}{2\beta_2} \tag{39}$$

$$S_i^* = \frac{-0.20407}{(2 \times -0.00061)}$$

$$S_i^* = 167.27$$

where S_i^* is the fiscal space estimate associated with maximal growth, according to our rough calculations.

This result would indicate that the turning point, the level of fiscal space that corresponds to the highest levels of growth, is approximately 167 percent of GDP. Bearing in mind that this is a simple back-of-the-envelope calculation of the turning points based on the point estimates, it appears that on average across our panels, highest growth is maximised when fiscal space is 166 percent of GDP, ceteris paribus. With countries' individual debt limits in our sample differing, this fiscal space estimate corresponds to a different level of debt for each country, allowing for at least a small amount of heterogeneity regarding thresholds. Given our limited time-series analysis (see Appendix Section 9.2.3) we know coefficient estimates vary across countries.

Table 11.0 shows this result for fiscal space's turning point in the context of each country's historical debt limit and average debt-to-GDP. Column (1) simply presents each country's

[70] Other regression coefficients can be easily substituted into equation (31).

individual debt limit, as calculated by our model (see Section 6.2). Column (3) presents a historical average of each country's debt-to-GDP ratios (see Section 4.2.1 for more information). To find the implied turning point for the debt-to-GDP ratio (shown in column (2)), we simply take each country's calculated debt limits, and subtract 167. While these calculations are crude, the main goal here is to bring our previous analysis into frame with the context of popular debt-growth literature. Our implied turning points are:

$$T_i = \overline{D}_i - S_i^* \tag{40}$$

where T_i is the implied turning point. If S_i^* is meaningful, then this measure would be a country's 'optimal' or growth-maximising debt-to-GDP ratio.

The results indicate that these debt ratios associated with maximum growth are on the lower end of the spectrum in relation to previous debt-growth literature. Previous literature holds empirically that turning points are roughly 90 percent of GDP (Reinhart & Rogoff, 2010). Our results show a number of advanced economies with a considerably lower debt-to-GDP level. However, our results are based off panel estimates for fiscal space turning points, so this may not be homogenous across all countries. For Table 11.0, the main conclusion that we can draw is from the distribution of estimates. Countries with high debt limits and turning points such as New Zealand and Norway may be able to achieve higher growth with higher sustainable debt, while countries with high debt limits but low turning points such as France or Japan, may be better off reducing debt.

Table 11.0: Debt-Growth Turning Points

	Static Debt Limit Historical	Implied Turning Point Historical	Debt-to-GDP ratio Historical
AUS	221.6	54.6	20.60
AUT	201.9	34.9	48.50
BEL	203.4	36.4	100.53
CAN	203.9	36.9	68.45
DNK	213.5	46.5	49.04
ESP	222.4	55.4	26.30
FIN	207.8	40.8	45.64
FRA	198.9	31.9	47.08
GER	198.1	31.1	59.04
GRC	212.9	45.9	38.55
IRL	236.1	69.1	68.83
ISL	231.8	64.8	120.45
ISR	193.5	26.5	95.18
ITA	189.1	22.1	82.66
JPN	184.4	17.4	16.48
KOR	231.3	64.3	57.60
NLD	210.3	43.3	44.16
NOR	250.3	83.3	45.31
NZL	240.8	73.8	67.73
POR	201.6	34.6	45.92
SWE	199.6	32.6	55.95
UK	197.3	30.3	42.62
USA	214.5	47.5	55.69

Static debt limits are taken from Table 7.0, specification (3). The implied turning points are simply the debt limits minus 167, following equation (31). For comparison purposes, the historical average debt-to-GDP ratios for each country has been taken from Table 1.0.The sample period is 1970-2007.

The calculations behind Table 11.0 are overly simplistic and should not be taken as accurate turning points. More, they are designed to reinforce the ideas presented earlier. Firstly, that while a country may have available fiscal space, higher growth can be associated with increasing or decreasing debt depending on exactly where the country lies relative to its debt limit and 'turning point'. Also, that even when a country can achieve higher growth by increasing public debt sustainably, it still matters 'how' the money is introduced into the economy, whether via investment or consumption.

6.4. Discussion and Policy Implications

Fiscal policy is a balancing act. On one side, the government provides stimulus to the economy in the form of spending. On the other, is hardship in the form of taxes or debt. Both debt and taxes can be used together to manage fiscal policy; however there are risks that can increase long-term pressures. For example, imposing higher taxes can limit growth. Also debt, while stimulating in nature, transfers these (interest) costs to future taxpayers. The challenge is how to stimulate the economy when needed, without compromising sustainability. With waning global growth, the economy clearly needs stimulus. However, using debt to finance government expenditure essentially means that future taxpayers are paying for government services used today. This further questions sustainability, but also what a government spends its money on in regards to immediate consumption compared to long-term investment.

The aim of this thesis was to accomplish a number of things. Firstly, we look to test the theory of 'fiscal fatigue' empirically. Secondly, we establish estimates of debt limits for advanced economies. And lastly, we examine the relationship between fiscal space and growth for these advanced economies. Overall, our empirical models indicate a number of results. We find support for Ghosh et al.'s (2013) posited cubic fiscal reaction function, which supports the theory of fiscal fatigue. Secondly, we find that country debt limits are heterogeneous across countries, but are larger than historical debt-to-GDP levels. This indicates that countries have a decent amount of 'wiggle-room' for policy or fiscal stimulus to occur without jeopardising sustainability. Lastly, it would appear there is a quadratic relationship between fiscal space and growth, indicating that the highest levels of growth are associated with countries that have little fiscal space, or debt is sustainably high. According to our rough calculations, fiscal space of 167 percent of GDP is associated with the highest growth rates. Comparing the implied turning points and historical debt-to-GDP levels in Table 10.0, our result would indicate that countries with debt-to-GDP levels above the 'turning point' may be able to achieve higher growth by lowering public debt.

In assessing these results, several points should be borne in mind. The reported estimates of fiscal space are against projected debt levels in 2015, and do not take account of possible contingent liabilities. Contingent liabilities, such as unfunded pension obligations and potential bank bailouts, by increasing actual debt, can worsen our fiscal space estimates substantially.

The debt limit and subsequent fiscal space estimates are by no means desirable or representative of optimal debt levels. Factors such as rollover risk mean that governments will want to ensure they do not exhaust their fiscal space and that their debt remains well below its calculated debt limit, i.e. ensuring sufficient fiscal space.[71]

An unsustainably high debt level is one that lies above the debt limit in our model. Obviously, going beyond this limit would have negative implications for an economy's growth, ranging from infinitely high servicing costs to market shut-outs. However, below this limit, while debt levels may be viewed by some as 'high' they are not unsustainably so. A number of policymakers think that high debt reduces growth. This view is in line with the results in the empirical literature, and finds that this relationship becomes particularly strong when public debt approaches 100% of GDP (Reinhart and Rogoff 2010a, 2010b; Kumar and Woo 2010; Cecchetti et al. 2011).

Fiscal space and debt-to-GDP, by construction, are negatively correlated across our sample (countries with lower debt typically have more fiscal space); however this may not be the case across the board. A country may have a better track record of fiscal adjustment; if so there is potential for this country to have more fiscal space and also be charged with a relatively lower risk premium, compared to a country with a poorer track record but lower debt. This could help explain the different treatments between, say, Greece and Japan. Greece has a lower level of debt than Japan, but faces increasing financing difficulties, indicating that their fiscal space is much less, even though they have a lower overall debt level. However, there a number of factors to consider, such as assessing our degree of economic ignorance, or realising that correlation does not imply causation. We believe that our findings are an important addition to the current debate on fiscal policy.

Do high levels of public debt reduce economic growth? This is an important policy question. But underlying this question is another – what constitutes a 'high' level of debt? This is what our concept of debt limits and fiscal space look to incorporate. Our results indicate a quadratic (inverse) relationship between fiscal space and growth, implying countries with ample amounts of fiscal space may achieve higher growth by increasing debt (and vice versa). But these relationships do not mean that countries can sustain any level of debt. What our results seem to indicate, however, is that the advanced economies in our sample are still below the country-specific debt limit, so debt is not at unsustainable levels. However, the threshold at which debt starts having a negative effect on growth is a different story; countries should consider that the implied debt levels associated with high growth may be above or below current levels. Our findings suggest that policy makers should diligently scrutinise the government debt situation before implementing fiscal programs as their effectiveness to boost economic activity or resolve external imbalances may not be guaranteed.

[71] Rollover risk is a risk associated with the refinancing of debt. Rollover risk is commonly faced by countries and companies when their debt is about to mature and needs to be rolled over into new debt. If interest rates rise adversely, they would have to refinance their debt at a higher rate and incur more interest charges in the future.

Our fiscal space and growth results hint at two different recommendations depending on where a country currently sits. At the first extreme, with high debt, obviously it makes sense to increase fiscal space. However, at the other extreme, countries with larger fiscal space can afford to sustainably increase their debt. It is more complicated for countries in the middle. A policy recommendation would require further research, focusing on such ideas of how the money should be allocated if a country were to decrease fiscal space (increase debt), or potential mechanisms for increasing fiscal space without jeopardising growth.

For advanced economies, the key question is how to rationally allocate the fiscal space that is available. The opportunity to enlarge future space through growth-stimulating budget policies is limited. However, fiscal space justifies policies that may worsen current financial imbalances but would promote growth and thereby enlarge the future scope of public expenditure. The opportunity to create fiscal space depends on variables such as the tax revenue/GDP ratio, spending on infrastructure and other investment, and budget rigidity. These variables differ greatly among middle-income countries, and so too does their capacity to create space. But the practice of budgeting is not changed by fiscal space. Resources are allocated for initiatives in the budget incrementally, and while the budget covers all government expenditure, decisions are made at the margin. Introducing the concept of fiscal space adds a longer term sustainable dimension to the current debt-growth literature. This bridges the gap to real life, where decisions require consideration of how current decisions impact future space, and growth.

Another important policy objective hinted by this thesis is restricting unproductive spending. For example, this may require military outlay cuts, subsidy reductions, wage restraints, or justification of civil service elements (e.g. the problem of ghost workers, etc.). Simultaneously, productive spending needs to be protected. There can be damaging social effects by not spending enough on important sectors, e.g. health. Reducing spending in these sectors could weaken the sectors; it would then be costly and time consuming to "rebuild" them. Outside the direct results from the thesis, other policy aims which could support our conclusions may be, for example, streamlining program implementation, and improving governance. With regards to debt financing, creditors could help by paring conditionality agreements, administrative overhead reductions, increased sector spending coordination, and, given the small number of country program managers, reduce the administrative overload.

Policymakers need to evaluate whether the social return from the allocation of debt justifies the cost, given that borrowing must be both serviced and repaid. Governments may choose to borrow without taking specific account of the direct returns from the increased spending, instead focusing on the obvious cost in the form of money borrowed. But they must take account of the benefits when assessing the overall sustainability of a program. Such assessments weigh a number of factors, including: prospective growth rates, export and remittance potential, predicted interest rate environment, predicted government revenues, existing debt composition (currency borrowed, interest rates, maturity, etc.), and new debt being considered.

If budget policies are well managed, countries will likely have greater potential for creating extra fiscal space. Our results prove useful as they reflect the importance of governments' choice of program financing and infrastructure allocation if spending is expanded to foster growth. Our estimated country-specific debt limits provide a marker for countries to assess their current debt position and determine the best course of action, or whether they should be 'worried'. The theory highlights some warning signs, such as increasing interest rates, which could occur before reaching the debt limit. The importance of sensible fiscal management to ensure a sustainable path for public debt is also emphasised.

A disturbing feature of ad-hoc sustainability is the apparent disconnect from practical politics. While political debates about sustainability are mostly about bounds on debt-GDP and/or deficit-GDP ratios, much of the academic literature has focused on real fiscal series and treats non-stationary debt-to-GDP ratios as unproblematic.

Ultimately, this paper should be used to help inform governments of their debt limit, and how much available fiscal space they have. At its core, this paper provides information on advanced economy debt limits which policymakers can consider to ensure policy and debt sustainability. However, precision at the country-level has to be taken cautiously. Furthermore, should the importance of the concept of fiscal space be questioned, our further empirical work on fiscal space and growth should warrant its consideration. We show that fiscal space fits in with popular debt-growth literature, while providing its own unique slant on debt assessment. It also poses the question of funding allocation; the returns to using up fiscal space depend on whether it funds such avenues as investment, or consumption.

7. Conclusion

This paper provides an addition to the current literature, and begins to bridge the gap between ideas of sustainability, debt, and growth. This paper contributes to the existing literature on fiscal sustainability by building upon a framework from Ghosh et al. (2013) that conceptualizes the notion of fiscal space—defined as the difference between current debt level and a (country-specific) debt limit, where the latter is the debt level beyond which fiscal solvency fails. We also extend this debt sustainability literature into the debt-growth nexus by investigating the relationship between fiscal space and growth. This is done to check if fiscal space fits empirically into what is now conventional non-linear debt-growth theory, and to provide a future framework for policy analysis.

We apply our framework empirically to a sample of 23 advanced economies over 1970–2007. Our findings indicate robust empirical support for the fiscal fatigue characteristic, after testing both a static and dynamic model. We also use these estimates to determine country-specific debt limits, and find that the debt limits vary across country. For example, the debt limit obtained for countries in the sample ranges between 150 to 250 percent of GDP, while the fiscal space estimates indicate limited or no available fiscal space for Greece, Iceland, Italy, Japan and Portugal, and ample space for Australia, Korea and the Nordic countries.

The main contribution from this thesis is extending the idea of fiscal space into the debt-growth literature, by combining fiscal space with growth theory. We find an inverse quadratic relationship between fiscal space and growth. This has a number of implications. Firstly, given the construction of fiscal space, it implies that fiscal space fits standard debt-growth non-linear theory, whereby higher debt (or lower fiscal space) begins to hamper growth rates. This would add robustness to the use of fiscal space as a policy tool in the future. Secondly, we find not only the level of government spending to matter for growth, but where the money is spent e.g. between consumption or potentially larger growth generation from investment. We also know of no current literature that has bridged the gap between debt sustainability and growth. Although some calculations are rather back-of-the-envelope, they are designed to further stimulate the debate on debt and growth, while addressing sustainability as a key concern for policymakers in the future.

For future analysis, we recommend investigation into a time-varying debt limit, further analysis into individual country characteristics, and theoretical development of fiscal space into the wider growth literature for policy relevance. Future studies could incorporate more recent data (2008-2016) and investigate any changes post-GFC. A robustness analysis should also be carried out, focusing on sub-sample regressions within the Euro-area, and separating out Portugal, Ireland, Greece, and Spain given their debt problems post-GFC.

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9. Appendix

9.1. Data Description

Variable	Data Description
Primary Balance-to-GDP	General government primary net lending or borrowing i.e. net lending/borrowing plus net interest payable/paid (interest expense minus interest revenue), as a percentage of GDP.
Debt-to-GDP	Gross debt as a percentage of GDP.
Output Gap	The difference between actual and potential (calculated using a Hodrick-Prescott filter with the 1600 as the standard value for the smoothing parameter[72]) real government output.
Government Expenditure Gap	Measure the effect of temporary fluctuations in government outlays (e.g. military spending).
Trade Openness	Sum of exports and imports as a percentage of GDP.
Present and Future age dependency ratio	The ratio of dependents (people younger than 15 or older than 64) to the working-age population (aged 15-64), with future age dependency ratio projected 20 years ahead.
Rule of Law	Captures perceptions of the extent to which agents have confidence in and abide by the rules of society.
Inflation	The log of a three-year lagged moving average of CPI inflation, following Ghosh et al. (2013). This is used to examine possible effects of inflation on the fiscal balance, due to, for example, bracket-creep effects, or greater fiscal effort to counter the effects of higher interest rates accompanying higher inflation.
Oil Prices	The log of (trend) oil price. Used for oil exporting countries only.
Non-fuel commodity prices	The log of (trend) non-fuel commodity price index. Used for non-fuel commodity exporting countries only.
Political Stability Risk	From the International Country Risk Guide (ICRG) dataset. Smaller (larger) values indicate higher (lower) risk.
IMF Arrangement	Binary variable equal to one if a country has an IMF support program in a given year, or zero if no support program is in place. This is a proxy for international influence on fiscal behaviours.
Fiscal Rules	Binary variable equal to one if a country has any type of fiscal rule (expenditure, revenue, balance budget, and debt) in a given year, or zero if otherwise.
Real Interest Rate	The lending rate adjusted for inflation as measured by a country's GDP deflator. However, the terms and conditions attached to lending rates differ by country, which could limit their comparability.

[72] A Hodrick-Prescott filter (HP filter) is a common mathematical tool used in macroeconomics to remove the cyclical component of a time series from raw data. However, it is sensitive to specification (annual vs. monthly data) and penalises variations in the growth rate of the trend component.

Real GDP per capita Growth Rate	The annual change in the logarithm of real GDP per capita. Real GDP per capita is the population average of the sum of gross value (2005 US dollars) added by all resident producers in the economy plus any product taxes, minus any subsidies not included in the value of the products. It is calculated without making deductions for depreciation of fabricated assets or for depletion and degradation of natural resources.
Government Investment	Gross capital formation as a percentage of GDP. Gross capital formation consists of outlays on additions to the fixed assets of the economy plus net changes in the level of inventories.[73]
Labour force participation rate	The proportion of the population aged 15 and older that is economically active: all people who supply labour for the production of goods and services during a specified period.
Unemployment	The share of the labour force that is without work but available for and seeking employment.
Government Consumption Expenditure	Includes all government current expenditures for purchases of goods and services (including compensation of employees). It also includes most expenditures on national defence and security, but excludes government military expenditures that are part of government capital formation.
Education	Total enrolment in tertiary education, regardless of age, expressed as a percentage of the total population of the five-year age group following on from secondary school leaving.
Life Expectancy	The number of years a new-born infant would live if prevailing patterns of mortality at the time of its birth were to stay the same throughout its life.
Population Growth	The annual percentage change in a country's population
Rule-of-Law index	Taken from World Bank's 'World Governance Indicators'. This indexes countries using an aggregate indicator between -2.5 to 2.5, and captures perceptions of the extent to which agents have confidence in and abide by the rules of society.

Our data have been obtained from a variety of sources. The initial variables used in our static fiscal reaction function were obtained from Ghosh et al. (2013) and the IMF. Further variables introduced in our dynamic specifications and fiscal space-growth regressions have been obtained from the World Bank and IMF.[74]

[73] Fixed assets include land improvements (fences, ditches, drains, and so on); plant, machinery, and equipment purchases; and the construction of roads, railways, and the like, including schools, offices, hospitals, private residential dwellings, and commercial and industrial buildings. Inventories are stocks of goods held by firms to meet temporary or unexpected fluctuations in production or sales, and "work in progress."

[74] Our data-set contains further variables that have the potential to affect primary balance (government cash surplus; Gini index; financial depth; terms-of-trade; exports-to-GDP ratio; and purchasing power parity) but due to limitations or gaps in data these have not been included in our analysis.

9.2. Additional Results

The following section contains additional results that, while not critical to our overall conclusions, do support them and are interesting variations to consider. Firstly, we have stationarity analysis in the form of time-series unit root testing (compared to panel). Secondly, further cointegration testing is undertaken on government revenue and government expenditure, as determined by theory. Third, we run individual time-series fiscal reaction functions, to compare to the panel regression in our thesis. Next, to check if it is appropriate to combine our advanced economies in a panel fiscal reaction function, we run sub-sample fiscal reaction functions to examine the behaviour of both low and high debt-observed countries. In terms of fiscal space and growth, we also run regular panel regressions, to compare to the most appropriate CCE empirical method results in our thesis.

9.2.1. Analysis of Time-Series Properties

9.2.1.1. Dickey-Fuller Time Series Results

The most common unit root test in time-series literature is an (augmented) Dickey-Fuller (ADF). An important practical issue for the implementation of the ADF test is the specification of the lag length, p. If p is too small, then the remaining serial correlation in the errors will bias the test. If p is too large, then the power of the test will suffer. Ng and Perron (1995) suggest the following data dependent lag length selection procedure that results in a stable size of the test and minimal power loss: Set an upper bound p_{max} for p. A useful rule of thumb for determining p_{max}, suggested by Schwert (2002), is

$$\rho_{max} = \left[12\left(\frac{n}{100}\right)^{1/4}\right] \tag{41}$$

where $[x]$ denotes the integer part of x. This choice allows ρ_{max} to grow with the sample so that the ADF test regressions are valid if the errors follow an ARMA process with unknown order. For our panel data set, each country has roughly 46 time periods, meaning $n = 46$

$$\rho_{max} = \left[12\left(\frac{46}{100}\right)^{1/4}\right]$$
$$\rho_{max} = 9$$

Therefore the maximum possible lag for which to begin unit root testing is 9. The next step, following Ng & Perron's (1995) process, is to determine the appropriate level of augmentation (lags) to include in the ADF test. To do this, we run an AIC, BIC, and HQIC lag tests on our individual country debt data, assessing goodness-of-fit. We estimate the ADF test regression with $p = p_{max}$. If the absolute value of the t-statistic for testing the significance of the last lagged difference is greater than 1.6[75], then set $p = p_{max}$ and perform the unit root test. Otherwise, reduce the lag length by one and repeat the process. This can be thought of as a form of general-to-specific testing.

Table 12.0: Debt-to-GDP DF-GLS Unit Root test

	Dickey Fuller GLS Unit Root Test, Debt-to-GDP	
p-value	**No Trend**	**Trend**
>0.1	United States, Belgium, Denmark, France, Germany, Italy, Norway, Canada, Japan, Finland, Greece, Iceland, Portugal, Australia, New Zealand, Israel, South Korea	United States, Austria, Belgium, Denmark, France, Italy, Netherlands, Norway, Sweden, Canada, Finland, Greece, Iceland, Portugal, Australia, Israel, South Korea
0.05<x<0.1	Austria, Netherlands, Sweden, Spain	Japan, Spain, New Zealand
0.01<x<0.05		Germany, Ireland
<0.01	United Kingdom, Ireland	United Kingdom

For each country we run a Dickey-Fuller Generalised-Least-Squares (DF-GLS) unit root test on debt-to-GDP, and are presented with a DF-GLS test statistic and the corresponding critical values at the 1%, 5% and 10% confidence level. The testing process automatically calculates Schwert's (1989) rule-of-thumb, and tests down following Ng & Perron (1995) to determine the appropriate lag. The results are presented in Table 12.0 above, where each country's result is based on its own significant lag specification.

The null hypothesis for the DF-GLS is that there is a unit root, while the alternative is that the time-series is stationary. We include both a trend and no-trend options for comparison. It appears the majority of countries fail to reject the null that there is a unit root. 17 countries fail to reject the null of a unit root even at the 10% level. There are a small number of marginal countries that could reject the null depending on inclusion of trend. Overall the results indicate that the majority of countries' debt-to-GDP appears to have a unit root.

[75] Corresponding approximately to a 10% two-sided critical value.

Table 13.0: Primary Balance DF-GLS Unit Root test

	Dickey Fuller GLS Unit Root Test, Primary Balance	
p-value	**No Trend**	**Trend**
>0.1	United States, Austria, Belgium, Germany, Italy, Netherlands, Norway, Japan, Finland, Greece, Iceland, Ireland, Portugal, Spain, South Korea	United States, Austria, Belgium, France, Germany, Italy, Netherland, Norway, Sweden, Canada, Japan, Finland, Greece, Iceland, Ireland, Portugal, Spain, South Korea
0.05<x<0.1	France, Canada	New Zealand
0.01<x<0.05	New Zealand	United Kingdom, Denmark, Israel
<0.01	United Kingdom, Denmark, Sweden, Australia, Israel	Australia

Following a similar process as above, we run a DF-GLS unit root test for each country's primary balance. The results are shown in Table 13.0 above. Again, for primary balance it appears the majority of countries fail to reject the null hypothesis of a unit root, regardless of trend inclusion. However, there are a number of marginal countries which could reject the null depending on significance of the p-value. This includes New Zealand, United Kingdom, Denmark, Israel, and Australia. Overall, the results indicate that the majority of countries' primary balance appears to have a unit root.

The results above determine that the majority of advanced economies have non-stationary properties in both debt-to-GDP and primary balance, which supports the panel unit root results in our thesis. To justify the use of panel regressions for both the fiscal reaction functions and fiscal space-growth regressions, we run cointegration testing to determine if non-stationarity is an issue.[76]

[76] See sections 4.2.3 or appendix section 9.2.2 for further information.

9.2.2. Government Revenue and Expenditure Cointegration

Following our cointegration testing for primary balance and debt-to-GDP (section 4.2.3), we also explore the avenue of cointegration between government revenues and expenditure. The idea of sustainability in this context would imply that revenues and expenditures should be cointegrated. To test this, we employ the panel Westerlund cointegration test.

Table 14.0: Panel Cointegration Results: Government Revenue and Expenditure

Statistic	**Value**	**Z-value**	**P-value**	**Robust P-value**
	(1 1) lag, constant, trend			
Gt	-2.578	-1.249	0.106	0.12
Ga	-11.583	0.302	0.619	0.26
Pt	-13.471	-3.819	0	0.02
Pa	-11.603	-2.089	0.018	0.06
	(1 2) lag, constant, trend			
Gt	-2.554	-1.106	0.134	0.5
Ga	-11.402	0.43	0.666	0.5
Pt	-13.471	-3.819	0	0
Pa	-11.603	-2.089	0.018	0
	(2 1) lag, constant, trend			
Gt	-2.554	-1.106	0.134	0
Ga	-11.402	0.43	0.666	0.25
Pt	-13.471	-3.819	0	0
Pa	-11.603	-2.089	0.018	0
	(2 2) lag, constant, trend			
Gt	-2.521	-0.912	0.181	0.02
Ga	-11.46	0.389	0.651	0.26
Pt	-11.9	-2.023	0.022	0.02
Pa	-10.307	-1.075	0.141	0.02

Note: Table 14.0 shows the 'group' and 'pooled' cointegration test results between government revenue and expenditure. Preliminary analysis warrants the inclusion of a constant term. From there we test a variety of lag specifications and trend inclusion.

There appears to be cointegration between government revenue and government expenditure. Again our focus is on robust (bootstrapped 50 times) p-values, due to potential cross-sectional dependence. For government revenue, and expenditure, because there is obviously a trend (see graphs above), we include a constant and trend, but vary lag lengths. Our results in Table 14.0 above indicate strong rejection of the null across the board, in terms of the pooled cointegration tests. However, the group tests give mixed results. Overall, we conclude that there is cointegration across the panel as a whole. However, within individual groups there may not be cointegration. This result is most well known in Quintos' (1995) condition for 'weak' sustainability, which assumes that government spending and revenues are each integrated of order one, and cointegrated with the same vector, satisfying the TC.

9.2.3. Time Series Fiscal Reaction Function

Table 15.0: Individual Country Time-series Fiscal Reaction Functions (Countries 1-6).

VARIABLES	(US)	(UK)	(AUT)	(BEL)	(DEN)	(FRA)
$Debt_{t-1}$	12.404664*	-31.46657***	-0.47265	-5.601825**	0.47050510	2.85700719*
	(6.482)	(7.937)	(12.638)	(2.795)	(0.537)	(1.553)
$Debt\_Square_{t-1}$	-0.1736051*	0.7802172***	-0.00980	0.049293**	-0.0098273	-0.0497845*
	(0.103)	(0.200)	(0.223)	(0.023)	(0.012)	(0.028)
$Debt\_Cubic_{t-1}$	0.00078132	-0.006394***	0.000154	-0.00014**	0.000041	0.0002800*
	(0.001)	(0.002)	(0.001)	(0.000)	(0.000)	(0.000)
Output Gap	0.461395***	0.101471	0.244394	0.751454***	1.16239***	-1.11958***
	(0.116)	(0.323)	(0.200)	(0.200)	(0.260)	(0.279)
Government Expenditure Gap	-1.19493***	-0.952038***	0.26421	-0.28173***	-0.345603	-0.46234***
	(0.189)	(0.341)	(0.240)	(0.060)	(0.312)	(0.107)
Trade Openness	-0.545179**	-0.34433308	-0.05006	0.1484547	-0.2447489	0.844503***
	(0.251)	(0.260)	(0.182)	(0.226)	(0.196)	(0.149)

Note: Dependent variable is primary balance to GDP ratio (percent); all model specifications are estimated using feasible GLS and include country fixed-effects. Estimation allows for AR(1) autocorrelation within panels (with common AR coefficient) and heteroskedasticity across panels.. This is essentially the same fiscal reaction function in Table 5.0, specification (4); however given the time-series nature, we have not included the remaining coefficients as they do not add to the narrative. Each column corresponds to a different country.

Table 16.0: Individual Country Time-series Fiscal Reaction Functions (Countries 7-12).

VARIABLES	(GER)	(ITA)	(NLD)	(NOR)	(SWE)	(CAN)
$Debt_{t-1}$	18.3569***	-0.429244	-0.606872	-0.6639151	-16.8890***	6.38383***
	(4.386)	(4.346)	(5.405)	(1.947)	(2.290)	(2.304)
$Debt\_Square_{t-1}$	-0.34856***	0.024118	0.028772	0.014497	0.263194***	-0.0777***
	(0.084)	(0.041)	(0.085)	(0.040)	(0.036)	(0.029)
$Debt\_Cubic_{t-1}$	0.002204***	-0.000138	-0.000229	-0.000121	-0.00131***	0.0003***
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Output Gap	0.290028	-0.26153	0.401152*	2.442827***	0.4050761	0.087553
	(0.179)	(0.426)	(0.223)	(0.466)	(0.384)	(0.137)
Government Expenditure Gap	1.41819***	-0.652***	0.13978	-1.42974***	-0.731342**	-0.097768
	(0.214)	(0.167)	(0.165)	(0.181)	(0.304)	(0.155)
Trade Openness	0.36482***	0.281702	0.4753**	1.076464***	-1.43638***	0.18966679
	(0.119)	(0.330)	(0.196)	(0.351)	(0.366)	(0.190)

Note: Dependent variable is primary balance to GDP ratio (percent); all model specifications are estimated using feasible GLS and include country fixed-effects. Estimation allows for AR(1) autocorrelation within panels (with common AR coefficient) and heteroskedasticity across panels.. This is essentially the same fiscal reaction function in Table 5.0, specification (4); however given the time-series nature, we have not included the remaining coefficients as they do not add to the narrative. Each column corresponds to a different country.

Table 17.0: Individual Country Time-series Fiscal Reaction Functions (Countries 13-18).

VARIABLES	(JPN)	(FIN)	(GRC)	(ICE)	(IRL)	(POR)
$Debt_{t-1}$	-0.1143324	0.5783009	-2.465511	8.174166***	0.1067633	26.43259***
	(0.226)	(0.605)	(4.861)	(0.991)	(0.328)	(1.233)
$Debt\_Square_{t-1}$	0.000934	-0.002991	0.034021	-0.19771***	0.002746	-0.29981***
	(0.002)	(0.016)	(0.063)	(0.025)	(0.004)	(0.018)
$Debt\_Cubic_{t-1}$	-0.000001	0.000006	-0.000145	0.00154***	-0.000017	0.001127***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Output Gap	0.995803***	1.45480***	-0.381551	0.092647	0.99518***	4.36083***
	(0.276)	(0.180)	(0.323)	(0.114)	(0.204)	(0.120)
Government Expenditure Gap	-0.607359***	-0.223458	-0.162002	0.386912	-0.31871***	-0.95852***
	(0.139)	(0.191)	(0.116)	(0.236)	(0.118)	(0.031)
Trade Openness	-0.608911***	-0.112868	0.243869	-0.55128***	-0.037908	-0.90775***
	(0.178)	(0.128)	(0.156)	(0.192)	(0.168)	(0.063)

Note: Dependent variable is primary balance to GDP ratio (percent); all model specifications are estimated using feasible GLS and include country fixed-effects. Estimation allows for AR(1) autocorrelation within panels (with common AR coefficient) and heteroskedasticity across panels.. This is essentially the same fiscal reaction function in Table 5.0, specification (4); however given the time-series nature, we have not included the remaining coefficients as they do not add to the narrative. Each column corresponds to a different country.

Table 18.0: Individual Country Time-series Fiscal Reaction Functions (Countries 19-23).

VARIABLES	(ESP)	(AUS)	(NZL)	(ISR)	(KOR)
$Debt_{t-1}$	1.91386514	-1.01690355	-5.5653678***	-0.31582093	-7.7112060***
	(2.529)	(1.316)	(1.575)	(0.206)	(0.000)
$Debt\_Square_{t-1}$	-0.03953740	-0.00235598	0.15054659***	0.00133661	0.53107780***
	(0.048)	(0.044)	(0.031)	(0.002)	(0.000)
$Debt\_Cubic_{t-1}$	0.00026711	0.00074412	-0.0011712***	-0.00000161	-0.00874***
	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)
Output Gap	0.34549206	0.44570218*	1.40965829***	0.05420133	2.44301698***
	(0.237)	(0.244)	(0.277)	(0.155)	(0.000)
Government Expenditure Gap	-0.15465414	0.27266247	0.53898554***	-0.32566240**	-2.2605099***
	(0.193)	(0.347)	(0.079)	(0.152)	(0.000)
Trade Openness	0.35927654***	-0.06852535	-0.5706846***	0.56284305***	-0.4874528***
	(0.136)	(0.166)	(0.194)	(0.154)	(0.000)

Note: Dependent variable is primary balance to GDP ratio (percent); all model specifications are estimated using feasible GLS and include country fixed-effects. Estimation allows for AR(1) autocorrelation within panels (with common AR coefficient) and heteroskedasticity across panels.. This is essentially the same fiscal reaction function in Table 5.0, specification (4); however given the time-series nature, we have not included the remaining coefficients as they do not add to the narrative. Each column corresponds to a different country.

9.2.4. Fiscal Reaction Function (Sub-Sample)

Table 19.0: Fiscal Reaction Function (sub-samples).

VARIABLES	(1)	(2)
$Debt_{t-1}$	0.0288	0.02663***
	(0.051)	(0.007)
$Debt\_Square_{t-1}$	0.0010**	
	(0.000)	
$Debt\_Cubic_{t-1}$	-0.00001**	
	(0.000)	
Output Gap	0.2395***	0.49109***
	(0.038)	(0.049)
Government Expenditure Gap	-0.2212***	-0.23796***
	(0.031)	(0.036)
Trade Openness	0.0643**	0.13295***
	(0.028)	(0.040)
Age Dependency	0.0045	0.08972
	(0.031)	(0.062)
Future Age Dependency	-0.1186***	-0.22358***
	(0.024)	(0.042)
Inflation	0.3295	6.27420***
	(1.316)	(1.524)
Oil Prices	32.2240***	7.47007***
	(8.966)	(2.563)
Non-fuel Commodity Prices	5.2017**	-2.46094
	(2.579)	(2.693)
Political Stability	0.0106	0.04568*
	(0.016)	(0.025)
IMF Arrangement	-0.9835	-0.94867
	(0.772)	(0.831)
Fiscal Rules	-0.3095	0.87617***
	(0.211)	(0.295)
Investment	0.8901***	0.15405***
	(0.216)	(0.055)
Banking Crisis	2.8028	-0.50239**
	(1.915)	(0.215)
Constant	-6.979*	-7.07017*
	(3.769)	(3.754)
Observations	372	564

Note: Dependent variable is primary balance to GDP ratio (percent); all model specifications are estimated using feasible GLS and include country fixed-effects. Estimation allows for AR(1) autocorrelation within panels (with common AR coefficient) and heteroskedasticity across panels.

Table 19.0 shows the results for a fiscal reaction function limited to sub-samples of countries with either high or low observed debt over our sample period. The regression is otherwise similar to those is Section 6.1. Specification (1) is our fiscal reaction function limited to high debt countries. These countries include Belgium, Italy, Canada, Japan, Greece, Ireland, and Israel. The criteria for these countries was selected using Table 1.0: If a country has a debt observation greater than 100 percent of GDP, then it is included. Our results in (1) indicate that this cubic reaction function holds. The coefficient on debt squared is significant and positive, while the coefficient on debt cubed is negative and significant. This supports the theory of fiscal fatigue setting in at high debt levels. Our results here apply to a limited sub-sample; however, the cubic function still holds in Tables 5.0 and 6.0. If our assumption that advanced economies would have similar behaviour at high debt levels holds, then this would imply that the cubic function would hold for the other countries if observed at equivalently high debt levels.

Specification (2) in Table 19.0 is limited to a sub-sample of all countries not observed at high levels of debt according to Table 1.0 (see Table 1.0 for other countries). As these countries have not been observed at high debt levels, we would not expect them to display fiscal fatigue, or a negative significant debt-cubed coefficient. Instead, at low debt levels, our theory assumes that the primary balance responds positively to lagged debt. Therefore, we include only the lagged debt term, leaving debt squared and debt cubed out. Our results in specification (2) indicate that debt has a significant positive coefficient. This result reinforce our theory for an initially positive response of the primary balance to rising debt. The fact that this result is significant also implies that the sub-sample of advanced economies in our regression do behave in a similar way at low debt levels, further justifying our assumption that they would be similar at high debt levels too.

9.3. Growth and Fiscal Space Robustness

To determine whether our growth and fiscal space quadratic relationship is robust, we run a progression of further regressions in GLS form to examine the relationship. Our robustness checks indicate different results to our proposed non-linear fiscal space and growth theory. In general, our checks indicate a negative relationship between fiscal space and growth, rather than the 'inverted-U' shape from our CCE regressions. Bearing in mind, these robustness checks are carried out using standard GLS regression methods, rather than the more appropriate CCE methods using in Section 6.3.[77] Overall, our robustness checks show support for a negative relationship between fiscal space and growth, where the highest growth levels are associated with low fiscal space. This would indicate that the relationship between growth and fiscal space requires further examination, and is sensitive to estimation method.

9.3.1. Growth and Fiscal Space

Table 20.0: Growth and (Static) Fiscal Space

Static Fiscal Space (Historical) and Growth

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
Fiscal Space	0.00871**	0.01219	0.01657*	-0.00927	0.00193	-0.00613
	(0.004)	(0.010)	(0.010)	(0.013)	(0.013)	(0.014)
Fiscal Space squared		-0.00002	-0.00009**	-0.00015***	-0.00030***	-0.00027***
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Inflation	-0.00048	0.18636	2.54111*	4.01405***	3.28867**	-1.96726
	(1.453)	(1.532)	(1.480)	(1.507)	(1.460)	(3.119)
Output Gap			0.52879***	0.58309***	0.60641***	0.60658***
			(0.035)	(0.037)	(0.038)	(0.038)
Fiscal Space$_{t-1}$				0.04498***	0.02232*	0.02843**
				(0.011)	(0.013)	(0.014)
Fiscal Space squared$_{t-1}$					0.00020***	0.00018***
					(0.000)	(0.000)
Inflation$_{t-1}$						5.92508*
						(3.044)

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1 The dependent variable is Δln(GDP per capita). All specifications are estimated by panel GLS. The fiscal space data is the 'historical' and 'static' series, which is our fullest data set. The models also includes country fixed effects. Please see Appendix (Section 9.1) for further information on data.

Initially, in Table 20.0, specification (1), we run a simple linear GLS regression with fiscal space and inflation as regressors on growth. This is the only specification for which we find a

[77] See Section 6.1 for brief outline on GLS methodology.

positive relationship between fiscal space and growth. Given the lack of dynamics and additional regressors, as well as consistent results for subsequent regressions, we do not give much weight to this result. Specifications (2) to (6) investigate a quadratic model allowing for a non-linear relationship between fiscal space and growth, as suggested by debt theory. Overall, these regressions do not indicate support for our fiscal space and growth theory. Rather, they imply that increasing fiscal space is associated with lower growth. This is indicated by the statistically significant, negative coefficient on our fiscal space squared term across most of our models, plus either a negative or insignificant coefficient on the linear fiscal space term. This indicates countries with lower fiscal space (and higher debt) achieve higher growth. Table 21.0 runs the same regression specifications as Table 20.0, but uses our dynamic fiscal space estimates as a regressor. Although we again do not find support for our quadratic theory, similar results to that of Table 20.0 imply that our dynamic fiscal space and static fiscal space calculations may be as robust as each other to draw information from.

Table 21.0: Growth and (Dynamic) Fiscal Space.

Dynamic Fiscal Space (Historical) and Growth						
VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
Fiscal Space	0.00871**	0.00420	0.00515	-0.01946	-0.01664	-0.02446*
	(0.004)	(0.009)	(0.009)	(0.012)	(0.012)	(0.013)
Fiscal Space squared		0.00002	-0.00004	-0.0001**	-0.000***	-0.000***
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Inflation	-0.00048	-0.32327	2.04924	3.66952**	2.74253*	-3.18358
	(1.453)	(1.559)	(1.505)	(1.551)	(1.518)	(3.174)
Output Gap			0.5243***	0.5751***	0.5947***	0.5956***
			(0.035)	(0.037)	(0.038)	(0.037)
Fiscal Space_{t-1}				0.0426***	0.02827**	0.0340***
				(0.012)	(0.012)	(0.013)
Fiscal Space squared_{t-1}					0.00018***	0.00015***
					(0.000)	(0.000)
Inflation_{t-1}						6.65241**
						(3.064)
Constant	3.5908***	3.6728***	5.2217***	3.7290***	3.9545***	3.9979***
	(1.393)	(1.399)	(1.320)	(1.290)	(1.264)	(1.263)

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1 The dependent variable is Δln(GDP per capita). All specifications are estimated by panel GLS. The fiscal space data is the 'historical' and 'dynamic' series. While not our fullest data-set, it provides a cross-check for our 'static' series. The models also includes country fixed effects. Please see Appendix (Section 9.1) for further information on data.

9.3.2. Solow Growth and Fiscal Space

Table 22.0: Growth (Solow) and (Static) Fiscal Space

Static Fiscal Space (Historical) and Growth (Solow)

VARIABLES	(1)	(2)	(3)	(4)
Fiscal Space	-0.00259	0.00762	0.00729	0.00988
	(0.013)	(0.014)	(0.014)	(0.012)
Fiscal Space squared	-0.00033***	-0.00020***	-0.00020***	-0.00026***
	(0.000)	(0.000)	(0.000)	(0.000)
Inflation	-4.17653	3.14873	3.11831	0.78146
	(2.829)	(3.222)	(3.225)	(2.805)
Output Gap	0.59478***	0.74983***	0.74993***	0.73638***
	(0.035)	(0.036)	(0.036)	(0.033)
Fiscal $Space_{t-1}$	0.01615	0.03953***	0.03932***	0.02188*
	(0.013)	(0.014)	(0.014)	(0.012)
Fiscal Space $squared_{t-1}$	0.00018***	0.00012*	0.00012*	0.00013**
	(0.000)	(0.000)	(0.000)	(0.000)
$Inflation_{t-1}$	2.01598	3.17831	3.20149	-1.99032
	(2.757)	(3.138)	(3.142)	(2.729)
$Investment_{t-1}$		-0.41133***	-0.41213***	-0.42757***
		(0.034)	(0.034)	(0.031)
Population Growth			0.00000	0.00000*
			(0.000)	(0.000)
Initial GDP per capita	-1.59264***			-1.86711***
	(0.142)			(0.139)

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1 The dependent variable is Δln(GDP per capita). All specifications are estimated by panel GLS. The fiscal space data is the 'historical' and 'static' series, which is our fullest data set. The model also includes country fixed effects. Initial GDP per capita is ln(GDP per $capita)_{t-1}$. Please see Appendix (Section 9.1) for information on data.

Extending upon our initial growth regressions in Table 20.0 and 21.0, we incorporate the baseline empirical growth regression following the neoclassical growth model in Table 22.0 and 23.0.[78] The empirical growth model is based on a conditional convergence equation that relates the GDP per capita growth rate to the initial level of income per capita, the investment/saving-to-GDP rate and the population growth rate. The model is augmented to include the level of fiscal space. Since we are interested in checking whether there exists a non-linear impact of government debt on growth, we use a quadratic equation in fiscal

[78] See Solow (1956).

space. Our empirical counterpart to this model includes population growth rate, the savings rate (proxied for by investment), and initial GDP.[79]

Table 23.0: Growth (Solow) and (Dynamic) Fiscal Space.

Dynamic Fiscal Space (Historical) and Growth (Solow)

VARIABLES	(1)	(2)	(3)	(4)
Fiscal Space	-0.02052*	-0.00567	-0.00614	-0.00294
	(0.012)	(0.013)	(0.013)	(0.011)
Fiscal Space squared	-0.00026***	-0.00015**	-0.00015**	-0.00021***
	(0.000)	(0.000)	(0.000)	(0.000)
Inflation	-4.75123*	2.45754	2.40310	0.62859
	(2.886)	(3.267)	(3.267)	(2.856)
Output Gap	0.58661***	0.74578***	0.74579***	0.73399***
	(0.035)	(0.036)	(0.036)	(0.033)
Fiscal Space$_{t-1}$	0.02456**	0.04331***	0.04301***	0.02901***
	(0.012)	(0.013)	(0.013)	(0.011)
Fiscal Space squared$_{t-1}$	0.00014**	0.00011*	0.00011*	0.00010**
	(0.000)	(0.000)	(0.000)	(0.000)
Inflation$_{t-1}$	2.57683	3.51897	3.56458	-1.73876
	(2.786)	(3.147)	(3.148)	(2.753)
Investment$_{t-1}$		-0.41816***	-0.41933***	-0.43581***
		(0.034)	(0.034)	(0.031)
Population Growth			0.00000	0.00000*
			(0.000)	(0.000)
Initial GDP per capita	-1.59718***			-1.88515***
	(0.145)			(0.141)
Constant	51.69454***	12.02558***	12.06749***	68.67579***
	(4.467)	(1.282)	(1.282)	(4.423)

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1 The dependent variable is Δln(GDP per capita). All specifications are estimated by panel GLS. The fiscal space data is the 'historical' and 'static' series, which is our fullest data set. The model also includes country fixed effects. Initial GDP per capita is ln(GDP per capita)$_{t-1}$. Please see Appendix (Section 9.1) for information on data.

The results of our Solow model reinforce the conclusions of our baseline regressions. Across specifications (1) to (4), using both static (Table 22.0) and dynamic (Table 23.0) fiscal space measures, we find consistent results for our variables of focus; fiscal space and fiscal space squared. Fiscal space appears to be insignificant across all four models, having a positive point estimate when more dynamics are added in. Our main finding is that fiscal space

[79] Note that while we have these main variables, there were data limitations in acquiring a proxy for depreciation of physical capital.

squared is negative and significant. This would indicate that, controlling for our Solow determinants of growth, the highest level of growth is still associated with low amounts of fiscal space. Then growth decreases as fiscal space is increased (or debt is paid down), meaning that the larger levels of growth are associated with countries having little/no fiscal space, or having debt-to-GDP as large as sustainably possible.

The remaining coefficients display similar results to above, with the coefficients on output gap, lagged fiscal space, and lagged fiscal space squared all being statistically significant and positive. In our model, the coefficient on lagged investment (proxy for savings) is negative and statistically significant. Although this is opposite to Moral-Benito (2010), and appears counterintuitive, there are studies in the literature that find similar results (Papanek, 1972). Firstly, investment could be limited by savings, so they move together contemporaneously; since high savings precedes an anticipated reduction in growth, investment will likely be inversely correlated with future growth. Another explanation is that because investment is less costly when growth is high, firms will increase their investment in the present if they expect growth to decline in the future.

Another prominent strand of growth literature is endogenous growth theory. Endogenous growth theory holds that investment in human capital, innovation, and knowledge are significant contributors to economic growth. Given this theory, following similar specifications to Moral-Benito (2010) with a wide variety of different variables being investigated, we further control for environmental variables via Barro regressions.

9.3.3. Barro Growth and Fiscal Space

Table 24.0: Growth (Barro) and (Static) Fiscal Space

	Static Fiscal Space (Historical) and Growth (Barro)				
VARIABLES	(1)	(2)	(3)	(4)	(5)
Fiscal Space	0.02085	0.02291	0.03195	0.00037	-0.09387**
	(0.031)	(0.031)	(0.028)	(0.029)	(0.042)
Fiscal Space squared	-0.00025**	-0.00025**	-0.00024**	-0.00021**	0.00007
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Inflation	-0.27139	-0.39017	-2.84848	-1.20413	-43.54913***
	(3.810)	(3.809)	(3.524)	(3.577)	(10.120)
Output Gap	0.73637***	0.73438***	0.35927***	0.31858***	0.24713***
	(0.035)	(0.035)	(0.036)	(0.035)	(0.041)
Fiscal Space$_{t-1}$	0.00789	0.00659	-0.02075	0.00511	0.07742*
	(0.030)	(0.030)	(0.028)	(0.028)	(0.044)
Fiscal Space squared$_{t-1}$	0.00014	0.00014	0.00017*	0.00015	-0.00003
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Inflation$_{t-1}$	-1.69886	-1.58739	0.08925	2.08327	24.83554***
	(3.173)	(3.181)	(2.903)	(2.988)	(9.172)
Investment$_{t-1}$	-0.44070***	-0.44096***	-0.69936***	-0.66456***	-0.73053***
	(0.034)	(0.034)	(0.035)	(0.034)	(0.047)
Population Growth	0.00000	0.00000	0.00000	-0.00000	0.00000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Initial GDP per capita	-2.09268***	-2.20954***	-0.98748***	-0.65741***	-1.58161***
	(0.234)	(0.301)	(0.237)	(0.225)	(0.399)
Education	0.00556	0.00200	0.00976	0.02017***	-0.00349
	(0.009)	(0.011)	(0.008)	(0.008)	(0.009)
Life Expectancy		0.06529	-0.15932**	-0.18393**	0.00105
		(0.110)	(0.081)	(0.076)	(0.103)
Investment			0.65944***	0.60693***	0.79175***
			(0.038)	(0.037)	(0.048)
Government Consumption				-0.42726***	-0.44078***
				(0.055)	(0.073)
Labour Force Participation					-0.05433
					(0.044)
Constant	74.16992***	72.47391***	45.56349***	44.50660***	57.50436***
	(6.200)	(6.956)	(5.243)	(4.830)	(6.306)

Robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$ The dependent variable is Δln(GDP per capita). All specifications are estimated by panel GLS. The fiscal space data is the 'historical' and 'static' series, which is our fullest data set. The model also includes country fixed effects. Initial GDP per capita is ln(GDP per capita)$_{t-1}$. Please see Appendix (Section 9.1) for information on data.

Table 25.0: Growth (Barro) and (Dynamic) Fiscal Space.

Dynamic Fiscal Space (Historical) and Growth (Barro)

VARIABLES	(1)	(2)	(3)	(4)	(5)
Fiscal Space	0.00515	0.00638	0.02506	-0.00331	-0.08180**
	(0.026)	(0.027)	(0.025)	(0.025)	(0.034)
Fiscal Space squared	-0.00020**	-0.00020**	-0.00024***	-0.00022**	0.00004
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Inflation	0.16694	0.03386	-3.32349	-2.02339	-43.3258***
	(3.757)	(3.756)	(3.507)	(3.564)	(9.992)
Output Gap	0.73434***	0.73247***	0.35497***	0.31370***	0.23234***
	(0.035)	(0.035)	(0.036)	(0.035)	(0.040)
Fiscal $Space_{t-1}$	0.01918	0.01815	-0.01752	0.00413	0.05590
	(0.025)	(0.025)	(0.024)	(0.024)	(0.035)
Fiscal Space $squared_{t-1}$	0.00010	0.00010	0.00018**	0.00017**	0.00005
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$Inflation_{t-1}$	-1.82128	-1.72130	0.33354	2.46250	24.15529***
	(3.165)	(3.174)	(2.907)	(3.000)	(9.055)
$Investment_{t-1}$	-0.44727***	-0.44768***	-0.70398***	-0.66853***	-0.73168***
	(0.034)	(0.034)	(0.034)	(0.034)	(0.047)
Population Growth	0.00000	0.00000	0.00000	-0.00000	0.00000*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Initial GDP per capita	-2.12508***	-2.22094***	-0.98535***	-0.65541***	-1.50024***
	(0.235)	(0.302)	(0.237)	(0.225)	(0.393)
Education	0.00599	0.00311	0.01031	0.02052***	-0.00317
	(0.009)	(0.011)	(0.008)	(0.008)	(0.009)
Life Expectancy		0.05271	-0.16948**	-0.19218**	-0.01076
		(0.109)	(0.080)	(0.076)	(0.101)
Investment			0.66179***	0.60968***	0.80225***
			(0.038)	(0.037)	(0.047)
Government Consumption				-0.42691***	-0.45544***
				(0.055)	(0.072)
Labour Force Participation					-0.05522
					(0.042)
Constant	75.23750***	73.93697***	46.18818***	44.89496***	56.31817***
	(6.180)	(6.911)	(5.191)	(4.765)	(6.134)

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1 The dependent variable is Δln(GDP per capita). All specifications are estimated by panel GLS. The fiscal space data is the 'historical' and 'static' series, which is our fullest data set. The model also includes country fixed effects. Initial GDP per capita is ln(GDP per $capita)_{t-1}$. Please see Appendix (Section 9.1) for information on data.

The results of our Barro models in Tables 24.0 and 25.0 again do not show support for our initial growth and fiscal space theory. Instead, they reinforce the conclusions from our other robustness checks: a negative correlation between fiscal space and growth. The majority of our specifications (both Tables 24.0 and 25.0, specifications (1) to (4)) have similar coefficient estimates to the models above. Fiscal space appears to have a positive point estimate, but is insignificant. Fiscal space squared is both negative and statistically significant. Overall this can be interpreted the same as above; that higher growth is associated with lower fiscal space, or countries operating at higher sustainable debt levels. This interpretation still holds in our last model, with labour force participation added in; however the coefficient on fiscal space itself is now negative and significant.

The output gap appears to have a significant and positive coefficient, which is to be expected, solidifying our interpretation of cyclicality. However, now our dynamics in the equation (lags of fiscal space and fiscal space squared) are insignificant. Lagged investment remains significant and negative, following our interpretation above.

We have added in a range of variables in the Barro specifications. Firstly is the initial 'state' variable of the lagged (log) GDP per capita. This would indicate support for the idea of conditional convergence; "the growth rate rises when the initial level of real per capita GDP is low" (Barro, 2003, pg.1). So countries with higher initial GDP per-capita experience lower growth, and countries with lower initial GDP per capita growth at fast rates, converging or 'catching-up' to those with higher GDP. Controlling for these differences does not change the significance of our fiscal space results.

Our Barro model includes proxies for human capital – education and life expectancy. Overall it appears that education is either insignificant, or has a slight positive effect on growth, while increase in life expectancy is associated with lower growth. This would imply a similar interpretation to our age dependency in the fiscal reaction function, whereas the population gets older, there is increased pay-outs of social security etc. from the government, lowering the amount of government spending on investment etc. and lowering tax revenues, leading to a negative effect on growth. Contemporaneous government investment has also been included, which is positive and significant, holding the conventional interpretation; an increase in investment means an increase in capital spending, which boosts aggregate demand and therefore growth. As far as policy implications go, we can delve deeper into our fiscal space question.

Both the Solow and Barro models in GLS form do not show support for our initial results and quadratic growth and fiscal space theory. Rather, they show that lower fiscal space is associated with higher growth, given the positive constant and negative fiscal space coefficients. This indicates that our theory may need further investigation. However, while these GLS regression models incorporate growth theory, they do not deal with such issues as cross-sectional dependence, non-stationarity, and heterogeneity in the data. This is why we put more weighting to our CCE results, concluding that fiscal space displays an inverse quadratic relationship with growth, and fits into current debt-growth theory.