

ISSN 0111-1760

**University of Otago**  
**Economics Discussion Papers**  
**No. 0306**

October 2003

---

## **Sensitivity of technical efficiency estimates to estimation approaches: An investigation using New Zealand dairy industry data<sup>\*</sup>**

By

Mohammad Jaforullah and Erandi Premachandra  
*University of Otago*                      *The University of New South Wales*

*Abstract*

Using data from the New Zealand dairy industry for the year 1993, this paper estimates farm-specific technical efficiencies and mean technical efficiency using three different estimation techniques under both constant returns to scale and variable returns to scale in production. The approaches used are the econometric stochastic production frontier (SPF), corrected ordinary least squares (COLS) and data envelopment analysis (DEA). Mean technical efficiency of the industry is found to be sensitive to the choice of estimation technique. In general, the SPF and DEA frontiers resulted in higher mean technical efficiency estimates than the COLS production frontier.

**Keywords:** Technical efficiency, dairy, production frontier, DEA, New Zealand.

Correspondence to:  
Dr Mohammad Jaforullah  
Senior Lecturer  
Department of Economics  
University of Otago  
PO Box 56, Dunedin  
NEW ZEALAND  
Telephone: +64 3 4879308  
Email: [mjaforullah@business.otago.ac.nz](mailto:mjaforullah@business.otago.ac.nz)  
Fax: +64 3 4798174

---

<sup>\*</sup> The authors would like to thank Dr Robert Alexander for helpful comments and suggestions that led to significant improvement of the paper. The authors are grateful to the New Zealand Dairy Board and the Livestock Improvement Corporation for making the data available. However, the authors are solely responsible for any errors remaining.

# **Sensitivity of technical efficiency estimates to estimation approaches: An investigation using New Zealand dairy industry data**

## **1. Introduction**

The primary focus of this paper is to investigate the sensitivity of technical efficiency measures to estimation techniques using data from the New Zealand dairy industry, the most important industry in the agricultural sector of New Zealand. This type of study has not previously been done in the New Zealand context.

In 1957 M. J. Farrell published a paper entitled “The Measurement of Productive Efficiency” which proved to be seminal. Farrell’s (1957) paper stimulated interest in the area of production frontier estimation and led to the development of several techniques for the measurement of technical, allocative and economic efficiencies. The stochastic production frontier (SPF), developed independently by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977), and data envelopment analysis (DEA), developed by Charnes, Cooper and Rhodes (1978), are two approaches that have been heavily used in the estimation of technical efficiency in production. The statistical deterministic production frontier, developed by Afriat (1972), has not been as popular. The stochastic and statistical approaches utilise a parametric function to represent the production frontier, while DEA, which is based on a linear programming technique, is a non-parametric method. All three methods can also be classified as either stochastic or deterministic. The production frontier in DEA and the one suggested by Afriat (1972) are deterministic in the sense that they assign any deviations from the frontier, even those due to random factors, to inefficiency. On the other hand, the SPF allows the production frontier to be sensitive to random shocks by including a conventional random error term in the specification of the production frontier. As a result, only deviations caused by controllable decisions are attributed to inefficiency.

Since none of the production frontier models used in empirical analyses of production efficiency is without its limitations, it is very important to make a careful choice of model. Coelli (1995) discusses the strengths and weaknesses of different types of

production frontier models. The main criticism of deterministic frontiers is that they rule out the possibility of a deviation from the frontier being caused by measurement error or other noise (such as bad weather). Therefore, any deviations from the estimated frontier are attributed to inefficiency. Econometric stochastic production frontiers, however, obviate this criticism. Furthermore, they provide a measure of the reliability of the technical efficiency estimates by means of the standard errors of the model parameters. However, this benefit comes at the cost of imposing assumptions about the functional form of the production technology and the distribution of the inefficiency term. Avoiding such assumptions is an advantage of the DEA approach.

Although there is a considerable amount of literature in the field of measurement of technical efficiency in production, only a small proportion of this literature is dedicated to comparison of measurement methods of technical efficiency. This proportion is even lower when studies concerning the dairy industry are considered. To our knowledge, only Bravo-Ureta and Rieger (1990) have examined the sensitivity of estimates of technical efficiencies of dairy farms to estimation methods. They used data from six states of the USA on 404 dairy farms for 1982 and 1983. They used four production frontier methods. Three of these methods used deterministic frontiers – linear programming, corrected ordinary least squares and maximum likelihood – and the other – the econometric stochastic production frontier – used a stochastic frontier. They found that estimates of technical efficiencies vary across frontier estimation methods. Studies comparing technical efficiencies from different estimation methods using data from other industries include Jaforullah (1999), Neff, Garcia and Nelson (1993), Sharma, Leung and Zaleski (1999) and Wadud and White (2000), among others. Jaforullah (1999) examined the sensitivity of the estimate of technical efficiency of the Bangladesh handloom textile industry to using parametric, non-parametric, deterministic and stochastic production frontiers. Neff, Garcia and Nelson (1993) compared estimates of technical efficiencies of Illinois grain farms from deterministic parametric frontier, stochastic parametric frontier and DEA. Sharma, Leung and Zaleski (1999) compared estimates of technical, allocative and economic efficiencies from DEA and parametric stochastic frontiers using farm level data from the swine industry of Hawaii. Wadud and White (2000) compared DEA and stochastic frontiers in terms of estimates of technical efficiency using farm level survey data for 150 rice farmers in two villages of Bangladesh. Since the findings of these studies as

to sensitivity of technical efficiency estimates to different methods are mixed, more research comparing technical efficiency measurements from alternative models is needed in order to determine the robustness of estimates from a particular model. The present study analyses the extent to which DEA, econometric stochastic production frontier and the statistical deterministic frontier vary from one another in measuring technical efficiency, using data from the New Zealand dairy industry.

Within the agricultural sector of New Zealand, the dairy industry is the most important industry. For the year ending 1999, 32% of total agricultural production came from the dairy industry (Statistics New Zealand 2001). The industry is highly export oriented. For the same year, dairy exports constituted 20.5% of all merchandise exports of New Zealand. The New Zealand Dairy Board, the exporter of New Zealand's dairy products, is a major player in the international market. New Zealand holds 26% of the international market share, with the European Union (EU) dominating at 40% (New Zealand Dairy Board 1996). Currently, New Zealand's main exporting competitors are the EU, Australia, Canada and the United States of America.

Since the establishment of the GATT Uruguay Round Agriculture agreement in 1995, there has been increased optimism within the dairy sector in New Zealand. This optimism has been reflected in an increase in gross dairy produce over the years since 1995. In 2001, the dairy sector's contribution to the New Zealand GDP was 6.3%. The trade reforms imposed by the Uruguay Round are seen as a critical turning point for the New Zealand dairy industry. Not only have these reforms made the dairy industry more competitive in the international arena, but they have also allowed the industry to become more efficient in terms of its production while maintaining cost levels well below the average world cost (New Zealand Dairy Board 1996). Therefore, the efficiency aspect of the industry is a key ingredient in maintaining and further increasing its competitiveness in the international arena.

The remainder of this paper is organised as follows: Section 2 discusses the basic models used in the study; Section 3 describes the data; Section 4 reports the empirical results from the three models and their comparison; Section 5 concludes the paper.

## 2. Theoretical Models

In specifying the models compared in this paper, it is assumed that a dairy farm produces output ( $Y$ ) using six inputs: labour ( $X_1$ ), capital ( $X_2$ ), total dairy herd ( $X_3$ ), animal health and herd testing ( $X_4$ ), feed supplements and grazing ( $X_5$ ), and fertilizer ( $X_6$ ). The source of data on these inputs is discussed in the next section. The models used in this study are described in this section. Three models are considered in this study. These are the statistical deterministic production frontier, stochastic production frontier and DEA. In both the statistical deterministic production frontier and stochastic frontier models, a Cobb-Douglas production function is used to represent the production technology used by the New Zealand dairy farmers. In defence of this choice, the following can be said. The Cobb-Douglas function has been the most commonly used function in the specification and estimation of production frontiers in empirical studies. It is attractive due to its simplicity and because of the logarithmic nature of the production function that makes econometric estimation of the parameters a very simple matter. It is true, as Yin (2000) points out, that this function may be criticised for its restrictive assumptions such as unitary elasticity of substitution and constant returns to scale and input elasticities, but alternatives such as translog production functions also have their own limitations such as being susceptible to multicollinearity and degrees of freedom problems. A study done by Kopp and Smith (1980) suggests that functional specification has only a small impact on measured efficiency. Furthermore, Coelli and Perelman (1999) points out that if an industry is not characterised by perfectly competitive producers, then the use of a Cobb-Douglas functional form is justified. Considering the New Zealand dairy industry is not perfectly competitive, the use of this functional form is justified.

### 2.1 *The statistical deterministic production frontier*

The statistical deterministic production frontier (Afriat 1972) representing Cobb-Douglas production technology characterised by variable returns to scale is specified as:

$$\ln Y_i = \beta_0 + \sum_{k=1}^6 \beta_k \ln X_{ki} - \varepsilon_i \quad i = 1, 2, \dots, n \quad (1)$$

In equation (1),  $Y_i$  represents  $i^{\text{th}}$  dairy farm's output and  $X_{ki}$  is the amount of the  $k^{\text{th}}$  input used by the  $i^{\text{th}}$  farm. Constant returns to scale in production is imposed via the following restriction on the parameters:

$$\sum_{k=1}^6 \beta_k = 1 \quad (2)$$

The production frontier in equation (1) is deterministic because it includes a one-sided non-negative error term  $\varepsilon_i$ , which is assumed to be independently and identically distributed and has a non-negative mean and constant variance. There are problems in using ordinary least squares (OLS) to estimate this production frontier. According to Greene (1980), while OLS provides best linear unbiased estimates of the slope parameters and appropriately computed standard errors, it does not provide an unbiased estimate of the intercept parameter  $\beta_0$ . The OLS estimator of  $\beta_0$  is biased downward. Due to this problem, it is possible for the estimated OLS residuals of the model to have the incorrect signs. Since the calculation of technical efficiency relies on these residuals being non-positive, Greene (1980) suggests a correction for this biasedness by shifting  $\hat{\beta}_0$ , the OLS estimator of  $\beta_0$ , upward by the largest positive OLS residual ( $e^*$ ). This two-step procedure is known as the corrected ordinary least squares (COLS) method.

The unbiased estimator of the intercept parameter is given by:

$$\hat{\beta}_0^* = \hat{\beta}_0 + e^* \quad (3)$$

This correction makes all the OLS residuals non-positive, implying that the estimates of  $\varepsilon_i$  are non-negative and none of the farms is more than 100 percent efficient. Technical efficiency ( $TE$ ) of the  $i^{\text{th}}$  farm is calculated by using the following equation:

$$TE_i = \exp(-\varepsilon_i) = \exp(e_i - e^*) \quad (4)$$

where  $e_i$  is the OLS residual for the  $i^{\text{th}}$  farm and  $e^*$  is as defined above.

## 2.2 The stochastic production frontier

The stochastic production frontier (SPF) representing Cobb-Douglas production technology characterised by variable returns to scale is specified as:

$$\ln Y_i = \beta_0 + \sum_{k=1}^6 \beta_k \ln X_{ki} + \phi_i \quad i = 1, 2, \dots, n \quad (5)$$

The variables in this equation are the same as in equation (1). The production frontier can be made to represent constant returns to scale production technology by imposing the restriction defined by equation (2). What differentiates this frontier from the deterministic production frontier in equation (1) is a two-sided stochastic component embedded in the disturbance term  $\phi_i$ . The error term in equation (5) is made up of two components:

$$\phi_i = v_i - u_i \quad (6)$$

The first component,  $v_i$ , is a two-sided conventional random error term that is independent of  $u_i$ , and is assumed to be distributed as  $N(0, \sigma_v^2)$ . This component is supposed to capture statistical noise (i.e. measurement error) and random exogenous shocks such as bad weather and machine breakdowns, etc. that disrupt production. The second component  $u_i$  is also a random variable, but unlike  $v_i$ , it is only a one-sided variable taking non-negative values. This term captures technical inefficiency of a dairy farm in producing output. As discussed earlier, one of the disadvantages of the SPF method is that its estimation requires explicit specification of the distribution of the inefficiency term  $u_i$ . There is no consensus among econometricians as to what specific distribution  $u_i$  should have. In previous empirical studies a variety of distributions, ranging from the single-parameter half-normal, exponential and truncated normal distributions to the two-parameter gamma distribution, has been used (see Jaforullah and Devlin (1996), Bravo-Ureta and Rieger (1990), and Battese (1992) and Sharma, Leung and Zaleski (1999)). In this paper, a half-normal distribution for  $u_i$  has been assumed in estimating the stochastic production frontier, i.e. it is assumed that

$$u_i = |U|, \text{ where } U \sim N(0, \sigma_u^2) \quad i = 1, 2, \dots, n \quad (7)$$

Before choosing this distribution, a log-likelihood ratio test was conducted to test the null hypothesis that the probability distribution of  $u_i$  is half-normal against the alternative hypothesis that its distribution is truncated normal. At the 5% level of significance the null hypothesis could not be rejected.

### 2.3 Data envelopment analysis

Data envelopment analysis (DEA) uses a non-parametric piecewise linear production frontier in estimating technical efficiency. A DEA model may be either input-oriented or output-oriented. Both output-oriented and input-oriented DEA models produce the same technical efficiency estimate for a farm under the assumption of constant returns to scale in production. Under the variable returns to scale, the estimates of technical efficiency will differ. However, Coelli (1995) claims that since linear programming does not suffer from statistical problems such as simultaneous equation bias, the choice of a measure does not affect the efficiency estimates significantly. In deciding on the orientation of a DEA model one should also consider over which variables decision making units (DMUs) have most control. If DMUs have more control over output variables than input variables, the DEA model should be output-oriented; otherwise, the model should be input-oriented. Agricultural farms, such as dairy farms, usually have more control over their inputs than their outputs. Considering this fact and Coelli's (1995) assertion, it was decided to use an input-oriented DEA model in the present study.

The input-oriented, non-parametric and deterministic DEA frontier characterised by constant returns to scale is specified as:

$$\begin{array}{ll} \text{Minimise} & \lambda_i \\ & \lambda_i, \mathbf{z} \\ \text{Subject to} & \\ & y_i \leq \mathbf{Yz} \\ & \mathbf{Xz} \leq \lambda_i \mathbf{x}_i \\ & \mathbf{z} \in R_+^N \end{array} \quad (8)$$



In specifying the above linear programming model it is assumed that there are  $N$  dairy farms or DMUs. As before, each farm produces a single output  $y$  using 6 inputs. Therefore,  $y_i$  is the output produced and  $\mathbf{x}_i$  is the  $(6 \times 1)$  vector of inputs used by the  $i^{\text{th}}$  DMU. Other variables can be defined as follows:  $\mathbf{Y}$  is a  $(1 \times N)$  output vector with element  $y_j$  representing the output of farm  $j$ ,  $\mathbf{X}$  is a  $(6 \times N)$  input matrix with element  $x_{kj}$  representing input  $k$  used by farm  $j$  and  $\mathbf{z}$  is an  $(N \times 1)$  vector, the non-zero elements of which identify the fully efficient farms and indicate their importance to the  $i^{\text{th}}$  farm. These fully efficient farms are the peers of the  $i^{\text{th}}$  farm. These peers may be different for different dairy farms. The scalar  $\lambda_i$  measures the technical efficiency of the  $i^{\text{th}}$  dairy farm and by construction  $(1 - \lambda_i)$  measures the technical inefficiency of the farm.  $\lambda_i$  can have any value between zero and one; a value of one indicating that the farm is on the frontier and 100 percent technically efficient, and a value of less than one indicating that the farm is technically inefficient and it can reduce all inputs by at least  $(1 - \lambda_i) \times 100$  percent without affecting output.

In order to incorporate the assumption of variable returns to scale in production into the DEA model, an additional constraint has to be included in the set of constraints of the above model, namely:

$$\mathbf{lz} = 1 \quad (9)$$

where  $\mathbf{l}$  is a  $(1 \times N)$  vector of ones and  $\mathbf{z}$  is as defined above.

### 3. New Zealand Dairy Industry Data

Data for the present study were obtained from the 1993 economic survey of factory-supplying dairy farmers conducted by the Livestock Improvement Corporation Ltd on behalf of the New Zealand Dairy Board. A sample of 452 factory supplying dairy farmers was randomly selected from throughout New Zealand. 82 of the selected farmers declined to take part in the survey and another 76 failed to meet the survey criteria such as having at least 30 cows, deriving at least 50% of their gross income from dairy activities, etc. The remaining 294 farmers were then asked to participate in

the survey, which was administered through interviews conducted by Dairy Board consulting officers. Of the 294 farmers surveyed, 30 provided data in a form that could not be processed. As a result, the final sample consisted of 264 farmers (Livestock Improvement Corporation 1993).

Dairy farms in New Zealand produce many outputs such as processed milk, milkfat, milk protein and milk solids. Although most dairy farmers are primarily focused on dairy production, some farms also produce crops and keep sheep and beef herds as part of their production activities. Therefore, the on-farm labour and capital may be used for purposes other than the production of dairy output. To capture multiple production activities of the dairy farms, it has been decided to use total farm revenue as a proxy for total output of a dairy farm. Total dairy farm revenue includes revenues from all on-farm activities, and sales of dairy products and stock. It should be noted that 66% of the sample farmers are completely dairy farmers, i.e., 100% of their revenues came from dairy activities, and the remaining farmers derived at least 70% of their revenues from dairy produce. This value-based approach to aggregating multiple products of farms has been used in many previous studies. For example, Neff, Garcia and Nelson (1993), Battese and Tessema (1993) and Harris (1993) among others have used this approach to solve the problem of multiple outputs of farms/firms.

The inputs that are important in the production of dairy farm revenues are taken to include labour, capital, total dairy herd, animal health and herd testing, feed supplements and grazing, and fertilizer. Labour is measured by the total number of worker-hours per week including paid and unpaid labour. Capital is measured by the closing book value of fixed assets, including the value of land and buildings. Total dairy herd is the number of cows related to dairy activities. Inputs of animal health and herd testing, feed supplements and grazing, and fertilizer are measured in terms of expenditures on them. Summary statistics of these variables for the sample are shown in Table 1.

**Table 1.** Descriptive statistics for the sample of 264 New Zealand dairy farmers.

Variable	Mean	Median	Standard deviation	Minimum	Maximum
Total revenue (\$000)	165	145	93	39	746
Labour (hours per week)	80	80	36	40	410
Fixed Assets (\$000)	360	313	302	4	2024
Total dairy herd	259	226	133	65	1066
Animal health (\$000)	9	8	6	0.5	34
Feed supplements and grazing (\$000)	9	7	8	0	50
Fertilisers (\$000)	12	10	12	0	85

#### 4. Empirical Results and Analysis

The estimates of the parameters of the statistical deterministic production frontier as specified by equation (1) and the stochastic production frontier (SPF) as specified by equation (5) are presented below in Table 2 for both assumptions of constant returns to scale (CRTS) and variable returns to scale (VRTS). The corrected ordinary least squares (COLS) method, as explained earlier, was used to obtain estimates of the parameters of the statistical deterministic production frontier models. In obtaining estimates of the parameters of the SPF models, the maximum likelihood method implemented in the computer program FRONTIER, Version 4.1, developed by Coelli (1996a) was used.

It can be seen from Table 2 that all estimated parameters have economically meaningful signs and almost all of them are statistically significant at the 1% level of significance, suggesting that the estimates are satisfactory. In the SPF models, the parameter  $\sigma^2$  is the sum of the variances of  $u$  and  $v$ , i.e.  $\sigma_u^2 + \sigma_v^2$ , and the parameter  $\gamma$  is the ratio of the variance of  $u$  to the sum of the variance of  $u$  and  $v$ ,  $\sigma_u^2 / \sigma^2$ . Estimates of these parameters are significant at the 1% level of significance under both constant returns to scale and variable returns to scale.

**Table 2.** Estimated stochastic production frontier (SPF) and corrected ordinary least squares (COLS) production frontier for the New Zealand dairy industry.

	COLS		SPF	
	VRTS	CRTS	VRTS	CRTS
Intercept	4.654*	4.794*	4.939*	5.032*
	(21.061)	(32.911)	(20.978)	(32.337)
$X_1$	0.151*	0.161*	0.152*	0.159*
	(3.453)	(13.079)	(3.505)	(13.536)
$X_2$	0.162*	0.512*	0.160*	0.519*
	(13.083)	(12.955)	(13.479)	(13.149)
$X_3$	0.520*	0.163*	0.524*	0.152*
	(12.773)	(5.641)	(12.883)	(5.371)
$X_4$	0.161*	0.017	0.151*	0.015
	(5.563)	(1.283)	(5.336)	(1.240)
$X_5$	0.018	0.017*	0.016	0.016*
	(1.373)	(2.563)	(1.295)	(2.446)
$X_6$	0.017*	0.015*		
	(2.458)	(2.380)		
$\sigma^2$			0.066*	0.067*
			(5.363)	(5.507)
$\gamma$			0.644*	0.654*
			(4.653)	(5.017)
<i>LLF</i>			55.126	55.027

Notes: Figures in parentheses are asymptotic  $t$  tests. \* Significant at the 1% level.

The fact that  $\gamma$  is statistically significantly different from zero implies that the effect of technical inefficiency plays an important role in the variation of observed dairy farm output. The estimated value of  $\gamma$  in the VRTS SPF model, which is 0.644, implies that 64.4% of the total variation in dairy farm output is due to technical inefficiency. Similarly, the value of  $\gamma$  in the CRTS SPF model, which is 0.654, implies that 65.4% of the total variation in dairy farm output is due to technical inefficiency.

The distributions of individual technical efficiency estimates from the three models under both the CRTS and VRTS assumptions are presented in Table 3 along with some descriptive statistics. Here it should be mentioned that the DEA models were solved by the computer program DEAP, Version 2.1 developed by Coelli (1996b).

**Table 3.** Frequency distributions of technical efficiency estimates from the SPF, the DEA and the COLS, under both CRTS and VRTS assumptions.

Technical efficiency	COLS (VRTS)	COLS (CRTS)	SPF (VRTS)	SPF (CRTS)	DEA (VRTS)	DEA (CRTS)
$0 < TE \leq 0.1$	0	0	0	0	0	0
$0.1 < TE \leq 0.2$	0	0	0	0	0	0
$0.2 < TE \leq 0.3$	0	0	0	0	0	0
$0.3 < TE \leq 0.4$	14	14	0	0	0	0
$0.4 < TE \leq 0.5$	52	46	0	0	1	2
$0.5 < TE \leq 0.6$	104	110	0	1	4	9
$0.6 < TE \leq 0.7$	66	65	9	10	24	40
$0.7 < TE \leq 0.8$	18	19	39	40	52	74
$0.8 < TE \leq 0.9$	7	7	151	151	58	53
$0.9 < TE \leq 1$	3	3	65	62	125	76
Total	264	264	264	264	264	264
Mean	0.569	0.573	0.855	0.853	0.86	0.807
Standard Deviation	0.112	0.113	0.065	0.067	0.125	0.139
Minimum	0.307	0.305	0.608	0.599	0.464	0.414
Maximum	1	1	0.958	0.958	1	1

The estimated mean technical efficiencies from the COLS frontier models are very similar under both CRTS and VRTS assumptions. The same holds for the SPF models. However, for the DEA model, the mean technical efficiency is greater for the VRTS assumption than for the CRTS assumption. The last outcome is not surprising. As Coelli (1996b) states, the DEA model under variable returns to scale envelopes the data points more tightly than under the constant returns to scale, thereby yielding higher mean technical efficiency (TE) scores relative to the CRTS model. As to the distribution of technical efficiency estimates, it seems that it is not particularly sensitive to the assumption about returns to scale except in the case of the DEA model.

A comparison of the distributions of TE estimates from different models shows that the distribution is relatively symmetric in the COLS model, while it is skewed to the left in both the SPF model and the DEA model. This fact is also obvious from Figures 1-3 that represent the distributions of TE estimates in Table 3. However, the DEA technical efficiency measures show significantly higher variability than the stochastic TE measures. The longer and fatter tail of the distribution associated with the DEA model indicates that there is more variability in the TE scores derived under the DEA approach relative to the SPF approach. In contrast to efficiency scores in the COLS model, the TE estimates of both the SPF and the DEA models are clustered around the upper end of the TE distributions, indicating that most dairy farms in New Zealand are near to or at full technical efficiency. However, no farm is one hundred percent efficient in the SPF models (ie. at the efficient frontier). This is due to the stochastic nature of the frontier; it allows for the possibility that part of the deviation of the observed output from the frontier may be due to noise or measurement errors.

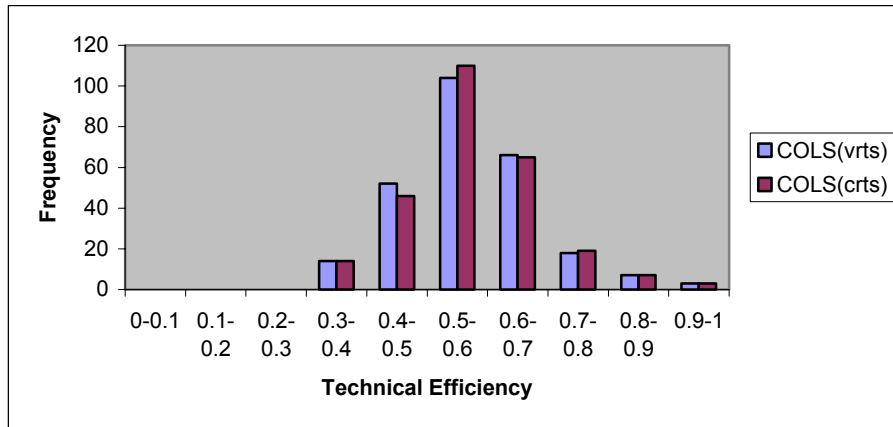


Figure 1 Histograms of the TE estimates from the COLS models.

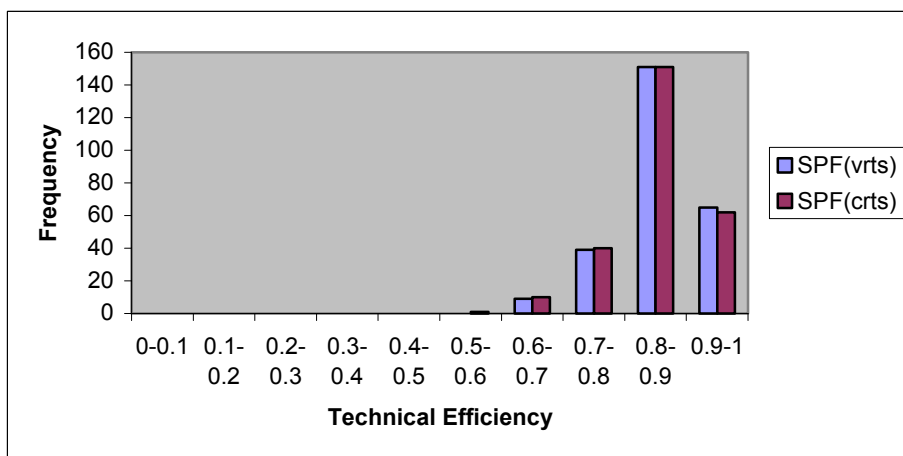


Figure 2 Histograms of the TE estimates from the SPF models.

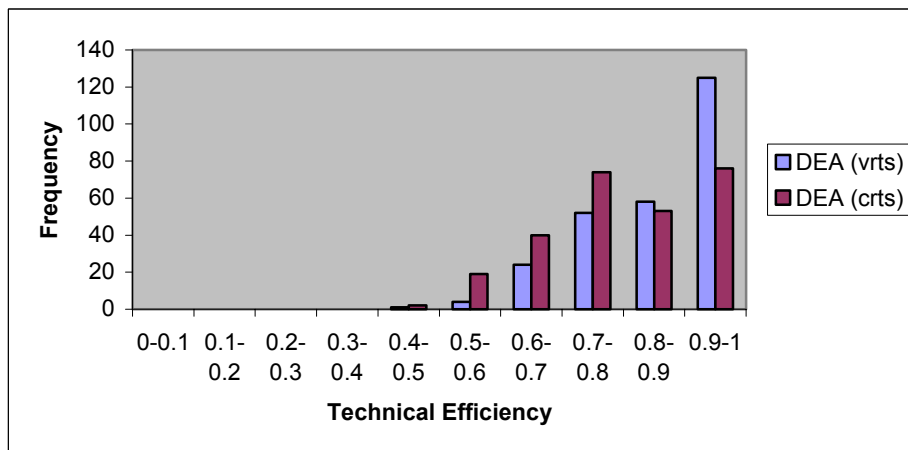


Figure 3 Histograms of the TE estimates from the DEA models.

From Table 3, it can be seen that the mean technical efficiency of New Zealand dairy industry is sensitive to model choice. Under the assumption of constant returns to

scale, the mean TE of the industry varies from 57.3% to 85.3% while under the assumption of variable returns to scale it varies from 56.9% to 86.9%. The COLS model produces the smallest mean technical efficiency while the DEA and SPF models produce distinctly higher mean technical efficiency for the New Zealand dairy industry under both scale assumptions. Under the assumption of VRTS, the DEA model has a higher mean technical efficiency for the New Zealand dairy industry than the SPF model while the reverse holds under the assumption of CRTS. Yin (2000) claims that the stochastic frontier in general leads to higher average technical efficiencies compared to the DEA frontiers. This claim is not supported by the results of the present study as mean TE generated by the DEA model for the New Zealand dairy industry is higher than that generated by the SPF model under variable returns to scale.

Statistical Z-tests or normal tests have been conducted in order to test whether the mean technical efficiencies obtained from the three models are significantly different from one another. These results are reported in Table 4.

**Table 4.** Hypothesis tests regarding the mean technical efficiencies ( $\mu_{TE}$ ) from the three models: COLS, SPF and DEA .

<i>Normal tests for the models incorporating variable returns to scale</i>			
Hypothesis			
$H_0 :$	$\mu_{TE}COLS = \mu_{TE}SPF$	$\mu_{TE}COLS = \mu_{TE}DEA$	$\mu_{TE}SPF = \mu_{TE}DEA$
$H_1 :$	$\mu_{TE}COLS \neq \mu_{TE}SPF$	$\mu_{TE}COLS \neq \mu_{TE}DEA$	$\mu_{TE}SPF \neq \mu_{TE}DEA$
Calculated Z statistic	-35.862	-29.062	-1.636
Decision	Reject $H_0$ at the 1% level of significance	Reject $H_0$ at the 1% level of significance	Do not reject $H_0$ at the 5% level of significance
<i>Normal tests for the models incorporating constant returns to scale</i>			
Hypothesis			
$H_0 :$	$\mu_{TE}COLS = \mu_{TE}SPF$	$\mu_{TE}COLS = \mu_{TE}DEA$	$\mu_{TE}SPF = \mu_{TE}DEA$
$H_1 :$	$\mu_{TE}COLS \neq \mu_{TE}SPF$	$\mu_{TE}COLS \neq \mu_{TE}DEA$	$\mu_{TE}SPF \neq \mu_{TE}DEA$
Calculated Z statistic	-34.817	-21.217	4.914
Decision	Reject $H_0$ at the 1% level of significance	Reject $H_0$ at the 1% level of significance	Reject $H_0$ at the 1% level of significance



The tests reject the null hypothesis that mean technical efficiencies from any two models are the same under both scale assumptions, except for the special case of the stochastic production frontier and DEA under the assumption of VRTS where the test fails to reject the null hypothesis at the 5% level of significance.

Both the ANOVA test and the Kruskal-Wallis test were also conducted in order to test the hypothesis that the mean technical efficiencies from the three models are the same against the alternative hypothesis that at least two of them differ from one another. As the ANOVA test requires the population variances to be equal in the three models, the results derived from this test alone may not be valid. Therefore, the Kruskal-Wallis test was also carried out. It does not require any assumptions regarding the normality or variances of the populations. These results are reported in Table 5. At the 5% level of significance, these tests reject the null hypothesis in favour of the alternative. These results further strengthen the findings from Table 4.

**Table 5.** Hypothesis tests regarding the mean technical efficiencies ( $\mu_{TE}$ ) from the three models: COLS, SPF and DEA.

Even though the statistical deterministic frontier, COLS, has produced technical efficiency measures that are quantitatively different from those of the stochastic production frontier and DEA, it may still be consistent with the other methods in ranking the individual farms in terms of efficiency. In many policy making situations, information on the ranking of farms in terms of efficiency may be more important than the quantitative estimates of technical efficiencies of farms. To assess the overall consistency of the three methods in ranking individual farms in terms of efficiency, the coefficient of Spearman rank-order correlation has been calculated between the three models. The estimates are presented in Tables 6 and 7.

Analysis of Variance (ANOVA) test		
	VRTS	CRTS
Hypothesis		
$H_0 :$	$\mu_{TE}COLS = \mu_{TE}SPF = \mu_{TE}DEA$	$\mu_{TE}COLS = \mu_{TE}SPF = \mu_{TE}DEA$
$H_1 :$	$\mu_{TE}COLS \neq \mu_{TE}SPF \neq \mu_{TE}DEA$	$\mu_{TE}COLS \neq \mu_{TE}SPF \neq \mu_{TE}DEA$
Calculated $F$ statistic		
	$F_{(2,789)} = 701.199$	$F_{(2,789)} = 489.982$
Decision	Reject $H_0$ at the 5% level of significance	Reject $H_0$ at the 5% level of significance
Kruskal-Wallis test		
Hypothesis		
$H_0 :$	$\mu_{TE}COLS = \mu_{TE}SPF = \mu_{TE}DEA$	$\mu_{TE}COLS = \mu_{TE}SPF = \mu_{TE}DEA$
$H_1 :$	$\mu_{TE}COLS \neq \mu_{TE}SPF \neq \mu_{TE}DEA$	$\mu_{TE}COLS \neq \mu_{TE}SPF \neq \mu_{TE}DEA$
Calculated $\chi^2$ test statistic		
	$\chi^2_{(2)} = 457.977$	$\chi^2_{(2)} = 419.018$
Decision	Reject $H_0$ at the 5% level of significance	Reject $H_0$ at the 5% level of significance

**Table 6.** Spearman rank correlation matrix of technical efficiency ranking obtained from the three models incorporating variable returns to scale.

	COLS	SPF	DEA
COLS	1		
SPF	0.99	1	
DEA	0.58	0.56	1

**Table 7.** Spearman rank correlation matrix of technical efficiency ranking obtained from the three models, incorporating constant returns to scale.

	COLS	SPF	DEA
COLS	1		
SPF	0.99	1	
DEA	0.75	0.74	1

The correlation coefficients are all significantly different from zero, as suggested by the Z statistic at the 5% level of significance. Under both scale assumptions, the TE estimates from the SPF and COLS models are the most highly correlated, while the correlation between the TE estimates from the SPF and DEA models are the least highly correlated. Since the correlation coefficients between the TE estimates from the three models are significantly different from zero and greater than 0.5, it can be concluded that the three models are consistent in their ranking of farms in terms of technical efficiency. However, they are more in accord in ordering farms under the CRTS assumption than under the VRTS assumption. Furthermore, the agreement between the COLS and SPF models in ranking farms is the greatest and almost perfect.

## 5. Summary and conclusions

This paper set out to compare the empirical performance of three popular approaches to estimation of technical efficiency in production: corrected ordinary least squares regression (COLS), stochastic production frontier (SPF) and data envelopment analysis (DEA). The comparison has focused on measuring the technical efficiency of dairy farms in New Zealand under two scale assumptions: constant returns to scale (CRTS) and variable returns to scale (VRTS). The general findings from this study indicate that estimates of technical efficiencies of individual dairy farms, and therefore the mean technical efficiency of the New Zealand dairy industry, are sensitive to the choice of production frontier estimation method. Of the three models considered for the dairy industry, the statistical deterministic frontier, i.e., COLS, produces the lowest mean technical efficiency while the SPF produces the highest

mean TE in general. However, it is not always the case that the SPF models produce a larger mean TE than the DEA models. The mean TE estimates from the SPF and the DEA models show that dairy farms in New Zealand are operating near to or at the efficient frontier. Individual farm TE estimates exhibit greater variability under both the CRTS DEA and the VRTS DEA models than under the COLS and SPF models. Although the SPF and DEA estimates conform to a large extent, ranking differences do exist between them. The three models are more consistent in their ranking of dairy farms in terms of technical efficiency under the assumption of CRTS than under the assumption of VRTS. The results also indicate that the choice of scale assumption does not significantly affect the mean technical efficiency estimate for the dairy industry.

The findings above are consistent with those of comparable studies done in the past. Jaforullah (1993) found the mean TE from the deterministic frontier to be lower than from the stochastic frontier. Neff, Garcia and Nelson (1993) also found the stochastic frontier to yield higher mean TE estimates compared to the deterministic models. They also found the correlation between the parametric measures to be very high, but the correlation between parametric and non-parametric models to be fairly low. Similar to the results derived in this paper, they found DEA to yield TE estimates that were more variable than those from the stochastic frontier. Wadud and White (2000) found a significantly higher level of mean technical efficiency under VRTS DEA than under the CRTS DEA and stochastic frontier models. They also found greater variability in the DEA models, but low correlation between the parametric and non-parametric models.

The above findings lead to the conclusion that if one aims at estimating mean technical efficiency of an industry, it is advisable that one uses different methods of efficiency estimation as opposed to a single method, as the measurement of technical efficiency is sensitive to the choice of estimation method. Such an approach will produce better information on the technical efficiency of the industry by producing a range within which the true technical efficiency may lie. The narrower the range, the more confident a researcher can be about the technical efficiency of the industry. However, if one is keen to use only one estimation method then, in choosing the method, one must consider the type of the industry under study, the type of data in

hand, the strengths and weaknesses of estimation methods and the objectives of the study. For example, if one intends to estimate the mean technical efficiency of an agricultural industry, the production frontier considered should be stochastic in nature as deviations from the frontier may easily be caused by random factors such as droughts, unexpected disruptions in the supply and demand for inputs and outputs, etc., over which farmers do not have any control.

## References

- Afriat, P. (1972). Efficiency estimation of production functions. *International Economic Review* 13: 568-598.
- Aigner, D.J., Lovell, C. A. K. and Schmidt, P. (1977). Formulation and estimation of stochastic frontier production function models. *Journal of Econometrics* 6: 21-27.
- Battese, G. E. (1992). Frontier production functions and technical efficiency: A survey of empirical applications in agricultural economics. *Agricultural Economics* 7: 185-208.
- Battese, G. E. and Tessema, G. A. (1993). Estimation of stochastic frontier production functions with time-varying parameters and technical efficiencies using panel data from Indian villages. *Agricultural Economics* 9: 313-333.
- Bravo-Ureta, B. E. and Rieger, L. (1990). Alternative production frontier methodologies and dairy farm efficiency. *Journal of Agricultural Economics* 41: 215-226.
- Charnes, A., Cooper, W. W. and Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operations Research* 2: 429-444.
- Coelli, T. J. (1995). Recent developments in frontier modelling and efficiency measurement. *Australian Journal of Agricultural Economics* 39: 219-245.
- Coelli, T. J. (1996a). A Guide to FRONTIER Version 4.1: A Computer Program for Stochastic Frontier Production and Cost Function Estimation. CEPA Working Paper 96/07, Department of Econometrics, University of New England. Armidale.
- Coelli, T. J. (1996b). A Guide to DEAP Version 2.1: A Data Envelopment Analysis (Computer) Program. CEPA Working Paper 96/08, Department of Econometrics, University of New England. Armidale.
- Coelli, T. J. and Perelman, S. (1999). A comparison of parametric and non-parametric distance functions: With application to European railways. *European Journal of Operational Research* 117: 326-339.
- Farrell, M. J. (1957). The measurement of productive efficiency. *Journal of the Royal Statistical Society A120*: 253-290.
- Greene, W. H. (1980). Maximum likelihood estimation of econometric frontier functions. *Journal of Econometrics* 13: 27-56.
- Harris, R. I. D. (1993). Measuring efficiency in New Zealand manufacturing in 1986/87 using a frontier production function approach. *New Zealand Economic Papers* 27: 57-79.
- Jaforullah, M. (1999). A comparison of production frontier methods using Bangladesh handloom textile industry data. *Economics Discussion Papers 9911*, University of Otago. Dunedin, Otago.
- Jaforullah, M. and Devlin, N. J. (1996). Technical efficiency in the New Zealand dairy industry: A frontier production function approach. *New Zealand Economic Papers* 30: 1-17.
- Kopp, R. J. and Smith, V. K. (1980). Frontier production function estimates for steam electric generation: A comparative analysis. *Southern Economic Journal* 47: 1049-59.

- Livestock Improvement Corporation (1993). 1993 Economic Survey of Factory Supply Dairy Farmers. Statistics Section, Livestock Improvement Corporation Limited: Hamilton.
- Meeusen, W. and van den Broeck, J. (1977). Efficiency estimation from Cobb-Douglas production functions with composed error. *International Economic Review* 18: 435-444.
- Neff, D. L., Garcia, P. and Nelson, C. H. (1993). Technical efficiency: A comparison of production frontier methods. *Journal of Agricultural Economics* 44: 479-489.
- New Zealand Dairy Board (1996). A Survey of the New Zealand Dairy Industry. Trade Strategy Section, New Zealand Dairy Board: Wellington.
- Statistics New Zealand (2001). New Zealand Official Yearbook. 102<sup>nd</sup> Edition, GP Publications. Wellington.
- Sharma, K. R., Leung, P. and Zaleski, H. M. (1999). Technical, allocative and economic efficiencies in swine production in Hawaii: A comparison of parametric and non-parametric approaches. *Agricultural Economics* 20: 23-35.
- Wadud, A. and White, B. (2000). Farm household efficiency in Bangladesh: A comparison of stochastic frontier and DEA methods. *Applied Economics* 32: 1665-1673.
- Yin, R. (2000). Alternative measurements of productive efficiency in the global bleached softwood pulp sector. *Forest Science* 46: 558-569.