PROFESSIONAL DEVELOPMENT FOR USING TECHNOLOGY IN MATHEMATICS TEACHING IN GHANA

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Abstract

Drawing on the framework of TPACK (technological pedagogical content knowledge) and the principles of effective mathematics pedagogy, this study set out with two aims. First, shifts in teachers’ technology dispositions (beliefs, attitudes, and knowledge) were explored after engagement in the professional development programme mediated with GeoGebra software. Second, typical features and nuances of the complexities of enacting effective mathematics pedagogy in a GeoGebra learning environment were examined.

Eleven in-service mathematics teachers from a senior high school in Ghana were engaged in a professional development programme for 12 months. They were introduced to the use of GeoGebra software in mathematics teaching, and then designed GeoGebra-based mathematics lessons, which they taught to their peers and subsequently their students in the mathematics classroom.

Self-report questionnaire, interviews, focus group discussions, lesson plans, and lesson observations were used for data collection. The results provided evidence that within GeoGebra-based mathematics lessons, teachers were able to enact, to different degrees, five practices central to effective mathematics pedagogy: creating mathematical setting, providing useful mathematical tasks, orchestrating mathematical discussions, making mathematical connections, and assessing students’ learning. Further analysis of the data provided evidence for theorising 31 core practices across these central themes of effective mathematics pedagogy. Following their engagement in the professional development, the teachers enacted these practices to greater or lesser extents. However, it was problematic for most teachers to effectively engage their students in deep mathematical discussion.

Engaging teachers to design and teach with GeoGebra in the mathematics content area offered a unique lens for understanding the shift in the teachers’ dispositions towards the use of technology in mathematics teaching and learning. As teachers engaged in using GeoGebra, their knowledge and perceived beliefs about the usefulness and nature of technology in mathematics education became profound. The teachers improved their technological pedagogical content knowledge (TPACK) during the study. Analysis of the components of TPACK showed that they improved their knowledge of the mathematics content as well as knowledge of technology, teaching, and students’ learning. However, their
intention to put new pedagogical approaches into classroom practice in the future depended on multiple contextual factors including administrative support, continual professional training, and provision of adequate technology facilities. The findings from this study have implications for Ghana’s senior high school mathematics education, TPACK, effective mathematics pedagogy, professional development, and research methodology.
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# Table of Contents

Abstract .......................................................................................................................... I  
Acknowledgements ......................................................................................................... III  
Table of Contents ........................................................................................................... V  
List of Tables .................................................................................................................. X  
List of Figures ................................................................................................................. XI  

CHAPTER ONE: INTRODUCTION  
My Motivation ................................................................................................................. 1  
Mathematics Curriculum and Technology ...................................................................... 3  
The Research .................................................................................................................... 5  
The Significance of the Study .......................................................................................... 6  
The Structure of the Thesis .............................................................................................. 6  

CHAPTER TWO: CONTEXT OF THE STUDY  
The Education System in Ghana ..................................................................................... 7  
Mathematics Education in Ghana .................................................................................... 9  
Technology in Ghana’s Education .................................................................................. 11  
Existence of ICT integration policy ................................................................................ 11  
Availability of ICT facilities ............................................................................................ 13  
Expertise .......................................................................................................................... 19  
Availability of professional development ........................................................................ 20  
Summary ......................................................................................................................... 23  

CHAPTER THREE: LITERATURE REVIEW  
Technology in Mathematics Education .......................................................................... 25  
Technology for doing mathematics ................................................................................ 27  
Technology for practicing mathematics skills ............................................................... 29  
Technology for developing mathematical concepts ....................................................... 31  
Theoretical Frameworks for Effective Technology Integration ....................................... 34  
Teachers’ beliefs ............................................................................................................. 35  
Teachers’ beliefs and technology integration .................................................................. 39  
Technology pedagogical content knowledge (TPACK) .................................................. 42  
Conceptualising effective mathematics pedagogy ......................................................... 48  
Professional Development .............................................................................................. 56  
Teacher Design Team Professional Development ......................................................... 58  
Potential of GeoGebra ..................................................................................................... 64
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>66</td>
</tr>
<tr>
<td>CHAPTER FOUR: METHODOLOGY AND RESEARCH DESIGN</td>
<td>68</td>
</tr>
<tr>
<td>Research Paradigm</td>
<td>68</td>
</tr>
<tr>
<td>Situating the Study in a Qualitative Research Methodology</td>
<td>70</td>
</tr>
<tr>
<td>Research Design</td>
<td>71</td>
</tr>
<tr>
<td>The Structure of the Professional Development</td>
<td>75</td>
</tr>
<tr>
<td>How the TPACK Framework was Applied in this Research</td>
<td>80</td>
</tr>
<tr>
<td>Ethical Considerations</td>
<td>81</td>
</tr>
<tr>
<td>The Research Setting</td>
<td>82</td>
</tr>
<tr>
<td>Participants</td>
<td>82</td>
</tr>
<tr>
<td>Cynthia</td>
<td>84</td>
</tr>
<tr>
<td>Prince</td>
<td>84</td>
</tr>
<tr>
<td>Mary</td>
<td>84</td>
</tr>
<tr>
<td>Gideon</td>
<td>85</td>
</tr>
<tr>
<td>Jonathan</td>
<td>85</td>
</tr>
<tr>
<td>Joshua</td>
<td>85</td>
</tr>
<tr>
<td>Sammy</td>
<td>86</td>
</tr>
<tr>
<td>Michael</td>
<td>86</td>
</tr>
<tr>
<td>Bernard</td>
<td>86</td>
</tr>
<tr>
<td>Peter</td>
<td>86</td>
</tr>
<tr>
<td>Martey</td>
<td>87</td>
</tr>
<tr>
<td>Methods for Data Collection</td>
<td>87</td>
</tr>
<tr>
<td>Semi-structured interviews</td>
<td>88</td>
</tr>
<tr>
<td>Focus group discussions</td>
<td>88</td>
</tr>
<tr>
<td>Self-report questionnaire</td>
<td>89</td>
</tr>
<tr>
<td>Observation of lesson enactment</td>
<td>90</td>
</tr>
<tr>
<td>Field notes</td>
<td>91</td>
</tr>
<tr>
<td>Data Analysis</td>
<td>91</td>
</tr>
<tr>
<td>Interviews and focus group discussions</td>
<td>92</td>
</tr>
<tr>
<td>Lesson enactment</td>
<td>94</td>
</tr>
<tr>
<td>Self-report questionnaire</td>
<td>95</td>
</tr>
<tr>
<td>Establishing the Trustworthiness of the Findings of the Study</td>
<td>96</td>
</tr>
<tr>
<td>Credibility</td>
<td>97</td>
</tr>
<tr>
<td>Transferability</td>
<td>100</td>
</tr>
</tbody>
</table>
CHAPTER FIVE: RESULTS

Teachers’ Initial Dispositions towards Technology ................................................................. 102
Perceived beliefs about the usefulness of technology in mathematics .................................. 103
Perceived beliefs about the nature and utilisation of technology ......................................... 108
Feelings towards technology .................................................................................................. 110
Intentions towards the use of technology ............................................................................... 110
Self-efficacy towards technology ............................................................................................ 111
Contextual influence ................................................................................................................ 114
Summary .................................................................................................................................. 116

Enactment of Effective Mathematics Pedagogy ............................................................ 117
Creating a mathematical setting ............................................................................................. 117
Useful mathematical tasks ....................................................................................................... 128
Mathematical discussion ........................................................................................................... 137
Mathematical connections ....................................................................................................... 144
Assessment of students’ learning ............................................................................................ 149
Summary .................................................................................................................................. 154

Shift in Teachers’ Technology Dispositions ....................................................................... 155
Changes in perceived usefulness of technology .................................................................. 155
Teachers’ affective attitudes towards technology ................................................................. 158
Behavioural attitudes toward technology ............................................................................... 160
Changes in teachers’ self-efficacy towards technology ........................................................... 163
Summary .................................................................................................................................. 179

CHAPTER SIX: DISCUSSION AND CONCLUSIONS .......................................................... 180

Teachers’ Initial Technology Dispositions ............................................................................ 181
Perceived beliefs of the usefulness of technology ................................................................. 181
Perceived nature and utilisation of technology .................................................................... 183
Perceived feelings and intention towards the use of technology ........................................ 184
Perceived self-efficacy with technology ............................................................................... 184
Perceived contextual influence ............................................................................................. 185

Enactment of Effective Mathematics Pedagogy ............................................................ 187
Creating a mathematical setting ............................................................................................. 187
Appendix L: Teachers perceived Belief about the Usefulness of Technology in Mathematics (themes extracted and organised from HyperRESEARCH) .......... 281
Appendix M: Proposed Framework for Analysing the Core Practice of Effective Mathematics Pedagogy (Based on the literature reviewed for the study) ......... 282
Appendix N: Sample Coding of Core Practices of Effective Mathematics Pedagogy (HyperRESEARCH output) ................................................................. 283
Appendix O: Sample Lesson Plan ................................................................. 288
Appendix P: Sample students’ worksheet ...................................................... 296
List of Tables

Table 3. 1 Elements of effective mathematics pedagogy .......................................................... 51
Table 4. 1 Activities of the GeoGebra introduction ................................................................. 78
Table 4. 2 Summary of data collection procedure and analysis .................................................. 91
Table 5. 1 Teachers’ awareness and use of mathematics software ......................................... 112
Table 5. 2 Teachers’ enactment of mathematical setting .......................................................... 118
Table 5. 3 Technologies the teachers adopted for their lessons ................................................. 119
Table 5. 4 Teachers’ enactment of mathematical task ............................................................. 128
Table 5. 5 Teachers’ enactment of mathematical discussion ..................................................... 138
Table 5. 6 Teachers enactment of mathematical connection .................................................... 145
Table 5. 7 Teachers’ enactment of assessment of students’ learning ....................................... 149
Table 5. 8 Changes in perceived usefulness of technology ...................................................... 155
Table 5. 9 Perceived change in the affective attitudes towards technology ............................... 159
Table 5. 10 Perceived changes behavioural attitudes towards technology .............................. 161
Table 5. 11 Perceived knowledge and use of technology ......................................................... 163
Table 6. 1 The core practices of effective mathematics pedagogy ........................................... 218
List of Figures

Figure 2. 1 Location of Ghana on Africa Map.................................................................7
Figure 2. 2 Students' performance in mathematics......................................................10
Figure 3. 1 Didactical functions of technology in mathematics education.........................27
Figure 3. 2 The TPACK framework (Koehler & Mishra, 2009)........................................43
Figure 3. 3 The interface of the GeoGebra window......................................................64
Figure 4. 1 Activities of the professional development................................................75
Figure 5. 1 Teachers’ perceived pedagogical importance of technology.............................103
Figure 5. 2 Teachers perceived nature and ways to use technology...................................108
Figure 5. 3 Teachers’ initial perceived hindrances to technology use................................114
Figure 5. 4 Rotation of object (Cynthia)........................................................................120
Figure 5. 5 Instruction for setting GeoGebra window (Cynthia).........................................121
Figure 5. 6 Students' task on rotation on rotation of object (Dorothy)............................122
Figure 5. 7 Illustration of real-life application quadratic function (Gideon).........................125
Figure 5. 8 Surface area of a cylinder (Bernard)............................................................130
Figure 5. 9 Distance-time graph (Joshua)........................................................................131
Figure 5. 10 Sum of interior angles polygon (Sammy).....................................................133
Figure 5. 11 Equation of a circle (Joshua).......................................................................136
Figure 5. 12 Students’ worksheet on area and perimeter of a rectangle (Martey)............138
Figure 5. 13 Sample solution on a task involving trigonometric ratios............................140
Figure 5. 14 Circle theorem (Peter)..............................................................................141
Figure 5. 15 Area and perimeter of rectangle (Martey)....................................................145
Figure 5. 16 Students’ task on area and perimeter of a rectangle (Martey).....................146
Figure 5. 17 Sample task on mensuration (Bernard).......................................................147
Figure 5. 18 Cylinder in the 3D graphics in GeoGebra (Bernard)......................................148
Figure 5. 19 Affordance of the checkbox in GeoGebra (Michael).....................................150
Figure 5. 20 Teachers’ conception of the transformation of trigonometric function...........167
CHAPTER ONE: INTRODUCTION

In this introductory chapter, I sketch the origins of this thesis. First, the rationale for the thesis is described through the lens of my personal motivation and experience in the study of mathematics. The second section briefly situates the rationale of the thesis in the mathematics curriculum and technology literature by identifying the tensions related to effective implementation of technology in the classroom. The approach of the professional development programme, GeoGebra, and the theoretical framework underpinning the study are briefly introduced. The research questions are then outlined. The chapter also details the significance of the study. The final section describes the structure of the subsequent chapters.

My Motivation
I grew up in a farming community of about 500 population in the Central Region of Ghana. The community had no primary school. Together with other children who grew up in the village I commuted on foot (about seven miles) each day for school. At primary five (age 11), I represented my school in an inter-zonal mathematics quiz competition. The competition comprised of 12 primary schools from three different towns. I placed third in the competition. My happiness was that, among the contestants, I was the only pupil in primary five. The rest were in primary six, a year ahead of me. As far as I can remember, that success marked the beginning of my interest in mathematics. My performance in the subject grew year by year. I developed an interest in assisting my peers who needed help in the subject from primary school. At the senior school level, I participated in a number of quiz competitions, which I won. At the College of Education level, where I was trained as a teacher in 2001-2004, I became the overall best student and mathematics was one of the subjects in which I had the highest score. This motivated me to read mathematics at a higher level. I completed university with first class honours in mathematics education and I was amongst the top two percent of the class. I repeated a similar performance in my master’s degree programme in mathematics education.

During my third year at university, I was introduced to the use of technology in mathematics teaching in a PhD research study conducted by one of our lecturers. I was among four participants in that study. I took a keen interest in learning how to use an Excel spreadsheet in mathematics teaching. This motivated me, during my master’s degree, to explore how
students learn quadratic functions through Excel spreadsheet-supported lessons. After my master’s degree programme, I became interested in GeoGebra software as well. I adopted these two technologies for classroom use and co-authored two papers, with one of the papers being based on my master’s degree thesis, and exploring the potential of an Excel spreadsheet on students’ learning of quadratic functions (Benning & Agyei, 2015). The other paper explored pre-service teachers’ views and their use of GeoGebra software (Agyei & Benning, 2015).

I have twelve years of teaching experience in mathematics prior to undertaking my PhD studies. I taught mathematics for five years in the school where the study took place. I became head of the mathematics department during the last two years. I organised three different professional learning programmes for the teachers in the department: (i) principles of test construction, administration, and analysis; (ii) problem posing as a method of teaching mathematics; and (iii) using technology to enhance effective mathematics teaching. The professional learning in technology was generic to create an awareness of the use of technology in the classroom.

My experience in using both the Excel spreadsheet and GeoGebra to support high school students to learn mathematics has made me form a belief that using technology in mathematics instruction has a far-reaching impact on students’ learning. For example, I realised these tools have the potential to initiate the students into communication which is a good starting point for meaningful mathematics learning. However, my primary worry was that most teachers in Ghana have yet to make full use of technology in their mathematics classroom (Agyei & Voogt, 2011; Mereku & Mereku, 2015). It has been a little over a decade since Ghana’s Ministry of Education, made it mandatory for teachers to use technology to assist students in exploring mathematics concepts. This mandate implies that professional development and positive teachers’ dispositions towards technology are critical in translating the curriculum intention into practice (Ertmer, 1999; Lawless & Pellegrino, 2007; Mishra & Koehler, 2006; Webb & Cox, 2004).

Consequently, this thesis sets out to explore how professional development may impact on the participant teachers’ dispositions towards the use of technology in mathematics teaching. The teachers were engaged in a one-year professional development programme at their own school, and were supported to explore the pedagogical use of GeoGebra in mathematics
teaching. In this thesis, teachers’ dispositions were operationalised to encompass their beliefs, attitudes, and knowledge towards the use of technology in mathematics teaching and learning. Each of these constructs will be conceptualised in Chapter Three.

Mathematics Curriculum and Technology

Technology can support students to achieve adequate mathematical proficiency when it is used with appropriate pedagogy (Bos, 2009). Loveless (1995) shared that technology has “the potential not only to support the current curriculum but also to enhance the experience and understanding of that curriculum and even extend thinking and learning in new ways” (p. 6). The National Council of Teachers of Mathematics, NCTM, (2003) confirmed that technology is an important tool for learning mathematics because it maximises students' understanding, stimulates their interest, and increases their proficiency in mathematics. These reasons are likely to have motivated many countries to include technology in their mathematics curriculum as an instructional tool (Lawless & Pellegrino, 2007; Levin & Wadmany, 2005). For example, the mathematics curriculum in Ghana expects teachers to use technologies such as a calculator and spreadsheet to enhance understanding of numerical computation and solve real-life problems. In addition, the curriculum requires teachers to adopt an instructional approach that assists students to (i) respond orally to questions about mathematics, (ii) discuss mathematics ideas and carry out mental computations, (iii) carry out practical and investigational mathematics activities, and (iv) work cooperatively with other students and develop interest in mathematics (Ministry of Education, Science and Sports, MOESS, 2007).

Similar to many countries, Ghana’s mathematics curriculum reflects major features of Vygotsky’s work, which places the process of acquiring mathematical knowledge within the sociocultural perspective and interactional settings (Herbel-Eisenmann, Meanney, Bishop & Metzuyanim, 2017; Vygotsky, 1978). When other factors (for example, school leadership, political will, learners’ demographic background, and parental support) are held constant, a teacher’s classroom practices significantly contribute to the variation of learning outcomes (Agyei & Voogt, 2011; Ertmer, 1999; Larbi-Apau & Moseley, 2012; Levin & Wadmany, 2005; Mereku & Mereku, 2015). Effective pedagogy in the mathematics classroom does not only promote students’ learning, but it also contributes to students’ cognitive and social
development (Marks, 2000). However, enhancing effective pedagogy in mathematics using technology persists as a challenge to educators. The introduction of technology into mathematics instruction requires teachers to have the professional competence to create a learning environment which encourages students to have shared responsibilities in constructing mathematical concepts (Bai & Ertmer, 2008; Ertmer & Ottenbreit-Leftwich, 2010; Hew & Brush, 2007; Ma, Lu, Turner, & Wan, 2008; Palak & Walls, 2009). For teachers to achieve productive mathematics instruction, it is important to strategically identify the core practices of mathematics instruction that are likely to improve teaching and learning with a particular audience (Jacobs & Spangler, 2017). One way to approach this is to examine models of effective pedagogy, specifically those proposed by Anthony and Walshaw (2007) and NCTM (2007).

Consistent with the argument of enacting effective pedagogy, particularly with technology, Mishra and Koehler (2006) espoused that teachers need a well-developed integrative knowledge of technology, pedagogy, and content (TPCK) to make it happen. Hechter, Phyfe, and Vermette (2012) argued that “the application of technological, pedagogical, and content knowledge principles should be understood under the broad contexts of school environments, individual teachers' previous experiences, and epistemological beliefs about teaching and learning” (p. 141). Their argument on technology integration echoed the stance of the Technology Acceptance Model (TAM) developed by Davis (1986). TAM is a regressive model which shows the causal relationship between external variables, perceived usefulness, perceived ease of use, attitude towards the use, and the individual’s behavioural intention of using a given technology (King & He, 2006).

Although Mishra and Koehler (2006) acknowledged the relevance of context (for instance, school environment and teachers’ personal orientations - prior experiences, beliefs, and attitudes) in effective implementation of technology, this component is less explored in most of the TPACK research studies (Chai, Koh, & Tsai, 2013). For teachers to appreciate the constraints and maximise the affordances of technology, it is argued in this thesis that their knowledge of technology integration needs to be developed alongside their beliefs and attitudes related to the use of technology (Ertmer & Ottenbreit-Leftwich, 2010; Rienties, Brouwer, & Lygo-Baker, 2013). Professional development is considered by many authors as effective approach to support teachers to improve their classroom practices (Antoniou & Kyriakides, 2013; Brinkerhoff, 2006; Cwikla, 2004; Mouza, Karchmer-Klein, Nandakumar,

The Research

This study is rooted in the interpretive paradigm of the case study research. It provides a response to the current need to support Ghanaian teachers to effectively use technology in their mathematics teaching (Agyei & Voogt, 2011; Buabeng-Andoh, 2012; Larbi-Apau & Moseley, 2012; Mereku & Mereku, 2015). The research study used GeoGebra (open-source mathematics software) in a professional development programme where teachers worked in collaborative design teams (facilitated by the researcher) to develop and enact technology-based mathematics lessons rooted in the principles of effective mathematics pedagogy (Anthony & Walshaw, 2007; NCTM, 2007) and the TPACK framework (Mishra & Koehler, 2006).

GeoGebra has been described as enhancing students’ conceptual and procedural knowledge (Jelatu, 2018; Jelatu, Sariyasa, & Ardana, 2018; Zulnaidi, & Zamri, 2017), problem-solving skills, and teachers’ reflective classroom practices (Bu, Mumba, & Alghazo, 2011; Bu, Mumba, & Henson, 2013). It is also consistent with a socio-constructivist teaching and learning approach where both teachers and students interact to construct mathematical concepts (Akkaya, Tatar, & Kağızmanlı, 2011; Saralar, İşıksal-Bostan, & Akyüz, 2018; Shadaan & Eu, 2013). Authors who have explored the effectiveness of GeoGebra, emphasised the need to introduce the software to mathematics teachers “so that students can explore the world of Mathematics in a wider [perspective] and [also] make the students able to think critically and creatively” (Arbain & Shukor, 2015, p. 213). It is hypothesised in this research study that engaging teachers to pedagogically interact with GeoGebra could facilitate the development of their dispositions towards the use of technology in mathematics teaching. Three research questions guided the study.

1. What were the teachers’ initial dispositions toward the use of technology in mathematics?

2. How did teachers enact effective mathematics pedagogy when using GeoGebra following their engagement in professional development?
3. How did the professional development impact on the teachers’ dispositions towards technology integration in mathematics?

The Significance of the Study
GeoGebra is free software and it covers a number of topics in mathematics curriculum. Several studies have verified its efficacy of supporting students to construct mathematics concepts both independently and socially (Akkaya, Tatar, & Kağızmanlı, 2011; Bu, Mumba, & Alghazo, 2011; Shadaan & Eu, 2013). What is now needed are for teachers to have the knowledge and skills to use it. This thesis is a response to this need. It is anticipated that the findings of this research will not only add to the existing body of knowledge related to the dispositions of teachers towards the use of technology, but will also widen the discussion of the awareness and potential approach of engaging teachers in professional development that promotes the enactment of the core practices of effective mathematics pedagogy. Particularly in Ghana, the results may provide some guidance for the government and Ministry of Education on the agenda they are pursuing regarding the development of professional competence of teachers to use technology in the classroom.

The Structure of the Thesis
The research activities and findings of this study are presented in the subsequent chapters. In Chapter Two, the study will be situated in a context by discussing the structure of education in Ghana as well as the introduction of ICT in Ghana’s education system. Chapter Three reviews relevant literature. The functions of technology in mathematics education will be discussed. The theoretical basis and principles of effective mathematics pedagogy are discussed in relation to technology integration frameworks. Chapter Four provides a description of the design of this study, including the research and analysis methods utilised. Chapter Five presents the findings of the study in relation to the research questions. The findings are then extensively discussed in Chapter Six. Following that, final conclusions are drawn and the implications and recommendations for practitioners and researchers are made. The thesis ends with my final thoughts.
CHAPTER TWO: CONTEXT OF THE STUDY

This chapter provides the context for the study. First, the education system and mathematics education in Ghana are discussed. Next, a snapshot of the current state of technology integration in the country is provided.

The Education System in Ghana

Ghana is located on the west coast of Africa and is just north of the equator (Figure 2.1). It has an area of 238,533 km², and shares borders with Burkina Faso (North), Cote d’Ivoire (West), and Togo (East). To the south is the sea (Gulf of Guinea). The country obtained her independence from the British in 1957 after years of colonial rule.

Prior to the arrival of the European settlers, knowledge and skills of competencies were acquired through oral transmission and apprenticeship. The Europeans arrived in the 16th century. They later introduced formal education, but it was mainly for spreading the Gospel and creating an elite group to run the colony. By 1881, one hundred and thirty-nine schools had been established, however, there was no uniformity in the operations of these schools. In 1882, the government initiated a plan to regulate the education system in the country. From 1882 to 1890, an Inspector of Schools was appointed to oversee the activities of the schools. The Office of the Director of Education was then created to carry the mantle. Currently, the main agency responsible for education delivery at the pre-tertiary level (basic schools and high schools) is the Ghana Education Service (GES) (Ekundayo, 2018).

Figure 2. 1 Location of Ghana on Africa Map
The education system in Ghana has gone through several reforms. For example, educational reform in 1987 was designed to increase access to basic education, shorten the pre-tertiary education structure from 17 years to 12 years, make education cost-effective, and improve the quality of education by making it more effective in regard to socio-economic conditions (Akyeampong, 2009). The current educational reform, which was implemented in 2007, placed key emphasis on the training of Ghanaian students to meet the demand of the technology information age (Adu-Gyamfi, Donkoh, & Addo, 2016).

In the current educational reform, the school age is usually from age 3 to 21 years. The educational structure comprises basic school, senior high school, and tertiary education. The basic school has three components: Pre-school (ages 3 to 5 years), primary school, which is equivalent to elementary school (ages 6 to 11 years), and junior school, which is equivalent to middle school (ages 12 to 14 years) (Adu-Gyamfi, Donkoh, & Addo, 2016). Therefore, it takes a Ghanaian child 12 years to complete basic education. Before a child completes basic school, he/she takes an external examination called the Basic Education Certificate Examination (BECE). The child must obtain an aggregate threshold mark in the BECE to qualify him/her to enter senior high school. Senior high school is equivalent to high school (ages 15 to 17 years) as used in many countries. The main programmes that are run in senior high school include General Science, Agricultural Science, General Arts, Visual Arts, Home Economics, Business, and Technical skills. There are other institutions which run purely vocational and technical programmes for students within this age bracket (ages 15 to 17 years). This means that a student could either go to Senior High School or Vocational/Technical School after the basic education.

The Tertiary Education/Institution is equivalent to a college or university (ages 18 to 21 years). Under the tertiary institution umbrella are polytechnics and universities. After completion of high school, students can opt to enrol in any programme offered by a polytechnic or university. There are also vocational, teacher, and nursing training institutions in Ghana. In 2008, 38 publicly-owned Teacher Training Institutions (TTIs), that offered certificate programs to prepare teachers for basic schools in Ghana, were elevated to tertiary status and re-designated as Colleges of Education (COEs) to offer tertiary programmes.

It is important to note that although an age bracket has been assigned to each level of education in Ghana, it does not necessarily restrict someone from entering any level. The
age for someone to enter any level of education in Ghana is most often dependent on the persons’ socio-economic background. In most cases, however, it is expected that by age 21 one should have completed tertiary level education in the country.

**Mathematics Education in Ghana**

The country’s mathematics curriculum was built on two premises: that every Ghanaian child “can learn mathematics and that all need to learn mathematics” (MOESS, 2007, p. ii). Therefore, mathematics is among the four core (compulsory) subjects each Ghanaian child is expected to study before entering tertiary level. The other core subjects are English Language, Integrated Science, and Environmental/Social Studies. Although different native languages and dialects are spoken in Ghana, English has been used as an official language from the time British colonised the country. It is predominantly used in both government and business activities. It is the formal language used for educational instruction and it is also studied as subject.

At the public primary school level, a teacher is allotted a class to teach all subjects, including mathematics. These teachers are trained generally by TTIs/COEs for basic schools. At the junior high school, teachers are assigned to teach a particular subject(s) which most often is/are dependent on the teacher’s strength and competency. Not all teachers who teach mathematics at this level are formally trained to teach the subject.

At the senior high school level, mathematics is taught as a core (compulsory/general) or elective subject and all students are expected to do core mathematics irrespective of their programme. The contents of the elective mathematics course go beyond that of the core mathematics and it is optional for students. Most mathematics teachers at the senior high school level are those who have received formal training from the two universities (University of Cape Coast and University of Education, Winneba) mandated to train mathematics teachers for that level in the country. There are some mathematics teachers at this level who read pure mathematics (BSc Mathematics) without education courses. Such teachers are compelled to obtain a Postgraduate Diploma in Education to enable them to upgrade to professional teacher status.
Ghanaian students’ performance in mathematics in recent years has not been deemed sufficient in both national and international examinations. There has been concern expressed about the students’ performance in the West African Senior Secondary Certificate Examination (WASSCE) and other international examination such as Trends in International Mathematics and Science Study (TIMSS) examinations (Anamuah-Mensah, Merekü, & Asabere-Ameyaw, 2006; Djangmah & Addae-Mensah, 2012; Fletcher, 2016). There is a fluctuation in the students’ performance in the WASSCE mathematics from 2007 to 2018 (see Figure 2.2). The students’ performance in the subject soared from 25.3% students passing in 2007 to 43.8% of students passing in 2011, 49.4% in 2012 and started decreasing from 36.6% in 2013 to 32.4% in 2014 and 24.0% in 2015. The performance began to increase in 2016 but the recent result in 2018 indicated a decline. It can be seen from these results that, for over a decade, the country’s best performance in mathematics is 49.4% in 2012. It is important to note that no examination was written in 2010 because the numbers of years students spend at senior high school was changed from three to four years. After 2012, it was reversed back to three years. This performance is worrying in the sense that more than half of the students would not get the chance to enter any of the tertiary institutions in the country.

Figure 2.2 Students’ performance in mathematics

Source: Compiled from Fletcher (2016) and Kumsah (2018)
There are several factors that account for students’ underachievement in mathematics in Ghana. Some of these factors include excessive use of a teacher-centred instructional approach (Anamuah-Mensah et al., 2006; Djangmah & Addae-Mensah, 2012; Fletcher, 2016; Frempong, 2010), inadequate teaching and learning resources (for instance, manipulative materials, drawing instruments, printing materials, calculators, computer algebra system, and dynamic geometry software), teachers’ weak content and pedagogical knowledge (Abreh, Owusu, & Amedahe, 2018), teaching to the test, weak computational skills by the students, inadequate preparation by the students, and students’ low confidence (Fletcher, 2016). Although there are many factors that contribute to students’ poor performance in mathematics, teachers’ classroom practices are a crucial factor in students’ performance (Hattie, 2012). The teachers’ content and pedagogical knowledge, beliefs, and attitudes have a direct influence on students’ mathematics learning (Wilkins, 2002). Therefore, there is the need to look for innovative and pragmatic approaches to teacher preparation and professional development, especially in this 21st century where the world is rapidly developing in terms of technology (Agyei, 2012; Anamuah-Mensah et al., 2006; Fletcher, 2016).

Technology in Ghana’s Education

Ghana has joined other countries in adopting the use of ICT for teaching and learning. To obtain a vivid picture about the nature of ICT adoption in Ghana’s education sector, extant local literature is reviewed here to help readers appreciate the strides the country has made, and possible ways forward, as far as the use of technology in teaching and learning is concerned. Also, the review will offer a useful basis for the reader to interpret the findings of the dispositions of the teachers in this research towards the use of technology in teaching and learning following their engagement in professional development. The review is presented in four sub-sections that are central to the effective use of technology in the classroom: the existence of ICT policy, the availability of ICT facilities, expertise, and the availability of professional development.

Existence of ICT integration policy

Overall, Ghana’s ICT in Education Policy is defined to reflect the National ICT policy referred to as the ICT for Accelerated Development (ICT4AD) policy. The ICT4AD policy,
the framework for socio-economic development for Vision 2020, was initiated by the Government of Ghana in 2001 and was approved and adopted in 2002 and 2004 respectively (Odongo, 2012). The Government of Ghana is committed to the transformation of the country’s economy into an information and knowledge-based economy and society through the provision of ICT expertise, resources, and research (Ministry of Education, 2008).

Among one of the key 14 pillars identified in the ICT4AD was to direct how ICT would be used to “facilitate education and learning within the educational system and promote e-learning and e-education as well as lifelong learning within the population at large” (Ministry of Education, 2008, p. 18). Thus, just like many developing and developed countries, Ghana acknowledges that socio-economic development can be a reality if ICT is allowed to play a pivotal role in widening the accessibility and improving the quality of education.

In pursuance of this agenda, Ghana’s Ministry of Education formulated an ICT in Education Policy. The primary goal was to enable graduates from Ghanaian institutions, both formal and non-formal, to “confidently and creatively use ICT tools and resources to develop requisite skills and knowledge needed to be active participants in the global knowledge economy by 2015” (Ministry of Education, 2008, p. 18). The seven thematic strategies outlined to achieve this broad goal were (i) the introduction of ICT into the curriculum, (ii) the development of digital content, (iii) the integration of ICT as management and administrative tool in educational settings, (iv) the provision of human development capacity, (vi) the provision of technical support, (vi) the provision of infrastructure, e-readiness and equitable access, and (vii) the provision of monitoring and evaluation (Ministry of Education, 2008).

In the area of the school curriculum, the recent Educational Reform 2007 re-affirmed the integration of ICT in education, not only as a subject to be learnt, but also as a pedagogical tool to facilitate effective teaching and learning through the provision of computer resources and capacity development of teachers. It is clear from this that the country is hopeful that the exploitation and adoption of ICT tools in education will reduce the current teacher-centred instructional approaches and promote a pedagogical approach where teachers and learners will be engaged in critical analysis, negotiation, and development of concepts to reflect 21st century teaching and learning (Ministry of Education, 2008, 2015).
Indeed, Ghana has a comprehensive ICT in education policy that addresses issues of leadership, infrastructure, ICT software, curriculum development, and human capacity development. This policy resonates with Kennisnet’s (2015) view of the effective use of technology in teaching and learning. Kennisnet argued that having an explicit vision is a necessary step to achieving effective pedagogical use of ICT tools. It is encouraging to note that recognition has not only been given to the provision of a formal national ICT policy and the distribution of ICT infrastructure and resources, but also how to provide continued professional development for teachers to gain pedagogical competence in the use of ICT tools (Ministry of Education, 2008).

Although it is consistent with Kinnesnet’s (2015) recommendations for Ghana to have a comprehensive ICT policy in education, there are concerns about the implementation of its intentions in mathematics (Agyei & Voogt, 2011; Mereku & Mereku, 2015). Though the mathematics curricula for various levels of pre-tertiary education contain statements about the use of ICT in teaching and learning, very little attention has been paid to instructional objectives and pedagogical activities that involve the use of ICT tools to conceptualise specific mathematical ideas. While the mathematics curricula for junior and senior high schools make a few mentions of using calculators, computers, and/or spreadsheets in specific mathematics instructions (for instance, decimal fractions, percentages, relations and functions, the trigonometric function, and the logarithm), that of the primary school makes no similar suggestions. It can also be argued that mere statements about the use of ICT in mathematics teaching and learning, without explicit instructional objectives and pedagogical activities involving the use of specific ICT tools or mathematical software, is insufficient to promote the whole agenda of effective ICT integration in mathematics.

**Availability of ICT facilities**

To integrate ICT effectively, teachers, school managers, support staff, and students need access to ICT facilities. In other words, they need appropriate computer hardware and software, sufficient internet connectivity and power supplies, appropriate teaching spaces and furniture, and support for maintaining the ICT equipment to ensure constant availability and accessibility (Ministry of Education, 2015). These issues are discussed using the following themes: government and development partners’ commitment, availability and accessibility of computer hardware and software, availability and accessibility of internet
connectivity, availability and accessibility of rooms and furniture, availability of power supply, and support for maintaining the ICT equipment.

Government and development partners’ commitment
The provision of ICT infrastructure in educational institutions is promoted in the Ghana Government’s plan of ensuring socio-economic development. For example, in 2003 the Government of Ghana secured a World Bank Group credit of US$40 million to support selected components of ICT for implementation towards the ICT4AD policy (Ghana News Agency, 2006). Aside from this, the Government has made another effort of introducing one laptop per child. The One Laptop per Child (OLPC) programme became one of the policy initiatives contained in the 2007 Budget Statement and Economic Policy of the Government of Ghana. Under the OLPC project, 60,000 laptop computers were purchased for distribution to basic schools throughout the country. Although the OLPC project sought to narrow the ‘digital divide’, most commentators on this initiative suggested that the distribution of laptops had a false start because there was no initial training for teachers, who play the central role in curriculum implantation. Secondly, the laptops the government provided did not have subject specific software applications to enable teachers and students to use them in teaching and learning (Buchele & Owusu-Aning, 2007; Mangesi, 2007; Owusu-Ansah, 2015).

Based on the ICT4AD policy, the Ministry of Education in Ghana included assurance of adequate and equitable provision, maintenance, and support of ICT infrastructure and resources for all levels of the education sector, in its initiatives. The Ministry has directly or indirectly involved other ministries, agencies, NGOs, and donor partners to ensure there was equitable distribution of ICT resources across all levels of education in the country (Ministry of Education, 2015). In 2012, the Ministry of Education, partnering with rLG, an ICT company in Ghana, initiated a ‘teacher laptop and ICT project’ where teachers received ICT professional training and free rLG laptop computers. Under the initiative, 65,000 teachers were expected to benefit from the package (Ghana Information Network for Knowledge Sharing, 2013). From 2012 to 2013, a total of 2,331 teachers in the Central Region of the country had benefited from the project (Ghana News Agency, 2014). Also, iEARN/SchoolNet Ghana initiated the Expanding Education Networking and Professional Development Programme in most parts of the country involving 250 schools. In addition, Accelon Ghana (broadband via satellite service provider in Africa), Standard Trust Bank
Ghana Limited (a financial institution in Ghana), and ICT Education Support Africa Foundation, instituted e-Education packages with the focus of providing affordable internet connectivity to basic and secondary schools in the country (Mangesi, 2007).

The ICT School Connectivity Project was also launched, through the Ghana Investment Fund for Electronic Communications (GIFEC), to provide computing infrastructure to 38 COEs, 37 public technical institutes, 510 public senior high schools, and 23,000 public basic schools in the country. The GIFEC was established in 2004 to support the Ministry of Education by providing ICT infrastructure to schools in Ghana, especially those in the rural area. The project was then expanded to provide ICT training to students and teachers. As at 2014, GIFEC had provided 524 educational institutions with a fully furnished and equipped ICT laboratory with accessories and internet access (38 COEs, 37 national vocational and technical institutions, 10 youth leadership training institutions, 240 senior secondary schools, 24 basic schools, 26 technical institutes, 24 community development institutes, 62 nursing training schools, and nine farm institutes) (Prempeh, 2014). In 2018, GIFEC provided training on instructional use of technology for 500 teachers at the basic schools. It also trained 4,500 students with coding skills through its ‘coding for kids’ programme. The Coding for Kids programme was designed to provide digital knowledge and skills to all children through the creation of websites, computer games, interactive arts, mobile apps, and animation stories, using programming languages such as JAVA, HTML, and CSS (Myjoyonline.com, November, 2018).

**Availability and accessibility of computer hardware and software**

In 2009, the Ministry of Education conducted a national survey involving 501 senior high schools (representing 97.6% of senior high schools in Ghana) to assess the status and implementation of the national ICT policy in education. The survey reported that, although 87% of the schools had computer laboratories, the majority of the facilities were below acceptable standards and could not be used to achieve pedagogical objectives. The survey further indicated that only 57% of the computers in the schools were functional (Ministry of Education, 2009). A more recent national survey (Ministry of Education, 2015) reported that the non-functional computers in the schools could be accounted for by a poor maintenance culture, lack of after installation support, and a proliferation of obsolete and inappropriate computers donated by development partners (Ministry of Education, 2015). For example, the survey indicated that most of the computers (54%) were Pentium I and Pentium II which
were below the minimum specifications set by the Ministry of Education. Other ICT equipment available in the schools were: printers (94%), photocopiers (61%), scanners (52%), TVs (53%), radios (33%), risographs (29%), digital cameras (17%), LCD projectors (5%), web cameras (2%), and video cameras (1%). The average ratio of students to computers at the national level was reported in the survey was 42:1. The region that had the best student-to-computer ratio was Volta Region, 33:1 and the worst region was Northern Region, 50:1. The Pan African Research Agenda on the Pedagogical Integration of ICT project conducted by Mereku et al. (2009), also reported that the ICT facilities available in the educational institutions included computers, compact discs (CDs) and mobile phones, and the average ratio of student-to-computer at senior high school level was 30:1.

The use of digital learning materials to mediate lesson delivery is on the increase worldwide (Mereku & Mereku, 2015). The Ghana ICT policy included in its plan the need to enhance effective lesson delivery and accessibility of curriculum materials across the length and breadth of the country through (i) the digitisation of the school curriculum and (ii) the creation of an educational portal/website (Ministry of Education, 2008). At the time this report was compiled, the idea of digitising learning materials had not been fully implemented by the nation, though some individual small-scale businesses had begun digitising subject content such as Mathematics, Biology, Chemistry, and Physics, for sale. The Ghana Mathematics Society (GMS) has also started digitising pre-tertiary mathematics content on tablets to aid mathematics instruction and learning, but this had yet to catch national attention.

In the recently reviewed ICT in education policy document it was also noted that there was a “general absence of rich content that was fit-to-purpose as far as national school curriculum was concerned” (Ministry of Education, 2015, p. 11). In order to catch up with countries such as The Netherlands, which as of 2014, had more than 70% of their teachers employing digital learning materials in their pedagogical practices (Kennisnet, 2015), the Curriculum Research and Development Division, under the Ghana Education Service, need to regularise the activities of individual small-scale businesses who are digitising learning materials.

The availability of teaching and learning software is another critical issue for effective ICT integration in pre-tertiary education in Ghana. The Pan African Research Agenda, in the Pedagogical Integration of ICT project conducted in 2009, indicated that the Microsoft
Office suite, Microsoft Encarta, AutoCard, CorelDraw, and Mavis Beacon were the only software that had been installed on computers in the senior high schools in the country. The AutoCard and CorelDraw software were only available on the computers of Technical Secondary Schools (Mereku et al., 2009). Thus, the most common educational software available and accessible in Ghanaian schools was the Microsoft Office suite and Microsoft Encarta (Ministry of Education, 2009). This implies that most Ghanaian schools did not have the requisite mathematical software for effective pedagogy. It is important that computers in schools have application software that embodies mathematical concepts installed. Without further financial resources, the country could begin with open source software such as GeoGebra, Maxima, Maxima, ExtCalc, R, QtiPlot, FreeMat, Octave, Sage, Maple, and Scilas (Wick, 2009).

Availability and accessibility of internet connectivity

Despite the government’s commitment to improving ICT facilities in pre-tertiary institutions, many schools in the country are yet to have internet connectivity. Findings from Natia and Al-hassan (2015) indicated that the ratio of student-to-computer connected to the internet in primary schools was 335:1 and in junior high schools was 186:1. The situation was not different in the senior high schools. The Ministry of Education (2009) baseline survey indicated that 111 out of 501 schools (representing 22.2%) had a local area network. Only 89 (17.7%) schools had internet and 80 of these were situated in the urban to semi-urban areas. Also, the survey reported that only 8.3% of the total number of computers in the Ghanaian second cycle schools were connected to the internet. It was also reported in the survey that it cost some institutions as much as US$1200 per month to get a reliable, high quality, and speedy internet service. Similarly, it is indicated in the final report on The Millennium Cities Initiative, a school-to-school connectivity project conducted in Ashanti Region of Ghana that (i) the country does not have the option for unlimited internet access and (ii) the cost of internet in the country is exorbitant (Kubis, 2014). Thus, for teachers and students to make appreciable use of interactive online exercises, video lessons, text files, and courseware, it is important the government and stakeholders in education make the internet more available and economically accessible.
Availability and accessibility of rooms and furniture

Dust and poor ventilation can cause frequent breakdown of ICT facilities. Therefore, the availability of well-furnished rooms is crucial in maintaining the longevity of any ICT facilities supplied to the schools. It was reported in the Ministry of Education’s (2009) survey that 67.2% of Ghanaian senior high schools had rooms that could be converted into computer laboratories. Many of the schools had the recommended capacity of 40 computers per room but the furniture was of a low standard for the purpose of a computer laboratory. Also, the rooms that were used for computer laboratories in most of the Ghanaian schools lacked appropriate cooling mechanisms which resulted in frequent breakdown of the ICT equipment (Ministry of Education, 2009).

The report from The Millennium Cities Initiative (MCI) project also indicated that typical Ghanaian classrooms contained 40-50 students with one teacher, and two or three students sitting together at a desk. The report identified that the classrooms in Ghana were not resourced digitally and the only appropriate means through which the teachers involved in the MCI project could share their mathematics and science lessons were through a computer/laptop and a projector they carried to the classroom (Kubis, 2014). Since the teachers in the MCI project were able to share their lessons with their students through a single computer/laptop and a projector, it is important to consider the context of the school. The availability of a computer and a projector alone in a classroom is not enough for effective technology integration. Students’ academic achievement is a function of several related factors, in which classrooms with high quality learning equipment plays a crucial role. For instance, the consistent success story of Finnish pupils over the past decades in PISA (Programme for International Student Assessment organised by OECD) in mathematics, science, and reading, is not only attributed to factors such as trust for the education policy; daily warm lunch; a health and dental service policy; support for children with special needs; emphasis on teacher qualifications, but also the provision of high quality learning equipment in their classrooms (Niemi, Kynäslahti, & Vahtivuori-Hänninen, 2013). Thus, it is crucial all people who are responsible for ensuring quality education in Ghana increase their efforts to ensure that classrooms are provided with needed learning facilities to match 21st-century teaching and learning. This would allow teachers and students to creatively apply technology to solve real-life problems.
Availability of power supply

The ICT facilities provided in schools will not be functional if there is no regular electricity supply. Ninety-eight percent (98%) of Ghanaian senior high schools are connected to the national grid (Ministry of Education, 2009), but as of 2015, irregular electricity supplies had become one of the greatest economic challenges the country is battling. This had effects on the operations of all sectors in the country. ICT equipment supplied to the schools broke down as a result of frequent power fluctuation (Kubis, 2014; Ministry of Education, 2009). Among some of the steps the government is taking to resolve the issue was to sign a US$920 million extended credit facility with the International Monetary Fund, in April 2015, to improve sources of power supply (Index Mundi, 2016).

Support for maintaining the ICT equipment

The national ICT policy document outlines the strategies of ensuring technical support, maintenance, and sustainability of the ICT equipment supplied to schools. Some of the strategies include: regular audit of ICT facilities, recruitment and training of ICT coordinators and laboratory technicians, and setting up regional and district technical support and maintenance centres (Ministry of Education, 2008). The situation on the ground indicated that little over half of the schools (51%) had a maintenance schedule for fixing broken computers (Ministry of Education, 2009). Less than half of the schools (49%) had trained staff to provide technical support (Ministry of Education, 2009).

Expertise

Teachers’ expertise in terms of basic ICT knowledge and skills are crucial in successful ICT integration in teaching and learning (Kennisnet, 2015; Mishra & Koehler, 2006). Included in the policy framework for Ghana’s ICT integration in education is the promotion of the expertise of subject-based teachers to use ICT tools pedagogically in their classrooms (Ministry of Education, 2008). Thus, it is important to examine literature that provides an understanding of the current level of teachers’ pedagogical expertise in using ICT in a specific subject such as mathematics. This needs to be done through analysis and synthesis of multiple sources of research and policy documents.

The available literature indicated that teachers are yet to fully implement technology in the mathematics classrooms in Ghana (Agyei & Voogt, 2011; Buabeng-Andoh, 2012; Mereku
& Mereku, 2015). There is evidence that teachers in Ghana do not have: sufficient knowledge about ways to integrate technology into a lesson, the skills of applying specific mathematical software in a lesson, or professional training opportunities for technology integration in mathematics teaching (Agyei & Voogt, 2011). The findings of the study conducted by Buabeng-Andoh (2012) indicated a mismatch between Ghanaian teachers’ beliefs and attitudes and their use of technology in the classroom. This result supported Agyei and Voogt (2011) who found that there was a very weak correlation between Ghanaian teachers’ overall attitudes towards computers (enjoyment, lack of anxiety, instructional productivity, professional development, interaction, and teachers’ understanding of the benefits of using technology) and their technology use in the classroom. Furthermore, the recent review conducted by the Ministry of Education (2015) indicated that there was an “absence of teachers sufficiently skilled to integrate the ICT into their subjects or projects in a contextual manner that would create lasting interest so desired to attract the learners” (p. 11). Thus, there are consistent reports indicating that the mathematics teachers in Ghana do not seem to have the professional competence (knowledge, beliefs, and attitudes) required for successful integration of ICT in mathematics teaching and learning. The nature of professional development teachers receive is a determinant of whether they will incorporate technology in their pedagogical decisions in the classroom (Bennison & Goos, 2010). Therefore, it is necessary to examine critically the kind of professional development available for teachers in Ghana.

**Availability of professional development**

The ICT4AD programme prioritised the provision of requisite professional training to improve human technical expertise, training of facilitators, and experts in the applications of ICT in education (Ministry of Education, 2008). In the past decade, several professional development interventions were provided by the Ministry of Education and Ghana Education Service, in conjunction with development partners, to promote teachers, school heads, and students’ competencies to make satisfactory use of ICT tools in educational settings. For example, APSnet facilitated exchange programmes with countries such as Denmark, Mexico, United States, and Great Britain to enhance Ghanaian teachers and students’ competency in ICT use. The programme was initiated and funded by UNESCO.

There is also an ongoing Senior High School Connectivity project, under the Secondary Education Improvement Project (SEIP), to promote school heads’ and teachers’ pedagogical
and professional competence in using ICT in school and classroom practices (Ministry of Education, 2014). The teachers enrolled in the Senior High School Connectivity project are expected to gain competencies in: (i) technology literacy, where teachers will gain basic knowledge and skills in ICT application related to their professional practices; and (ii) technology infusion, where the teachers are expected to use technology in their classroom practices.

Subject associations in the country have also begun responding to these needs by promoting the awareness and use of technology in the mathematics classroom (Mereku, 2010). For example, the Mathematical Association of Ghana (MAG) brainstormed the challenges facing the teaching and learning of mathematics in this technological age in their 38th National Conference/Workshop held at University of Development Studies, Wa on 14th -18th August 2012. As part of the activities to mark the 38th anniversary, a one-day professional training was organised for the teachers on how to use GeoGebra software in mathematics teaching. In 2013, the Ghana Mathematics Society (GMS) organised a training session on the use of Sketch Pad in mathematics for teachers and students who participated in a two-week mathematics camp, which was meant to expose the participants to practical approaches to mathematics teaching and learning.

The institutions that train mathematics teachers for pre-tertiary education have included courses in their curricula to promote professional competence in the use of technology (Asiedu-Addo, Apawu, Owusu-Ansah, & Armah, 2016). In Ghana, there are 38 public and two private COEs (six-semester programme) and two public universities (eight-semester programme) that train teachers for basic and senior high schools respectively. University of Education, Winneba, and University of Cape Coast are the two universities that train mathematics teachers for the country. The mathematics curricula of these institutions are basically structured to enable the prospective teachers: (i) grasp the mathematical concepts through mathematics content courses, (ii) develop pedagogical knowledge in mathematics teaching and learning through mathematics methodology courses, and (iii) acquire professional competence through teaching practice and educational and professional studies.

At the University of Cape Coast, the prospective mathematics teachers at the undergraduate level received only two computer literacy courses (a one-credit and a two-credit hour course in the first and second semester, respectively), which exposed them to basic computing skills
(Agyei, 2012). At the master’s degree level, prospective mathematics teachers receive a course in Computer Application in Mathematics education (one hour credit course). The Mathematics Education Department in the University of Education, Winneba offers the following ICT courses to undergraduate prospective mathematics teachers: ICT Systems and Tools for Mathematics Teachers, Fundamentals of Computer Programming, Courseware Design and Development Using Multimedia Tools, Computer Applications for Teaching and Learning Mathematics, Introduction to Computer Programming for Mathematics Teachers, and Web Technologies for Mathematics Teachers. At the graduate level, while the Master of Education (Med) students took Instructional Technology for the Teaching of Mathematics, Master of Philosophy students take Computers and Algorithms in Mathematics Education (Asiedu-Addo, Apawu, Owusu-Ansah, & Armah, 2016). Thus, unlike the University of Cape Coast, the Mathematics Education Department in the University of Education, Winneba has more ICT courses in their mathematics curriculum because their mathematics education programme mainly focused on mathematics and ICT education. At the University of Cape Coast, the prospective mathematics teachers could choose disciplines such as physics, chemistry, geography, or economics as a minor course.

It can be observed from the discussion above that Ghana is making strides in providing technology professional development for both in-service and prospective mathematics teachers. It can be argued, however, that the kind of technology knowledge the teachers receive through the programmes organised by the Ghana Education Service, in conjunction with development partners, is so generic that it does not adequately prepare the teachers to pedagogically use the technology in their specific subject area (Agyei & Voogt, 2011). There is also an issue of teachers not transferring their ICT knowledge and skills into actual classroom practice after they had received formal training. For example, Asiedu-Addo et al. (2016) conducted a follow-up study to explore how the teachers who graduated from mathematics education institutions and had received professional training in the use of ICT implemented it at their various workplaces. The results from 48 in-service teachers who participated in the study showed that only 42% incorporated ICT into their mathematics teaching, and the software they most frequently used was the Microsoft Office suite (Excel, PowerPoint, and Word). The study further indicated that limited technology resources was a key factor, which accounted for the majority of teachers being unable to transfer their ICT knowledge and skills into their practice. Also, a study conducted by Apawu (2011) indicated
that little attention is paid to how the tutors in the COEs used the range of mathematical software available in training their prospective mathematics teachers.

Baldwin and Ford (1988) summed up that, in order to reduce the gap between the knowledge learned and its implementation in the workplace, there needs to be a good understanding of trainee characteristics (for instance, ability, personality, beliefs, attitudes, and motivation), training characteristics (principles of learning, sequencing, and training content), and the work environment (external resources).

It is without question that the factors enumerated in the previous paragraph (and/or elsewhere) are crucial for the successful implementation of technology in teaching and learning, however, the teachers themselves also have critical role to play. The proverb, ‘You can lead a horse to water but you can’t make it drink’, is applicable in considering the implementation of curriculum innovations. While technology competence (knowledge and skills) and access to technology are useful during the first stages of technology adoption, empirical evidence suggests that teachers’ attitudes (expression of will) are critical to make technology integration happen (Christensen & Knezek, 2008). Voogt, Knezek, Cox, Knezek, and ten Brummelhuis (2013) asserted that “technology has the potential to transform schools, and teachers need to be able to actively participate in that process” (p. 5). Thus, the expression of teachers’ readiness (will) in the implementation of curriculum changes is key. Teachers have the onus to avail themselves of professional training and subsequently adopt the knowledge they acquire into practice. They also need to walk by the dispositions they espouse (Ertmer, Ottenbreit-Leftwich, Sadik, Sendurur, & Sendurur, 2012). For example, having the notion that technology is useful in promoting interactive instruction is not enough. A teacher who holds such a disposition needs to act upon it, by finding approaches that can support him or her to enact such pedagogical belief. Voogt, Knezek, Cox, Knezek, and ten Brummelhuis (2013) reiterated that “teachers who see themselves as ‘entrepreneurs’ or take leadership roles are more likely to use ICT in creative ways than other teachers” (p. 5).

**Summary**

This chapter described the state of technology integration in Ghana’s education sector. The synthesis indicates that the country has a comprehensive ICT policy document emphasising the key factors for effective technology integration in teaching and learning. Though the
country has made strides in the provision of ICT facilities, expertise, and the availability of professional development, improvement in these areas is still needed to realise successful implementation of ICT in education. This thesis supports the argument that teachers need to develop sufficient competencies in applying technology in mathematics instruction. With relevant professional competence, teachers can appreciate the affordances and constraints of technology integration at their workplaces. Most importantly, teachers need to be educated about the working environment as they go through professional training so that they will be resilient when they face difficulties in implementing technology in their classrooms.
CHAPTER THREE: LITERATURE REVIEW

As outlined in Chapter One, the research in this thesis explored how professional development influenced teachers’ dispositions toward the use of technology in mathematics teaching. It also explored how teachers enacted effective mathematics pedagogy in a GeoGebra learning environment. To put the thesis in perspective, this chapter presents and discusses the literature on which the thesis is grounded. First, the roles of technology in mathematics education with specific focus on teaching and learning will be discussed. The discussion is carried out with the purpose of drawing attention to the questions we need to ask in order to maximise the affordances of technology to improve classroom practices. Second, with specific focus on the technology, pedagogy, and content knowledge (TPACK) framework, the theoretical underpinnings of effective pedagogical use of technology in mathematics will be provided. Unresolved issues related to professional development for teachers will also be articulated. Teachers’ beliefs, attitudes, and knowledge towards the use of technology will be discussed. Finally, discussion will be presented on GeoGebra as a tool for mediating professional development related to technology integration.

Technology in Mathematics Education

Every field defines technology differently. This sometimes creates confusion (Luppicini, 2005). For example, engineers and social scientists have varying uses of the term ‘technology’. Engineers view technology as the process of material construction based on systematic engineering knowledge of how to design artefacts. In this perspective, technology is closely associated with machines or some kind of physical system. On the other hand, social scientists’ view of technology goes beyond material construction to include the significance of the operative use of technology in the social environment. From the social science perspective, Luppicini (2005) referred to technology as the “organization of knowledge for the achievement of practical purposes as well as any tool or technique of doing or making, by which capability is extended” (p. 104). This definition is consistent with Earle (2002), who explained that “computer technology is merely one possibility in the selection of media and the delivery mode - part of the instructional design process - not the end but merely one of several means to the end” (p. 6). Thus, from a social science perspective, integrating technology is not about technology - it is primarily about content and effective instructional practices (Luppicini, 2005). This thesis defined technology from
a social science perspective to include tools (hardware, software, and other media) with
which we deliver content and implement instructional practices in better ways (Earle 2002;
Hew & Brush, 2007). This definition is in line with how technology is operationalised in
Ghana’s ICT policy document (as discussed in Chapter Two). That document considered
technology to encompass hardware (computers, embedded processes in equipment, and
handheld portables such as smartphones and tablets), software applications (desktop, mobile
and web applications, and cloud services) and externally moderated social media and
networks (open or closed groups, and information and data sharing systems) which have
educational contents and curricular resources (Ministry of Education, 2015).

Much educational literature has identified the potential benefits of these technologies in
administration (Anderson & Dexter, 2005), research (Cuff, 2014; Smith, 2018), and teaching
and learning (Raja & Nagasubramani, 2018; Stosic, 2015). With particular focus on teaching
and learning in mathematics, many researchers have consistently argued that when
technology is used with appropriate pedagogy, it has the potential to improve students’
achievement (Almeqdadi, 2000; Benning & Agyei, 2015; Hannafin, Truxaw, Vermillion, &
Liu, 2008; Hoong & Khoh, 2003; Saha, Ayub, & Tarmizi 2010), problem-solving skills,
mathematical thinking (Idris, 2009; Niess, 2005; Ogwel, 2009; Sacristán et al., 2009),
conceptual development (Akkaya, et al., 2011), and attitudes, interest (Neurath & Stephens,
2006; Sanders, 1998), and engagement in mathematics (Attard & Curry, 2012; Geiger,

In the following sub-sections, the usefulness of technology in mathematics is explored in
detail and unresolved issues and challenges in implementing effective technology in teaching
and learning are articulated. The structure of the sub-sections was informed by Drijvers,
Boon, and van Reeuwijk’s (2010) categorisation of the pedagogical usefulness of
technology: (i) a tool for doing mathematics, (ii) a tool for practicing mathematics skills, and
(iii) a tool for developing mathematics concepts (see Figure 3.1). This trio functionality of
technology, of course, is not exhaustive, and it is also not mutually exclusive. This scheme
was adopted for its simplicity and for clarity of the presentation. It was also useful in
grouping the pedagogical types of use of technology that different authors have articulated
in connection with theories of learning.
Technology for doing mathematics

Technology as a tool for doing mathematics refers to offloading work that could be done manually. That is, the technology is used to assist students to carry out routine procedures such as the expansion of algebraic expression, handling large datasets, factorising polynomial functions with higher degree functions, and drawing graphs (Jupri, Drijvers, & van den Heuvel-Panhuizen, 2015). The emphasis is that the student knows how to perform the task by hand, but may choose to use technology to perform the task to save time in order to focus on the important aspects of mathematics learning (Roschelle, Noss, Blikstein, & Jackiw, 2017).

Among some of the early research work on computers in mathematics teaching and learning, Palmiter (1991) compared the performance of students using a computer algebra system to learn calculus to the performance of students using paper-and-pencil computations. Palmiter reported two findings. First, the computer-assisted students outperformed their counterparts on a test involving conceptual knowledge of calculus. Second, on a calculus computational exam, the computer-assisted students were again superior. Two teachers were involved in the study, each taught a different class. Palmiter acknowledged that the variance in the students’ performance in conceptual knowledge in calculus could have been explained by the differences in the teaching style, and not necessarily the intervention (computer approach of instruction), though both teachers used the same instructional manual and subject content, and also met weekly to compare notes. However, the author was not able to dismiss the affordance of the computer algebra system in supporting the students to perform more accurately, and more quickly, when solving problems involving integral calculus. Similar results had earlier been reported by Heid (1988), who used computer algebra, table tools, and graphing tools, to facilitate conceptual development in calculus. In addition, Heid reported that the computer-assisted students felt confident about their work with the computer helping them to concentrate on the problem-solving process. Calder, Brown,
Hanley, and Darby (2006) found that students taught with spreadsheets were able to progress more quickly into exploring larger numbers and decimals. The visual features in the spreadsheet prompted learners to “pose a new conjecture, reset their sub-goal, and then allowed them to easily investigate the idea of doubling the numbers” (p. 111). This, according to the authors, could make the students less frightened to take risks in exploring mathematical concepts.

The affordance of technology to provide multiple representations of a concept has been reported by many researchers (Heid & Blume, 2008; Moyer-Packenham & Westenskow, 2013; Reimer & Moyer, 2005). For example, students have the opportunity to see the dynamic features of minima and rates of change more clearly when produced by technology than through symbolic representation. According to Heid and Blume (2008) “once students identify a feature of representation that is particularly salient in their thinking, technology can afford opportunity for exploration” (p. 61). A virtual manipulative applet has the capability of connecting dynamic visual images with abstract symbols (written words and numerical symbols); a limitation of concrete manipulatives and pictorial images on textbook pages (Reimer & Moyer, 2005). Reimer and Moyer recounted that exposing students to exploring fractions through visual computer images could enhance their ability to explain and represent their thinking using pictorial models.

The illustrations above clearly ascertain the usefulness of technology to outsource the tasks students perform in the classroom (Moyer-Packenham & Westenskow, 2013). Roschelle et al. (2017) used the term divisions of labour to describe situations where students share part of their learning tasks with technology. However, Roschelle et al. admonished us to apply this division of labour with caution. Using technology as a productive tool to ensure efficiency does not fully exhaust the expectation of mathematics curricula. In line with Artigue’s (2002) argument of instrumental genesis (that is, a tool becoming an instrument), students and/or teachers are expected to use technology to transform mathematics activities where the focus will be on the “coevolution of the partnership between a person and a tool in doing mathematics” (Roschelle et al., 2017, p. 858). For example, Brown (2005) argued that technology has the capacity “to transform mathematics, mathematical activity, and mathematics thinking” (p.185), but it cannot be considered independently from the learning environment. The realisation of the affordances of the technology is embedded in the classroom context and managed by the teachers (Brown, 2005).
Technology for practicing mathematics skills

Technology for practicing skills refers to technology that offers immediate feedback to students’ responses, solutions, and strategies (Bokhove, 2010; Roschelle et al., 2017). This affordance of technology enables teachers to offer formative assessment to students (Ingram et al., 2016) and it also enables students to pace their own learning (Heitink, Voogt, Verplanken, van Braak, & Fisser, 2016). For example, Ingram et al. (2016) illustrated how the Show and Tell App was used by teachers to facilitate formative assessment in the mathematics classroom. Show and Tell is an App that can capture voice and writing in real time. This afforded teachers the chance to collect detailed evidence about students’ mathematical thinking and learning and they then further used the information captured on the device to address students’ misconceptions through remedial teaching (Ingram et al., 2016). Similarly, Reimer and Moyer (2005) highlighted the immediate and specific feedback students received while using the virtual manipulatives applet (dynamic visual images on computer). The specific instances of feedback in written form on the computer served the function of correcting or highlighting students’ errors and making students more aware of their own misconceptions (Moyer-Packenham & Westenskow, 2013). This feedback supported the students on how to write fraction notions accurately using numbers and words.

It has also been identified that offering technology based learning could be one method of improving the retention of students (Amaral & Shank, 2010; De Freitas, Morgan, & Gibson, 2015; Fozdar & Kumar, 2007; Roschelle et al., 2017). Roschelle et al. (2017) argued, from a cognitive perspective, that technology has the potential of enhancing learning and retention because it has the capacity of creating space where students have the opportunity of returning to the skills practiced in an earlier session on a regular schedule. The online system where students practice mathematics test questions could provide prompt feedback and students might not need to wait for a teacher before progressing to the next stage of mathematical task or skill. For example, the online tutoring site Khan Academy offers students the opportunity to practice exercises, watch instructional videos, and has a personalised learning dashboard that empowers learners to study at their own pace both at school and at home (Light & Pierson, 2014; Zengin, 2017). Similarly, Amaral and Shank (2010) found that online learning allowed their students to spend more time on tasks outside the classroom, which increased their success and course retention. A meta-analysis carried out by Harper and Milman (2016) identified some areas where technology proved useful in student learning. They synthesised empirical studies conducted between 2004 and 2014 on 1:1
technology in K-12 classrooms. They reported that tablet-based activity narrowed the achievement gap in geometry between high-achieving and low-achieving students in post-treatment test performances. One reason that accounted for the success of the students was that they were able to use the tablet most frequently for specific learning tasks, such as taking notes and searching for information via the internet, which enriched their learning experiences.

Technology may also be used to identify and address non-cognitive behaviours of students in mathematics learning. According to Taylor and Galligan (2006) “knowledge is not an entity external to the student, but is constructed within the cognitive structure of every individual … it is fundamentally personal and dependent on the experiences in prior and current learning environments and social interactions” (p. 12). This implies that anything in the learning environment that has the potential of disengaging students from learning ought be identified and removed. The learning sciences community acknowledges the importance of affect in students’ learning and has begun using sensors and technologies such as biosensors, eye-trackers, speech recognition, and automated video analysis that can automatically detect and respond to students’ off screen behaviour (Arroyo et al., 2014; Calvo & D’Mello, 2010; Conati & Maclaren 2009; Cooper, Muldner, Arroyo, Woolf, & Burleson, 2010).

Research exploring the relationship between technology and students’ affect (e.g., emotions, engagement, and attitudes) towards mathematics learning is gaining recognition (Arroyo et al., 2014; Ingram et al., 2016; Taylor & Galligan, 2006). Taylor and Galligan (2006) had illustrative evidence of the effectiveness of multimedia alternatives (video, audio, animations, diary, interactive examples, and self-assessment) on students’ cognitive and affective attitudes towards mathematics. The authors indicated that these technologies encouraged students to (i) feel part of the group they worked in, (ii) reflect on their feelings and beliefs about mathematics, (iii) use authentic problem-solving skills, and (iv) build their confidence through mathematics practice and self-assessment. The authors added that students who feared, or who felt daunted, confused, or terrified by doing mathematics, became spirited when they began using computer to do mathematics. Similarly, Arroyo et al. (2014) elucidated that Wayang Outpost (a multimedia-based intelligent tutoring system) could be used to address the individual needs, emotions, cognitive states, and metacognitive skills of learners.
Using technology to enhance students’ practicing of mathematics skills (procedural fluency) fits well in the mathematics education curriculum. Policy makers welcome this approach because students need to have a mastery of technology and mathematics skills to function in their future careers (Roschelle, Noss, Blikstein, & Jackiw, 2017). However, these skills need to be built through integration with the other four strands of mathematical proficiency: conceptual understanding, strategic competence, adaptive reasoning, and productive disposition (Harris, 2005; National Research Council, 2001). Thus, a meaningful adaptive use of technology is required to achieve this mathematical proficiency.

**Technology for developing mathematical concepts**

Using technology for developing concepts refers to creating a learning environment that evokes a specific thinking process and guides the development of mathematical thinking in students (Jupri et al., 2015). Pedagogically, researchers are consistent about the need to appropriate technology to (i) provide opportunities for students to visualise mathematics concept in a dynamic way and (ii) generate various examples of mathematics concepts for provoking exploration, generalisation, and mathematical thinking (Calder et al., 2006; Jupri et al., 2015).

A large body of research has examined the effectiveness of technology supporting the development of mathematics concepts. For example, Saha et al. (2010) and Akkaya et al. (2011) found that learning in the GeoGebra environment had a significant impact on students’ achievements because it provided multiple images for students to visualise mathematics concepts. It improved students’ spatial skills of exploring the relationship between geometric objects and algebraic expressions. Akkaya et al. (2011) added that the GeoGebra instructional approach has the possibility of supporting students to internalise the basic logic of a mathematical concept. Spreadsheets have also been demonstrated to be useful in students’ mathematics learning outcomes (Benning & Agyei, 2015; Dettori, Garuti, & Lemut, 2001; Jones, 2005; Neurath & Stephens, 2006; Niess, 2005; Niess, Sadri, & Lee, 2007). Neurath and Stephens (2006) confirmed that using an Excel spreadsheet in a high school algebra class could improve the interest of students who initially detested the learning of algebra. According to Niess (2005), spreadsheets offer dynamic modelling capabilities that led toward their use as a mathematical problem-solving tool with the capacity for
engaging students in higher order thinking skills that supported them in exploring beyond initial solutions.

Classrooms which are digitally enhanced have the potential to improve students’ interactive learning (Attard & Curry, 2012; Calder et al., 2006; Geiger et al., 2012; Ingram et al., 2016). Ingram et al. (2016) explored the effectiveness of the Show and Tell app on students’ engagement with problem solving. They found the app supported students to work both independently and cooperatively on problem solving tasks. Calder et al. (2006) also recounted that the ability of spreadsheets to produce prompt output could initiate students into mathematical discourse where they interactively reasoned and iteratively formulated their argument about mathematics concepts. Thus, the interactive nature of these software applications offered students opportunities to develop links among concepts (Funnell, Marsh, & Thomas, 1995), and made their mathematical thinking more visible (Ingram et al., 2016).

From the foregoing discussion, technology seems to provide an alternative instructional approach consistent with the work of developmental cognitive (Piaget, 1926) and socio-cognitive theories (Vygotsky, 1978). From the viewpoint of the proponents of these theories, small group learning increases achievement because it offers “opportunities for students to discuss, debate, and present their own and hear one another's perspectives… in their discussions of the content, cognitive conflicts will arise, inadequate reasoning will be exposed, and enriched understanding will emerge” (Springer, Stanne, & Donovan, 1999, p. 25).

The introduction of mobile technology apps in mathematics instruction does not only promote students’ engagement and confidence in mathematics learning, but it makes the students feel motivated and they develop a sense of ownership and autonomy over their learning (Bray & Tangney, 2015). Harper and Milman (2016) pointed out that introducing laptops into middle school classrooms fundamentally changed the nature of classroom activities where the devices were used to promote small group work, thus enhancing collaboration and effective communication.

There are other studies that have indicated that technology-based instruction can sometimes not be effective for students’ learning. Campuzano, Dynarski, Agodini, and Rall (2009)
reported no significant effect size when they conducted a rigorous study of the conditions and practices under which software products (for example, reading software, Cognitive Tutor Algebra 1, and Larson Algebra 1) were used to enhance student academic achievement. Also, Shiah, Mastropieri, Scruggs, and Fulk (1995) developed a computer tutorial programme as a prototype intervention to assist elementary students who had learning disabilities in mathematics. They found that there was no significant difference in the performance in the paper and pencil test involving word-problems between the computer users and non-computer users. Baker, Gearhart, and Herman (1994) assessed the effectiveness of interactive technologies on teaching and learning. Though it was shown in their study that the technology heightened students’ attitudes, reasoning, and problem-solving levels, the technology-assisted students did not outperform non-technology assisted students on standardised tests including vocabulary, reading comprehension, and mathematics concepts. Björkwall and Engblom (2010), Hur and Oh (2012), Carr (2012), and Liu, Lin, Tsai, and Paas (2012) reported that a 1:1 technology programme had no significant improvement in students’ academic achievement.

Harper and Milman (2016) indicated that a 1:1 technology programme faced challenges such as insufficient technical support, and inadequate professional development and training for teachers. They further indicated that teachers’ perceptions impeded the implementation of 1:1 technology programme. The contextual factors identified by Harper and Milman suggested that technology is effective in students’ achievement, however, it cannot substitute for the ideal classroom practice of the teachers (Kozma, 2001; Li & Ma, 2010). This implies technology could be only a component of effective classroom practices.

There is a ongoing issue within the educational research community of how to get teachers and students to use technology beyond routine work (Artigue, 2002; Drijvers, 2015). That is, utilising the changes in representational forms which technology makes available, challenging us to rethink the kind of instructional decisions appropriate for mathematics learning. Focussing on the teacher factor, Artigue (2002) and Drijvers (2015) articulated the complexities involved in bridging the disjuncture between the procedural and conceptual adoption of technology in teaching and learning of mathematics. Artigue (2002) pointed out that it is easy to see a pragmatic value (procedural skills/getting answers) of technology-assisted instructions, but it may be hard to grasp their epistemic value (developing conceptual understanding). Difficulties were obvious when teachers engaged students in
mathematical discourse in a technology learning environment because it requires them to have knowledge which goes beyond the standard mathematics culture. They need to intertwine standard mathematics knowledge and knowledge about the technology. Drijvers (2015) summarised his views as:

The integration of technology in mathematics education is not a panacea that reduces the importance of the teacher. Rather, the teacher has to orchestrate learning, for example by synthesizing the results of technology-rich activities, highlighting fruitful tool techniques, and relating the experiences within the technological environment to paper-and-pencil skills or to other mathematical activities. To be able to do so, a process of professional development is required, which includes the teacher’s own instrumental genesis, or, in terms of the TPCK model, the development of his technological and pedagogical content knowledge (p.148).

From this quotation, the important question we need to explore is how the role of the teacher changes when he or she uses technology to enact effective mathematics pedagogy, following their engagement in professional development. The following section explores in detail the theoretical underpinnings of effective integration of technology in mathematics.

**Theoretical Frameworks for Effective Technology Integration**

Research in education has made substantial progress in identifying crucial aspects that limit effective implementation of curriculum innovation in the classroom (Beswick, 2005; Ertmer, Ottenbreit-Leftwich, Sadik, Sendurur, & Sendurur, 2012). The teacher is just one part of, and contributor to, the effective implementation of technology in the teaching and learning of mathematics. In the mix of several contextual factors, the importance of teachers’ beliefs has been acknowledged by many researchers of technology integration in education (Bebell, Russel, & O’Dwyer 2004; Ruthven, Hennessy, & Deaney, 2005; van Braak, Tondeur, & Valcke, 2004).
**Teachers’ beliefs**

Teachers’ beliefs have received considerable attention in mathematics education research because of their centrality and prediction of teachers’ behaviours in the classroom (Beswick, 2005; Ertmer, Ottenbreit-Leftwich, Sadik, Sendurur, & Sendurur, 2012; Green, 1971; Nespor, 1987; Pajares, 1992). However, the characterisation of the concept of beliefs and its related constructs is fuzzy in the literature (Beswick, 2005). Terminologies such as attitudes, knowledge, values, conception, views, impressions, perceptions, judgement, predispositions, dispositions, and so forth have been conflated (Morgan, 2004) or used interchangeably to connote belief constructs (Pajares, 1992). For example, some authors think it is part of affect (McLeod, 1992), others consider beliefs to be part of knowledge (Furinghetti, 1996; Philipp, 2007), others characterised beliefs as part of attitudes (Grigutsch, 1998), and some consider it as part of conceptions (Pehkonen & Pietilä, 2003; Thompson, 1992). Some of the constructs are particularly associated with belief when it is operationalised from different perspectives. For example, emotions, values, and attitudes are mentioned when belief is defined from an affective perspective (Hannula, Evans, Philippou, & Zan 2004); self-efficacy, defined from a perspective of social cognitive theory (Bandura, 1986, 2010); and self-concept and self-esteem, when defined from a perspective of phenomenological and humanistic theories (Pajares, 1992). This all makes the definition of beliefs “the game of player’s choice” (Pajares, 1992, p. 309), making it difficult to understand and compare research studies (Pehkonen & Pietilä, 2003).

Debate has been keen on distinguishing knowledge from beliefs, a daunting task with which education researchers are battling (Chinn & Samarapungavan, 2001; Morgan, 2004). Furinghetti and Pehkonen (2002) argued that it is crucial to distinguish the type of knowledge (either objective or subjective) from which beliefs are defined. Beliefs have been defined by many authors as a subjective element of knowledge that an individual considers true and important in relation to a specific subject, tool, or approach (Pehkonen & Pietilä, 2003; Petko, 2012; Raymond, 1997). Thus, beliefs are relative and they are dependent on the person’s history, emotions, values, and environment.

According to Philipp (2007), knowledge is beliefs held with certainty or justified true belief. When knowledge is defined from Plato’s point of view of ‘justified truth’, then it becomes objective, factual, and infallible, and it is constructed independent of the environment. The constructivists diverge from the notion of knowledge being independent of human nature.
and they argue that everyone has a distinct perspective of viewing the world and hence knowledge is constructed through human interaction with the environment (that is, subjective knowledge). From the discussion in this section, it could be argued that when beliefs are defined through a distinction between a belief and a knowledge system, it creates an idealised view of knowledge which is less affective and less episodic (Österholm, 2010). Such definition also lacks a theoretical perspective which makes it difficult for readers to interpret.

Nespor (1987) identified four characteristic features when describing teachers’ beliefs: existential presumption, alterativity, affective and evaluative loading, and episodic structure. Existential presumptions are unquestionable and are the personal truth values an individual has formed about physical and social reality (Pajares, 1992). Such beliefs are deeply personal and difficult to be altered through persuasion. Such beliefs, according Pajares (1992), are usually taken for granted; they are usually formed by chance, intensive experience, or repeated practice. They include beliefs about what an individual and others cherish. Existential presumptions are perceived by the individual as unchangeable values that exist beyond the control of their knowledge (Nespor, 1987).

The alternative feature of beliefs occurs when an individual attempts to create an ideal situation based on conditions different from the physical and social reality (Nespor, 1987). For example, when an individual feels uncomfortable with an existing practice, he or she could create a new approach towards carrying out the task. In such a situation, “beliefs serve as means of defining goals and tasks, whereas knowledge systems come into play where goals and the paths to their attainment are well-defined” (Nespor, 1987, p. 319).

The affective and evaluative components operate more strongly with belief than knowledge (Nespor, 1987). That is, feelings, moods, and subjective evaluations based on personal preferences seem to “operate independently of the cognition associated with knowledge” (Pajares, 1992, p. 309). Thus, knowledge of a domain can be conceptually distinguished from feelings about that domain. What this means is that teachers could acquire the knowledge and skills of using, for example, GeoGebra, independent of his or her feelings (likes or dislikes, what excites or bores, what is important or trivial) towards GeoGebra. On the other hand, these beliefs and attitudes could influence the way a teacher acquires the knowledge and skills of using GeoGebra, and how it would be used eventually in the
classroom (Nespor, 1987). That is, the value that the teachers place on GeoGebra determines the extent to which they would use it.

With regards to the fourth feature of the teachers’ beliefs, Nespor (1987) distinguished between knowledge and beliefs, arguing that knowledge information is semantically stored, whereas beliefs reside in episodic memory, drawn from experience or cultural sources of knowledge transmission. Thus, according to Nespor, teachers usually craft their beliefs through their past experiences, events, or episodes, which hugely inform the way they enact subsequent practices in the classroom. From Nespor’s argument, beliefs tend to be unbounded. That is, they can be “readily extended to apply to phenomena that may be unrelated to the context” (Ertmer, 2005, p. 30). This enables teachers to have a pool from which they can draw a large amount of knowledge to address unusual experiences they encounter.

Ernest (1989) identified that teachers’ theoretical and practical knowledge are closely linked, and that these are associated with beliefs and attitudes. Aspects of theoretical knowledge (and associated beliefs and attitudes) form the basis for components of practical knowledge (and associated beliefs and attitudes), which relate to some aspect of classroom teaching practice. The distinction between these two types of knowledge is that the theoretical knowledge involves ‘knowing that’, which is more structured and largely acquired prior to, or away from, the practice of teaching. This level includes knowledge of subject matter and about the use of instructional tools (for example, technology), beliefs about and attitudes towards mathematics/instructional tools, knowledge of education, educational principles, and awareness of curriculum innovations. In contrast, practical knowledge involves ‘knowing how’, which is largely acquired from the practice of teaching. This level includes knowledge of teaching mathematics, models of teaching and learning mathematics, knowledge of classroom organisation and of the school context of teaching, and attitudes towards the teaching of mathematics. Attitudes to mathematics and its teaching involves teacher's personal reactions to experiences in the model of teaching and learning mathematics, compounded with other contextual factors.

The foregoing discussion demonstrates the difficulty in distinguishing beliefs from related constructs (Beswick, 2012; Furinghetti & Pehkonen, 2002). Furinghetti and Pehkonen (2002) shared that attempts by many authors to define beliefs, and to distinguish them from
related constructs such as attitudes and knowledge, have yielded little consensus. The
general agreement is that “teachers’ beliefs about the nature of knowledge and learning are
directly or indirectly related to performance mediated by cognitive processes, motivation,
attitudes, behaviour, efforts, and so forth” (Kim, Kim, Lee, Spector, & DeMeester, 2013, p.
78). Some authors have argued for the equivalence of beliefs and knowledge (Beswick,
2012). According to Beswick (2012) “beliefs are taken to be indistinguishable from
knowledge” (p. 128) because they are constructed in a similar way to how knowledge is
constructed.

Despite the struggle in reaching consensus, a unique distinction between beliefs and
knowledge is that while knowledge needs to be proved beyond reasonable doubt, beliefs do
not necessarily need to be, particularly when they are operationalised subjectively (Philipp,
2007). In other words, beliefs are formed based on conviction, and knowledge is formed
based on consensuality. Philipp argued that “What is knowledge for one may be belief for
another, depending upon whether one holds the conception as beyond question” (p. 259).
described non-consensuality as a feature of a belief system comprising propositions,
concepts, arguments, and so forth, which individuals or outsiders recognise but consider
disputable. Nestor further suggested that “belief systems are less malleable or dynamic than
knowledge systems. Knowledge accumulates and changes according to relatively well-
established canons of argument. Beliefs, by contrast, are relatively static” (p. 321).

Some authors have argued that it is unwarranted to uniformly have a distinct or precise
definition for each of the constructs (Kul, 2012), because teachers’ beliefs, knowledge and
practices are culturally sensitive and they can change at any time (Philipp, 2007). On the
other hand, other authors have shared that it is important to have precise definitions to guide
future research (McLeod & McLeod, 2002; Morgan, 2004; Pajares, 1992; Thompson, 1992).
The argument in this thesis agrees with the latter, that any attempt to provide definition, no
matter how porous that definition, could provide a necessary step in educational research.
For example, a distinctive definition for these constructs could help determine areas which
need research focus, the formulation of research questions (McLeod & McLeod, 2002;
Thompson, 1992), or the selection of appropriate research methodology and design (Pajares,
In the context of this research, beliefs are considered in line with McLeod and McLeod’s (2002) perspective. McLeod and McLeod defined beliefs in terms of their relations with nearby concepts, including knowledge and different affective constructs. They suggested that beliefs have strong relationships to both affective and cognitive processes that influence the practices in mathematics education. In this thesis, belief is interpreted as the attribution of some sort of truth to systems of propositions or other cognitive configurations (Hannula et al., 2004). It comprises teachers’ self-efficacy, values, emotions, and attitudes towards the use of technology in mathematics teaching and learning. Self-efficacy is interpreted in a psychological sense to imply teachers’ knowledge and ability to use technology to enact mathematics lessons (Ernest, 1989). According to Bandura (2010), “perceived self-efficacy is concerned with people’s beliefs in their ability to influence events that affect their lives. This core belief is the foundation of human motivation, performance accomplishments, and emotional well-being” (p. 1). Attitude is explained in an affective sense to imply teachers’ feelings, willingness, and values towards the use of technology in mathematics teaching and learning (Hannula et al., 2004; McLeod & McLeod, 2002).

**Teachers’ beliefs and technology integration**

The previous section sketched the complexities of reaching consensus in the definition of belief constructs. Similar difficulties exist in the literature regarding technology integration in mathematics education (Hermans, Tondeur, van Braak, & Valcke, 2008; Kim et al., 2013). Despite the inconsistency in operationalising beliefs, similar to the discussion in the previous section, the focus has been on the exploration of how belief constructs influence technology integration. For example, Ertmer and Ottenbreit-Leftwich (2010) described the influence of self-efficacy beliefs, knowledge, pedagogical beliefs, and cultural contexts on the implementation of technology in teaching and learning. They pointed out that teachers’ knowledge of technology only did not explain effective use of technology in teaching and learning. Rather, teachers needed to feel confident using that knowledge to facilitate student learning. Kim et al. (2013) argued that self-efficacy, beliefs about the value of technology, and beliefs about teaching and learning with technology, were crucial in technology integration, however, the fundamental beliefs that teachers bring into teaching provides useful understanding on how different technology is integrated (Bebell, Russel, & O’Dwyer 2004; Ruthven, Hennessy, & Deaney, 2005; van Braak, Tondeur, & Valcke, 2004).
Most of the empirical studies that have explored the relationships between teachers’ beliefs and their technology practices have reported mixed findings (e.g., Ertmer et al., 2012; Tondeur, van Braak, Ertmer, & Ottenbreit-Leftwich, 2017). Evidence indicates that teachers who hold constructivist beliefs tend to use technology frequently, and they usually use it to orchestrate a student-centred instructional approach, rather than their counterparts who hold teacher-centred beliefs of teaching and learning (Tondeur, van Braak, Ertmer, & Ottenbreit-Leftwich, 2017). This is not the case for all teachers. Some studies have indicated a mismatch between teachers’ constructivist beliefs and their actual practice with technology (Chen, 2008; Judson, 2006; So & Kim, 2009). For example, Chen (2008) explored twelve Taiwanese high school teachers’ construct beliefs and their classroom practices with technology. The author identified three interrelated reasons that accounted for inconsistencies between the teachers’ expressed beliefs and their technology practices: the influence of external factors, limited or improper theoretical understanding, and other conflicting beliefs. Admiraal et al. (2017) confirmed that there is a non-linear relationship between teachers’ pedagogical beliefs and the actual practice of technology in the classroom.

Several empirical studies have explored the predictive strength among various constructs of teachers’ beliefs towards the use of technology (Petko, 2012; Prestridge, 2012; Sandholtz, Ringstaff, & Dwyer, 1997; Stols & Kriek, 2011). Stols and Kriek (2011) established that teachers’ attitudes, subjective norms, and perceived behaviour, influenced their intention of using dynamic geometry software in their classrooms to develop concepts in the context of transformations, functions, or geometry.

Petko (2012) explored the teachers’ pedagogical beliefs and their use of digital media in the classroom through the use of the will (teachers’ personal beliefs), skill (teachers’ ability to use the technology), and tool (technical infrastructure) model developed by Christensen and Knezek (2002). The will, skill, and tool model is a theoretical framework that offers explanations for conditions under which teachers would use technology in their classroom. The correlation analysis performed indicated the existence of significant positive correlations between the will, skill, and tool constructs, and combined frequency and diversity of technology use in teaching. The multiple regression analysis demonstrated that the five sub-factors: teachers’ technology competency, accessibility of technology, teachers’ responsibilities, perceived usefulness of technology, and teachers’ constructivist beliefs accounted for 60% of the explained variance of using technology in teaching.
Agyei and Voogt (2011) adopted the same model: Teachers’ will (positive attitude), skill (technology competence), and tool (access to technology tools). Among all three constructs, skill was identified as the strongest predictor of effective technology integration in the classroom. The study also indicated that pre-service teachers were significantly more anxious and less technology competent than in-service teachers. Larbi-Apau and Moseley (2012) validated the sub-constructs (affective, perceived control, perceived usefulness, and behavioural attitude) of the overall computer attitude scale developed by Selwyn (1997). He found that all four sub-constructs and the overall computer attitude had high internal consistencies and significant construct validity. Hierarchically, it was identified that affective attitude (strongest influence), perceived usefulness, behaviour, and perceived behavioural attitudes predicted teachers’ computer use.

The above studies are inconclusive about which belief constructs are most predictive of technology use in the classroom. This is consistent with the complex relationship among these constructs. The results from these studies were reported through self-report surveys (see, for instance, Agyei & Voogt, 2011; Petko, 2012), which may have led to a common method variance, a situation that may inflate the true associations among the belief constructs, resulting in spurious significant findings. However, there is one key thing we can learn from the studies. The interplay of the constructs offers us useful insights into the direction of professional development teachers need. The following paragraph elaborates on this point.

Curriculum innovation requires teachers to deal with a complex set of interrelated variables including their own personal dispositions, students’ learning, school environment, assessment requirement, and so forth. This implies that when teachers’ beliefs are explored narrowly, it will not examine them enough to deal with the multi-dimensional factors of adopting technology in the classroom. For example, Hermans et al. (2008) argued that “the more a belief is related to other beliefs constructs, the more it is positioned at the centre of the belief system and the less this belief is subject to change” (p. 1501). Teachers’ affective attitudes (fear, discomfort, and hesitation), perceived control attitudes (ease or difficulty of technology use), perceived useful attitudes (degree of relevance in improving job performance), and behavioural attitudes (intentions and actions) informed the extent to which they would apply technology in their classroom (Huang & Liaw, 2005). What this
means is that a teacher could perceive GeoGebra, for example, to be useful in supporting his or her students to learn cooperatively, however, he or she may not attempt to use it if he or she feels anxious about his or her inability to problem-solve GeoGebra related issues (Howard, 2011). Teachers having the required knowledge and skills could assuage the fear, discomfort, and the hesitation attached to the use of the technology, and hence their attitude towards using technology would be improved, which would then lead to changes in their classroom practices (Larbi-Apau & Moseley, 2012; Myers & Halpin, 2002). Similarly, Sang, Valcke, van Braak, and Tondeur (2010) shared that “teachers’ positive experiences with computers; teachers’ comfort with computers; specific beliefs related to the use of computers as an instructional tool; number of workshops attended; the challenge subscale of the work preference inventory; assistance from others; and teaching efficacy” (p. 105) were crucial to build holistically teachers’ belief in technology integration. Although professional development has been proposed as a crucial step to initiate teachers into the use of technology in classroom practice, there is tension about what constitutes effective professional development. The discussion on professional development will be addressed in the next section.

Technology pedagogical content knowledge (TPACK)
The concept of technological pedagogical and content knowledge (TPACK) first appeared in an educational journal in 2003 (Lundeberg, Bergland, Klyczek, & Hoffman, 2003). It is an extension of pedagogical content knowledge (PCK), introduced by Shulman (1986). The framework was originally called TPCK but for easy pronunciation, it is now called TPACK (pronounced T-Pack; Thompson & Mishra, 2007). TPACK is a framework Mishra and Koehler (2006) introduced to help us understand and describe the types of knowledge teachers need for effective use of technology in education (Arora & Pany, 2018; Chai et al., 2013; Koh & Chai, 2014). TPACK is composed of three core domains of knowledge: Technological knowledge (TK), Pedagogical knowledge (PK), and Content knowledge (CK). The interaction of these three core domains of knowledge gives rise to pedagogical content knowledge (PCK), technological content knowledge (TCK), technological content knowledge (TPK), and the TPACK (see Figure 3.2).
Technological Knowledge (TK) refers to teachers’ knowledge about how to use ICT hardware, software, and associated peripherals. It enables and facilitates teachers to understand information technology, apply it properly, identify useful technologies, and continually adapt to changes in technology (Chai et al., 2010, 2013; Koehler & Mishra, 2009).

Pedagogical Knowledge (PK) refers to teachers’ knowledge about students’ learning, instructional methods, different educational theories, and learning assessment, to teach a subject matter without reference to content. It involves understanding the processes and practices or methods of teaching and learning, including knowledge about understanding the nature of the students, strategies for evaluating students, understanding cognitive, social, and development of theories, and how they apply to the students in the classroom (Chai, et al., 2010, 2013; Koehler & Mishra, 2009).

Content Knowledge (CK) is teachers’ knowledge of the subject matter, without consideration of how to teach the subject matter (Chai et al., 2010, 2013). Shulman (1986) described content knowledge as including the knowledge of concepts, theories, ideas, organisational framework, and knowledge of evidence and proof, as well as practices and approaches towards developing such knowledge. It includes conjectures, axioms, patterns, and logical argument. Schmidt et al. (2009) maintained that teachers must know about the content they are going to teach and how the nature of knowledge is different for various content areas.
**Pedagogical Content Knowledge (PCK)** is teachers’ knowledge of representing content knowledge and adopting pedagogical strategies to make the specific content/topic more understandable for the learners. Shulman (1986) described PCK as including "an understanding of how particular topics, problems, or issues are organised, presented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (p. 8). Hill, Ball, and Schilling (2008) explained teachers’ mathematics PCK as encompassing knowledge of students’ mathematical thinking and learning. Thus, PCK describes teachers’ knowledge of students’ understandings, thinking, and learning with regards particular subject matter topics. It involves teachers’ knowledge of curriculum and curriculum materials with learning and teaching subject matter topics, and knowledge of instructional strategies and representations for teaching and learning particular content topics (Chai et al., 2010, 2013; Grossman, 1990).

**Technological Pedagogical Knowledge (TPK)** is teachers’ knowledge of the existence and specifications of various technologies to enable teaching approaches without reference towards subject matter (Chai et al., 2013). That is, the ability to choose a technological tool based on its fitness for the learning activity. Some examples of TPK are using spreadsheets to conduct didactic lessons, using Webquest to conduct project-based lessons, using simulation and animation to catch students’ attention in the classroom, and adopting practice-and-drill activities downloaded from the internet in the classroom.

**Technological Content Knowledge (TCK)** is teachers’ knowledge about how to use technology to represent and research the content in different ways, without consideration of teaching. That is, it is the ability to put content and technology together. Examples of TCK include knowledge about online dictionaries, subject-specific websites and subject-specific ICT tools (for example, GeoGebra, Novasoft, and Geometer’s Sketchpad) (Chai et al., 2013).

**Technological Pedagogical Content Knowledge (TPACK)** is the knowledge of using various technologies to enhance knowledge creation of specific subject content. It is also the knowledge of how technologies are used to bridge students’ pre-existing experiences to new ones, and using computer applications to assist students who have some difficulties in learning certain subject-matter (Koehler & Mishra, 2009). True technology integration is about understanding and negotiating the relationships between technology, pedagogy, and content knowledge (Chai et al., 2013; Koehler & Mishra, 2009).
Contextual factors: The broken circle in Figure 3.2, which surrounds the interlocking circles, represents the contextual factors (such as institutional supports, teachers’ prior experience, perceived beliefs, and attitudes towards technology) necessary for effective implementation of technology to effectively teach a specific content area. Teachers’ understanding of the subtleties surrounding the use of technology in a learning environment is a key for effective technology integration. It helps them appreciate the affordances and constraints of various technologies and the appropriate way to use the tools to maximise students’ learning (Kereluik, Mishra, & Koehler, 2011; Koehler & Mishra, 2009; Warren, Lee, & Najmi, 2014). As an example, technology integration in mathematics education in Ghana is at its beginning stage and most of the classrooms are not sophisticatedly resourced. This means that the computers in the ICT laboratories are not installed with adequate mathematics software (Agyei & Voogt, 2011). Therefore, teachers’ knowledge about this situation will enable them to choose a particular technology that will match their context to enhance effective students’ learning (Zhao, Pugh, Sheldon & Byers, 2002) within that context.

In addressing contextual issues related to TPACK studies, Niess (2007) extended Grossman’s (Grossman, 1989, 1990) four central components of PCK towards TPACK to describe the knowledge and beliefs teachers required to enact mathematics instructions with technology:

1. This conception is what the teacher knows and believes about the nature of mathematics, what is important for students to learn, and how technology supports learning mathematics. These foundations of the teacher’s knowledge and beliefs about teaching mathematics with technology serve as a basis for his or her decisions about classroom instruction (objective, strategies, assignments, curriculum and text, and evaluation of student learning).

2. Knowledge of students’ understandings, thinking, and learning in mathematics with technology. In this area, the teacher relies on and operates from knowledge about how students learn mathematics/science with technologies and believes that technologies are useful in learning appropriate mathematics/science.

3. Knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics. With respect to the curriculum, the teacher discusses and implements various technologies available for teaching particular
topics and how the topics and ideas in a technology-enhanced environment with concern for how the activities are organized, scaffolded, structured, and assessed throughout the curriculum.

4. Knowledge of instructional strategies and representations for teaching and learning mathematics with technologies. With respect to teaching and learning, the teacher adapts mathematical representations with technologies in multiple ways to meet specific instructional goals and the needs of the breadth of learners in the class (p. 4).

The description of TPACK provided above offers us insights into the structure and content of the professional development that teachers need to orchestrate pedagogical change using technology. The professional development should provide opportunities for teachers to reflect critically on their classroom practices and to develop new knowledge and beliefs about subject matter, pedagogy, and students’ thinking and learning (Borko & Putnam, 1996; Darling-Hammond & McLaughlin, 1995; Doering, Veletsianos, Scharber, & Miller, 2009; Guskey, 2002; Heitink et al., 2016; Kopcha, 2012; Putnam & Borko, 2000). Dagienė (2011) remarked that effective integration of technology into the classroom depends on the ability of teachers to structure the learning environment in non-traditional ways, to merge the new technology with new pedagogy, and to develop socially active classrooms by encouraging a cooperative interaction, collaborative learning, and group work. Any professional development programme that concentrates on only technological, pedagogical, or content knowledge is likely to fail. Therefore, “effective teacher educational and professional development needs to craft systematic, long-term educational experiences where the participants can engage fruitfully in all three of these knowledge bases in an integrated manner” (Koehler, Mishra, Kereluik, Shin, & Graham, 2014, p. 109).

Although TPACK is young in educational research, several scholars have developed and adopted a number of instruments for it: a self-report survey (Archambault & Barnett, 2010; Lux, 2010; Schmidt et al., 2009), an observation checklist (Hofer, Grandgenett, Harris, & Swan, 2011), performance assessment rubrics (Harris, Grandgenett, & Hofer, 2010), and levels of TPACK development (Niess et al., 2009). These instruments provide approaches to investigate the knowledge and skills teachers need for effective use of technology in the classroom. The self-report survey is usually in the form of a Likert scale which is used to assess teacher’ competence in the various constructs in TPACK. The observation checklist is used to observe teachers’ level of TPACK development as they go through professional
development programme. The performance assessment rubrics are used to evaluate teachers’ TPACK by directly examining their performance on a teaching task. This teaching task could be preparation of a lesson plan, classroom management, assessment practices, or enactment of an ICT-based lesson. Measurements such as interviews and open-ended questions could provide in-depth understanding of teachers’ knowledge, experiences, and beliefs about issues in situated contexts (Yeh, Hsu, Wu, & Chien, 2017).

Furthermore, the five levels: recognising, accepting, adopting, exploring, and advancing identified by Niess et al. (2009) provides a useful lens for describing the developmental progression of teachers’ TPACK when they are engaged in professional development or course work. Recognising is when teachers can use the technology and recognise the alignment of the technology with subject matter content, yet do not integrate the technology in teaching and learning of the content. Accepting is when teachers form a favourable or unfavourable attitude toward teaching and learning specific content topics with an appropriate technology. Adapting is when teachers engage in activities that lead to a choice to adopt or reject teaching and learning specific content topics with an appropriate technology. Exploring is when teachers actively integrate teaching and learning of specific content topics with an appropriate technology. Advancing is when teachers redesign the curricula and evaluate the results of their decision to integrate teaching and learning specific content topics with an appropriate technology.

TPACK is a growing framework and therefore there are certain areas that need attention. Some concerns have been raised about the relevance of each construct and the relationship between the constructs. Many researchers have indicated these constructs interact in highly complex ways (Angeli & Valanides, 2009; Cavanagh & Koehler, 2013; Graham, 2011) but Mishra and Koehler failed to provide adequate explanation of the relationship existing between these constructs (Angeli & Valanides, 2009; Archambault & Barnett, 2010; Cox, 2008; Graham, 2011). Also, there are mixed arguments about which of the knowledge domains significantly contribute to teachers’ TPACK development. Some researchers mentioned that teachers who lack both pedagogical knowledge and classroom experience find it difficult to apply TPACK (Niess, Lee, Sadri, & Suharwoto, 2006; Orlando & Attard, 2016; Pamuk, 2012; Pierson, 2001; Tømte, Enochsson, Buskqvist, & Kårstein, 2015; Valtonen, Kukkonen, & Wulff, 2006). Brunetto and Kontorovich (2017) argued that TPACK has been rarely used in the case of in-service teachers who have pedagogical experience.
Brunetto and Kontorovich strengthened their argument by indicating that developing “teachers’ TPACK is inseparable from integrating technology in the classrooms”, and thus prior teaching experience is key (p. 121). On the other hand, some authors shared that teachers’ computer self-efficacy significantly predicts effective implementation of technology in the classroom (Littrell, Zagumny, & Zagumny, 2005; Niess, 2011; Petrogiannis, 2010; Wang & Chen, 2006; Zhao et al., 2002). For example, Niess (2011) shared the view that in-service teachers did not learn their mathematics content with technology and for that matter “they do not have essential experiences in learning with these technologies…they have not been prepared to engage in the strategic thinking for knowing when, where, and how to use domain-specific knowledge and strategies for teaching with the technologies” (p. 308).

Increased knowledge in a single component of TPACK may not be sufficient to stimulate change in practice (Chai et al., 2013). Therefore, what needs to be done is an emphasis on training that supports teachers to gain growth in the connectedness and complexity of TPACK as a whole (Baya’a & Daher, 2015; Brunetto & Kontorovich, 2017; Koehler & Mishra, 2009). To address this situation, according to Niess (2011) educational researchers have a number of questions to which they need to respond:

What is a TPACK-based teacher preparation program? How do pre-service and in-service teachers develop TPACK? What are the essential experiences needed for integrating pedagogy, content, and technology within the educational contexts? How is a teacher’s TPACK recognized? Are there different levels of TPACK? What is the effect on a teacher’s TPACK as new technologies are introduced for incorporation in their curriculum? How is a teacher’s TPACK assessed? (p. 308).

**Conceptualising effective mathematics pedagogy**

There is some debate about what makes mathematics teaching and learning effective (Franke, Kazemi, & Battey, 2007; Jacobs & Spangler, 2017). Teaching mathematics in the classroom not only comprises the complex interactions between curriculum, assessment, and pedagogy, but also involves engagement of students, teachers, administrators, and schools in contexts that vary each day (Franke et al., 2007). For example, in fashioning the pedagogy
which is capable of achieving positive learning, teachers are required not only to focus on the activities that engage students in mathematical thinking, but they also need to figure out how those activities relate to the knowledge of students’ identities, histories, culture, and school environment (Jacobs & Spangler, 2017; Stein, Engle, Smith, & Hughes, 2008). These factors make it difficult to claim a particular instructional approach or pedagogical model as a stand-alone solution for improving mathematical proficiency.

For teachers to achieve productive mathematics instruction, it is important to strategically identify the core practices of mathematics instruction that are likely to improve teaching and learning with a particular audience (Jacobs & Spangler, 2017). One way to approach this is to examine two major frameworks that seek to explain effective pedagogy, in this case through comparing those proposed by Anthony and Walshaw (2007) and NCTM (2007). Three reasons informed the adoption of these models (Anthony & Walshaw, 2007; NCTM, 2007) in this thesis. Both models have an explicit focus on student-centred learning, as in Vygotsky’s sociocultural theory where the mathematics classroom is considered as a social space to embrace students, the teacher, the subject of learning, and engaged activity (Middleton, Jansen, & Goldin, 2017; Vygotsky, 1978). The combined model has the advantage of compensating the weaknesses of each other. Both models have been demonstrated to apply to both technological and non-technological innovations in mathematics education in terms of critiquing those innovations with reference to their ability in enabling teachers to enhance effective pedagogy (Andrews, Ryve, Hemmi, & Sayers, 2014; Hunter & Back, 2011; Ingram et al., 2016; Ke, Kang, & Liu, 2016; Kwon & Orrill, 2009; Scott, Clarkson, & McDonough, 2011; Sharma, 2015). The instructional practices that are central to Anthony and Walshaw (2007) and NCTM’s (2007) models of effective pedagogy have some commonalities with other literature on teaching and learning in mathematics.

Numerous scholars have made extensive efforts to develop models for evaluating effective pedagogy, however, the descriptions in most of these models overlap and lack common terminologies (Jacobs & Spangler, 2017). Several terms have been used to connote effective pedagogy: standard authentic instruction (Newmann & Wehlage, 1993), professional standards for teaching mathematics (Martin & Speer, 2009), and active learning of mathematics (Smith, 1999). The common notion among these models of effective pedagogy is the creation of a learning environment which makes both teachers and students
participative meaning makers of mathematical concepts. In this study, effective pedagogy is conceptualised to align with Anthony and Walshaw’s (2007) perspective of effective mathematics teaching and learning. They described effective mathematics teaching as utilising the kinds of pedagogical approaches that engage learners to achieve desired learning outcomes. The desirable learning outcomes espoused by these authors encompass students’ ability to demonstrate the five strands of interrelated mathematical proficiency: conceptual understanding (comprehension of mathematical concepts, operation, and relation), procedural fluency (skills in carrying out procedures flexibly, accurately, efficiently, and appropriately), strategic competence (ability to formulate, represent, and solve mathematical problems), adaptive reasoning (capacity for logical thought, reflection, explanation, and justification), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy) (National Research Council, 2001).

From the perspective of a classroom as a community of practice, Anthony and Walshaw (2007, 2009), in a review, suggested 10 principles of pedagogical practices that facilitate learning for diverse learners (see Table 3.1). Both models (Anthony & Walshaw, 2007; NCTM, 2007) emphasise the engagement of students in open-ended tasks to extend their mathematical proficiency, as advocated by the National Research Council (2001). For example, engaging students in classroom dialogue, that is mathematical communication (Anthony & Walshaw, 2007; NCTM, 2007), has the likelihood of promoting students’ proficiency in adaptive reasoning where they could extend their mathematical knowledge to unfamiliar situation in their settings (National Research Council, 2001). Franke et al. (2007) noted that mathematics learning arises from a student’s participation and interactions within the social space of a classroom that provides autonomy for students to share mathematical ideas.

In a similar review, Gervasoni, Hunter, Bicknell, and Sexton (2012) identified that creating a powerful learning environment and selecting rich tasks are among factors that promote successful mathematics learning. In their review, Hertzog and O’Rode (2011) identified problem-solving, explanation, representation, and mathematical connections as core practices in mathematics instruction that have the potential of promoting mathematical proficiency. In their earlier studies, Newmann and Wehlage (1993) identified that authentic instruction in social studies and mathematics should focus on engaging students in higher-
order thinking activities, developing depth of knowledge, substantive conversation, as well as providing opportunities for students to connect the concept they have learned to the world, and offering social support for student achievement.

Table 3.1 Elements of effective mathematics pedagogy

<table>
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<tr>
<td>An ethic of care</td>
<td>• Choosing “good” problems that:</td>
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<tr>
<td>- creating a classroom community that promotes the needs of individual students</td>
<td>- Exploring of important mathematical concepts,</td>
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<tr>
<td>Arranging for learning</td>
<td>- providing student the chance to solidify and extend their knowledge.</td>
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<td>Building on students’ thinking</td>
<td>Making connections</td>
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<td>Worthwhile mathematical tasks</td>
<td>• Encouraging students to explore multiple solutions.</td>
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<td></td>
<td>• Encouraging students to think more deeply about:</td>
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<td></td>
<td>- the problems they are solving</td>
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<td></td>
<td>- making connections with other ideas within mathematics</td>
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<td>Assessment for learning</td>
<td>• Assessing questioning techniques to facilitate students’ learning and reasoning.</td>
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<td>Mathematical communication</td>
<td>• Assessing students’ understanding by:</td>
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<td></td>
<td>- listening to discussions</td>
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<td></td>
<td>- asking students to justify their responses</td>
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<td>• Creating a variety of opportunities such as</td>
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<td></td>
<td>- group work and</td>
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<td></td>
<td>- class discussions for students to communicate mathematically.</td>
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<tr>
<td>Mathematical language</td>
<td>• Modelling appropriate mathematical language and strategies for solving challenging mathematical problem.</td>
</tr>
<tr>
<td>Tools and representations</td>
<td>• Using multiple representations to foster a variety of mathematical perspectives.</td>
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<tr>
<td>Teacher knowledge and learning</td>
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The knowledge of teachers plays a central role in effective pedagogy. Unlike Martin and Speer (2009), Anthony and Walshaw (2007) explicitly included teacher knowledge in their model of effective pedagogy in mathematics. Although teacher knowledge is not specifically
included in most of the models of effective pedagogy (for instance, Martin & Speer, 2009),
the basic assumption is that teachers need to have a depth of knowledge of mathematics
content, as well as the understanding of students’ learning, before they can orchestrate an
effective mathematics lesson (Anthony & Walshaw, 2007; Ernest, 1989; Franke et al., 2007;
Gervasoni et al., 2012; Ingram, Linsell, & Offen, 2018; Shulman, 1987). Gervasoni et al. (2012), in a review, considered three issues critical in powerful pedagogical actions in
mathematics education. Among the three issues, teachers’ knowledge, belief, and attitudes
are the foundations of the practices of teaching of mathematics (Ernest, 1989). The other two
issues Gervasoni et al. included in effective pedagogy were (i) creating a powerful learning
environment and (ii) selecting tasks and models that promote deep learning. Teacher
knowledge on the worthwhile mathematical tasks and classroom learning environment are
critical factors in successful mathematics learning. With regards to the introduction of
technology in mathematics education, Mishra and Koehler (2006) stressed the need for
teachers to have integrated knowledge of technology, pedagogy, and content. This thesis
argues that teachers’ professional competence should encompass not only their knowledge
in the content, pedagogy, and the tool (including technology), but their beliefs and attitudes
towards the mathematics content, pedagogy, and the tool.

From the foregoing discussion, creation of a mathematical setting, useful mathematical
tasks, mathematical discussion, mathematical connections, and assessment of students’
learning could enhance students to develop mathematical proficiency where they could
become curious about mathematical ideas and further contextualise concepts in a new
setting. The difficulty here is that these practices of mathematics instruction are generic and
underspecified (Franke et al., 2007). Teachers who do not have any experience in a particular
learning environment, for example using technology to mediate a mathematics lesson, would
find it difficult to enact these practices meaningfully. Therefore, there is a need to theorise
core practices of each of these components of effective pedagogy within the specific domain
of mathematical concept, pedagogy, and tool (technology). Before I move on to discuss the
central themes of effective mathematics pedagogy, there is a particular construct that needs
to be defined.

The construct of engagement is mentioned multiple times in this discussion because of its
centrality to the theme of effective mathematics pedagogy, and because of its
interrelatedness with belief constructs such as attitudes, affective, and cognitive (Kong,
Wong, & Lam, 2003). In this thesis, engagement is used loosely to imply both teachers’ perceptions and enacted practices that promote students’ feeling, participation, thinking, and achievement in mathematics learning. For in-depth discussion on engagement, readers are referred to Jansen and Middleton (2011), Middleton, Jansen, and Golden (2017), and Shernoff (2013).

Creation of a mathematical setting

The first category of effective mathematics pedagogy pertains to setting up the environment to induct students into mathematics learning. It involves the actions the teachers take before, during, and after the lesson. Creation of a mathematical setting is conceptualised to encompass the skills and knowledge of the teacher to set up the learning environment to hook and sustain the students’ interest and attention throughout the mathematics lesson. The concept of an ethic of care (Anthony & Walshaw, 2007) reflects the considerations the teachers must make about the learning environment (classroom communities) to ensure that students have the opportunities to construct positive relationships through interactions and dialogue (Attard, 2011; Ingram, 2013). This is especially the case when innovation involves the use of a certain pedagogy or tool (for example, technology). Without enough preparation, and setting up the learning environment, achieving meaningful learning outcomes would be difficult. There is a need to go beyond instructional practices that focus only on mathematical thinking. Teachers need to open up their instructional approach to cater for students’ identities, histories, and cultural and school experiences, all in relation to the mathematical tasks in which the students are engaged (Franke et al., 2007; Ingram, 2013; Jacobs & Spangler, 2017). Franke et al. (2007) reiterated that particular norms for classroom communities are important to ensure (i) value is placed on ideas and methods, (ii) autonomy in choosing and sharing mathematical ideas, (iii) appreciating the value of mistakes as a site for learning for everyone, and (iv) renegotiation of mathematical ideas.

Useful mathematics tasks

The usefulness of the task is how the objectives and activities included in the lesson engage students in exploring and understanding mathematical concepts, procedures, and/or relationships. According to Ingram (2013), mathematical tasks presented to students can either engage or disengage them. Thus, it is crucial to structure mathematical activities that promote perseverance, mathematical intimacy and integrity, independence, concentration, cooperation, and reflection. An important aspect of effective pedagogy is the provision of
an important mathematical task (Hiebert & Grouws, 2007) that offers opportunities for students to extend their mathematical knowledge and thinking and skills of problem-solving (Anthony & Walshaw, 2007; Martin & Speer, 2009; NCTM, 2007). The teacher needs to carefully select teaching and learning aids appropriate for students’ cognitive levels. According to Hiebert and Grouws (2007), one of the features of classroom mathematics teaching that facilitate students’ conceptual development (mathematical proficiency) is engaging students in struggling with important mathematics tasks.

Students easily assimilate new mathematical concepts when it conflicts less with their existing knowledge (schema). Effective teachers provide worthwhile mathematical tasks that enable students to use their existing experiences to generate new mathematical concepts (Anthony & Walshaw, 2007). This implies that teachers require a solid understanding of their students’ cognitive levels as well as their existing knowledge and experiences. During exploration, teachers provide common based activities that facilitate changes in procedural and conceptual proficiency in mathematics by identifying concepts/misconception, process, and skills. Students experiment with mathematical ideas, record observations and solutions, make conjectures, and represent mathematical ideas through diagrams and graphs. Teachers guide students’ participation by encouraging dialogue within small groups. Teachers ask questions to challenge how students are making sense of the mathematical concept (Anthony & Walshaw, 2007; Martin & Speer, 2009; NCTM, 2007). Teachers need to have adequate knowledge of the use of curriculum resources (for example, technology, syllabus, and textbook). The teacher needs to design and enact a catchy introduction to hook (engage) and sustain the interest and attention of the students throughout the mathematics task (Anthony & Walshaw, 2007).

**Mathematical discussion**

Mathematical discussion is central in an engaged activity of mathematics learning. It involves creating opportunities for students to communicate their mathematical dispositions. In-depth mathematical discussion has commonly been described in literature as indicating effective pedagogy in mathematics (Franke et al., 2007; Jacobs & Spangler, 2017; Kazemi & Stipek, 2009; Stein et al., 2008). In the words of Franke et al. (2007):

> How teachers and students talk with one another in the social context of the classroom is critical to what students learn about mathematics. What it means to
do and learn mathematics is enacted through the discursive practices that form in the classroom (p. 230).

Effective teachers create a learning environment to facilitate classroom dialogue which emphasises mathematical argument where conclusions are reached through agreement between students and teacher (Anthony & Walshaw, 2007). Franke et al. (2007) reiterated that asking students to solve any problem does not only enhance concept formation, but it also promotes students’ reflective and analytical thought. However, teachers most often find it challenging to create a high-quality mathematical discussion environment that fosters reflective and analytical thought (Leinhardt & Steele, 2005; Stein et al., 2008).

Consequently, the instruction pattern dominant in most classroom discussion is initiation, response, and evaluation (IRE). The teacher dominates the discussion by initiating a question, students then provide responses, and teachers evaluate the responses (Franke et al., 2007). Most teachers do not make judicious choices of making the talker and the listener(s) negotiate mathematical meaning (Franke et al., 2007). Teachers need to monitor students’ responses, walk between desks to check both group and individual work, carefully select a student’s or group’s work for whole class discussion to enhance students’ mathematical thinking. Teachers have the advanced role of identifying possible student misconceptions, as well as responses he or she will provide to solidify and extend students’ mathematical knowledge (Martin & Speer, 2009; NCTM, 2007). For effective mathematical discussion, students need to be offered the opportunities to explain and justify their solutions to the satisfaction of each member in the mathematics classroom. Teachers have a role to play in ensuring that social norms are set to guide mathematical discussions. Teachers need to set rules that promote respect, encouragement, sharing, and equitable distribution of mathematics resources. Students need to be taught to develop good listening skills and judge other’s ideas constructively. Teachers need to encourage students to learn how to use appropriate mathematical terms, language, definitions, and methods to build their argument (Anthony & Walshaw, 2007; Stein et al., 2008).

**Mathematical connection**

The mathematical connectivity is how the teacher engages students to use complex and non-algorithmic thinking (for instance, using an unrehearsed approach or pathway to generate multiple solutions for a problem). The need to focus mathematics instruction more on
problem-solving, applications, and higher-level skills was recommended over three and a half decades ago (Cockcroft, 1982; NCTM, 1980). The recent literature reiterates the relevance of including open-ended tasks to enhance students to make sense of the new mathematical concept and skills (Anthony & Walshaw, 2007, 2009; Jacobs & Spangler, 2017; Martin & Speer, 2009; NCTM, 2007). According to Anthony and Walshaw (2007), effective teachers support students in creating connections between different ways of solving problems, between mathematical representations and topics, and between mathematics and everyday life. A mathematical task that has multiple solutions is useful in probing students’ thinking and expansion of mathematical knowledge. It also helps to elicit the varied strategies students use to carry out the same task.

Assessment of students’ learning

Assessment in mathematics learning is a very broad topic. Assessment is used to monitor students’ mathematical learning, inform the teacher’s future instruction, update parents about their wards’ learning progress, determine mathematical knowledge of a country’s students, and inform policy direction in education. In this thesis, assessment is operationalised as the creation of a learning environment where formative feedback and feedforward from the teacher and students are promoted to monitor the progress of students’ learning in a specific mathematical task. Effective teachers support their students’ learning by providing students with appropriate feedback about their thinking, by helping them learn to self- and peer-assess, and by using students’ thinking to sequence mathematics instruction (Anthony & Walshaw, 2009; Martin & Speer, 2009; NCTM, 2007; Suurtamm, et al., 2016).

Professional Development

The kinds of activities teachers are engaged in during technology professional development determines the extent to which they will include technology in their pedagogical decisions (Niederhauser et al., 2018). For example, traditional (one-shot workshop training) professional development has for considerable time received criticism because it (i) may lack collaboration and coordination with the teachers; (ii) is too fragmented, short-term, and lacks follow-up activities; (iii) lacks a message and relevance to what actually happens in the teachers’ classrooms; (iv) lacks cutting-edge instructional and communication technology integration; (v) is not aligned with recent developments in curriculum,
assessment, methodology, and technological advances; and (vi) lacks incentives and does not respond to teachers’ needs and concerns (Barufaldi, 1997). Sztajn, Borko, and Smith (2017) identified that a gap exists between what is known about professional development and what teachers actually experience in professional development. For example, the authors pointed out that most professional development offered to teachers is not systematically investigated and the activities included in it is not informed by research findings.

A paradigm change in professional development has been proposed by many authors to offset the limitations in traditional professional development. Whitcomb, Borko, and Liston (2009) suggested that professional development organised to help teachers to create authentic solutions for a classroom problem should be “situated in practice, focused on student learning, embedded in professional communities, sustainable and scalable, and both supported and accompanied by carefully designed research” (p. 208). Koehler, Mishra, and Yahya (2007) and Webb, Robertson, and Fluck (2005) espoused that a professional development model where teachers are engaged in technology-oriented activities is one of the key steps to enhance teachers’ knowledge and skills to use technologies to teach mathematics. Koellner, Jacobs, and Borko (2011) affirmed that in order to provide result-yielding professional development, the activities included in it should (i) foster active teacher participation in the learning process, (ii) use teachers’ own classroom practices to provide a basis for their learning, and (iii) provide a supporting professional community to enhance teachers’ learning.

Meanwhile, Kul (2012) invited researchers to adopt a longitudinal research approach to investigate a professional development model in the school context. Sztajn et al. (2017) confirmed that the professional development offered to teachers should be systematically investigated and the activities should be informed by research findings. They further reiterated that professional development conducted in the school setting could promote “close collaboration between researchers and practitioners in design and implementation [of curriculum innovations which] can help increase the likelihood of participants’ buy in and successful scale-up” (p. 818). The elements in professional development suggested by these authors reflect the philosophical assumption of the interpretive research paradigm, where both the researcher and the research participants are interdependent in solving classroom related problems. These characteristics of professional development resonate with the one Williamson-Leadley (2015) identified: that effective professional development should be
characterised by (i) a job-related issue that represents a collective concern, (ii) participants being involved in the planning activities of the professional development, (iii) problem solving through a collaborative approach, and (iv) provision of support after the initial training.

Trigueros and Lazano (2012) added that providing teachers the opportunity to work in groups inside and outside school is an indication of effective professional development. Goos and Bennison (2008) extrapolated from their study that teachers are often interested in professional development that models planning and pedagogy. Such programmes, according to Goos and Bennison, help teachers to meaningfully orchestrate technology-based lessons to enhance students in conceptualising mathematical ideas. They shared that such an approach would clarify any link, or lack thereof, between the teachers’ beliefs and actual practice in a mathematics classroom. Thus, Agyei (2012) recommended that using a technology, which is readily available, with the potential of supporting students’ higher-order thinking in mathematics is key to successful intervention in integrating technology. By this, he means that to fully develop the teachers’ competency in technology integration, teachers’ knowledge and use of particular mathematical software is of the essence. That software should be easily adaptable, user-friendly, and invoke critical thinking.

From the foregoing discussion, effective professional development for technology integration in mathematics teaching needs to focus on the use of a specific technological tool, accessibility of mathematical software, and develop teachers’ technological knowledge related to their existing pedagogical content knowledge (TPACK) (Akkoç, 2011; Garet, Porter, Desimone, Birman, & Yoon, 2001; Goos & Bennison, 2008; Mishra & Koehler, 2006).

**Teacher Design Team Professional Development**

One of the professional development models that has been responding to this paradigm change is the teacher design team professional development model. Several technology professional development models have adopted this approach to engage teachers to rethink classroom practices that will bring genuine and long-lasting improvement in students’ learning (Kafyulilo, Fisser, & Voogt, 2011, 2014, 2016; Koehler et al., 2007). The teacher design team approach involves at least two teachers from the same or related disciplines coming together on a regular basis with a common aim to “(re)design and enact (a part of)
their common curriculum” (Handelzalts, 2009, p. 7). Learning occurs through the process of reflection on practice (Rogers, 2001; Schön, 1983). This approach involves teachers being invited to work as a team to implement innovation in the curriculum, and providing them with the creative space to collaboratively think about the challenges associated with their subject area and to further work on new strategies to improve their practices (Kafyulilo et al., 2011; Soebari, 2012). Several studies have illustrative examples of how the teacher design team professional development model has been adopted to improve the professional knowledge and practices of teachers. For the purpose of illustration, the following paragraphs annotate some of the studies to highlight different approaches different authors have used. The closing paragraphs illuminate issues related to teacher design team professional development.

Development and enactment of teachers’ TPACK have been demonstrated to improve through teacher design team professional development. Alayyar, Fisser, and Voogt (2012) engaged 78 Kuwaiti pre-service science teachers in teams of three to four members to design an ICT solution for an authentic problem they faced during in-school training. The pre-service teachers were put into two groups: human support (HS) and blended support (BS). For the human support group, experts in ICT, pedagogy, and content supported the pre-service science teachers through face to face interactions to design ICT-based lessons that addressed pedagogical problems. In a blended support group, the teacher design teams (TDs) worked on their own through an online support environment; the experts did not attend to them unless they (TDs) needed assistance. The intervention lasted 12 weeks, two hours per week. The participants’ pre-test and post-test attitudes towards ICT, ICT skills, and TPACK were evaluated through the answering of questionnaires, interviews, and peer and expert analysis of the artefacts the individual teams designed. Evidence triangulated from this data indicated the participant teachers had increased scores in their attitudes towards ICT, ICT skills, and TPACK, however, the blended support group had superior scores. Alayyar et al. (2012) therefore, concluded that blended conditions for supporting design teams is a more desirable approach of honing teachers’ technological, content, and pedagogical knowledge than human support.

There is a direct correlation between learning by design team and three instructional strategies: problem-driven, solution-driven, and procedural-driven. Lahti and Seitamaa-Hakkarainen (2014) established this linkage by exploring the pedagogical aspects of virtual
designing in a higher education context. They were specifically determined to understand the kind of pedagogical guidance the student teachers could provide through learning by design teams. The study involved 53 industrial design students from four Finish Universities and four tutors (who had 20 years of teaching experience) from Aalto University, Helsinki. The students worked in teams of three or four members to design a plastic product for the Design Forum Finland's exhibition. They were expected to achieve this by engaging in discussion and brainstorming their ideas to improve the artefact they were developing. The tutors’ role in the study was to structure and orchestrate a collaborative design team learning environment for the students using Moodle. Moodle is an open source learning management system that contains tutorials on how to use different kinds of software such as a matrix of different ICT applications with suitable teaching methods, and lesson plans that integrate ICT. Moodle also served as a portal platform where teachers shared their documents, discussed their ideas, and reflected on their thoughts. Moodle provided an environment where the design teams could report the progress of their artefacts. It was found, using the qualitative content analysis of the teacher design teams’ notes recorded in the database of Moodle, that three teaching strategies - problem-driven, solution-driven, and procedural-driven - occurred simultaneously during the design process. The solution-driven strategy was identified to be predominant in the student design teams’ reports.

Supporting teacher design teams with technology seems to be promising in bringing desirable changes in classroom practices. This is because it was observed in Lahti and Seitamaa-Hakkarainen’s (2014) study that as teachers collaboratively worked in a technology environment with experts, it afforded them opportunities to (i) define the context of the artefact they were developing in terms of its usability, materials and techniques required (i.e., problem-driven); (ii) generate, test, and re-evaluate new ideas of tackling the problem (solution driven); and (iii) organise the design process expected to be followed (procedural-driven). This result confirms that engaging teachers in a technology programme oriented around collaborative design of artefacts could bring authentic solutions to the practical problems in education (Dillenbourg, Järvelä, & Fischer, 2009; Littleton, Scanlon, & Sharples, 2012).

Furthermore, a teacher design team approach of professional development is promising in providing opportunities for teachers to create a context of practice and reflection for personal and cultural changes desired to systematically reform the school curriculum and integrate
technology in teaching (Waddoups, Wentworth, & Earle, 2004). Waddoups et al. (2004) described case studies of curriculum design teams that enabled them to have a situated evaluation of the processes of personal and institutional change associated with technology integration initiatives at Brigham Young University, United States of America. The initiatives were a response to the federal grant programme formulated to Prepare Tomorrow’s Teachers to use Technology (PT³). Waddoups et al. (2004) used observations to collect data from the PT³ project managers on how they organised, supported, and trained design team members to collaboratively create technology-enhanced curricula. They also collected data from the design team members through interviews, questionnaires, and analysis of team products. The study participants were faculty members who were at the beginning stage of integrating technology into their instruction. It was found in the study that working in collaborative design teams brought desirable results in educational practices because it afforded the teachers opportunities to (i) share their common goal regarding technology integration, (ii) persevere when the task of designing a product became difficult, and (ii) contextualise the technology to meet their own needs (Waddoups et al., 2004).

Kafyulilo et al. (2016) investigated the impact of teacher design teams as a professional development arrangement for developing technology, knowledge, and skills among 12 Tanzanian in-service science teachers aged between 26 and 42 years. The teachers were grouped (three members per group) according to their subject area (chemistry, biology, and physics) and each team was tasked to collaboratively design and teach a technology-enhanced lesson. The teacher design team professional development model employed was characterised by an introductory workshop, lesson preparation in design teams, lesson implementation in the classroom, and reflection on the designed and implemented lesson. Five different instruments: a TPACK survey, an interview guide, focus group discussions, a researchers’ logbook, and an observation checklist were used to triangulate the data. The pre- and post-test measurements indicated an increase in teachers’ technology integration knowledge and skills. It was found also that the teachers were able to share knowledge, skills, and experience through working in collaborative design teams.

It has also been identified that teacher design team does not only stimulate and support teachers to learn, but makes them active consumers of technological tools, initiates them to be designers of technology resources, and makes them claim the local ownership of technology resources (Agyei & Voogt, 2012). Agyei and Voogt reported on four pre-service
teachers who worked in two design teams to develop and teach technology-based lessons for the first time in their teacher preparation. The data the researchers collected through interviews, questionnaires, and the researchers’ logbook indicated that enacting a technology integration programme in a teacher design team setting has the potential of enhancing teachers’ technological, pedagogical, and content knowledge (TPACK).

From the above discussion, there is agreement in the literature that using teacher design teams as professional development provides an environment for teachers to creatively share their knowledge, skills, and experience, build their capacity, develop technology-based lessons, and make changes in the curriculum with the aim of solving classroom problems. Teachers learning through practice was apparent in the above studies. However, all these potentials benefits may not be realised because of the challenges associated with the use of teacher design team professional development. For instance, Waddoups et al. (2004) identified that engaging teachers in design teams provided them the opportunity to progress, but technical and infrastructural problems could plague the implementation efforts of the teachers. Waddoups et al. further indicated that many of the design teams are most often temporary and the teams tend to collapse after the professional development arrangement, thus making it difficult to coordinate and sustain the activities of the design teams. Lahti and Seitamaa-Hakkaraainen (2014) also pointed out that teacher design teams indeed permit teachers the opportunity to employ a variety of interactions and methods in technology integration, but it is very difficult to implement sophisticated pedagogical knowledge in technology-enhanced learning. The teachers require comprehensive knowledge and skills to balance their pedagogical ideas with the affordances and constraints associated with the technology. Also, teachers’ prior knowledge and beliefs, as well as time requirements, could be an impediment to transferring the experiences gained through collaborative design teams (Agyei & Voogt, 2012).

The literature reviewed in this section indicates that most of the studies that have been conducted on technology integration, through the use of teacher design teams, are centred on pre-service teachers and that there are very few studies that have been conducted in Africa, particularly in the Sub-Saharan African countries like Ghana. The few studies conducted in Sub-Saharan African include those by Agyei and Voogt (2012) and Kafyulilo et al. (2016), which used Ghanaian pre-service mathematics teachers and Tanzanians in-service science teachers respectively. Moreover, the teacher design team approach has its
own challenges; there is the need to have agreed methodological guidelines for effective implementation. Although technology integration is context sensitive, having agreed guidelines could help future users to adapt it to suit their cultural needs.

Using a technological tool to mediate professional development for technology integration seems ideal. All the researchers in the literature reviewed in this section used a technological tool(s) to facilitate the professional development programme they organised. For example, Waddoups et al. (2004) used iMovie and PowerPoint to support teachers (faculty members) improve their assessment practices in the classroom. The iMovie training the participants received helped them understand how they could digitise and manipulate video. The skills the faculty members gained assisted them in coaching their student teachers to simplify their portfolio development process. The PowerPoint training also assisted the faculty members help their students on how to organise videos and artefacts into portfolios. Lahti and Seitamaa-Hakkarainen (2014) employed Moodle as a technological tool to mediate teacher design team professional development.

In the Sub-Saharan African countries, some of the technological tools that have been used in teacher design team professional development include Excel spreadsheets by Agyei and Voogt (2012) in Ghana, and PowerPoint and recorded video by Kafyulilo et al. (2016) in Tanzania. In Agyei and Voogt’s study, the pre-service teachers collaboratively used Excel spreadsheets to design activity-based lessons in mathematics. The teachers in Kafyulilo et al.’s study developed animated science lessons through the use of PowerPoint and recorded videos.

The tools of iMovie, Excel spreadsheet, PowerPoint, Moodle, and others are very powerful technological tools for mediating professional development in the technology integration campaign, but their usage is quite context-sensitive. For instance, in most of the Sub-Saharan African countries, particularly Ghana where mathematics classrooms are not fully installed with digital technologies such as computers, a sound system, interactive whiteboard, and the internet, it will be very difficult to use a digital device such as Moodle to mediate a professional development programme. The teachers would not be able to use it in their actual classroom after the professional development because the internet is not easily accessed in the mathematics classroom in Ghana.
Potential of GeoGebra

Figure 3.3 shows the interface of the GeoGebra window. It is a technological tool which has gained popularity in mathematics education because it is easily accessible, user-friendly, and cost-effective for technology integration professional development. It covers a lot of mathematics contents, such as algebra, calculus, statistics, vectors, and geometry. It has a distinctive online community platform which supports both teachers and students to access existing instructional materials. It also has spreadsheet features and GeoGebra apps can also be installed on Android smartphones. Thus, teachers and students are able to use it at school or home if computers are not available. These features make GeoGebra usable for mathematics teaching from primary school through university (Hohenwarter, Jarvis, & Lavicza, 2009).

![Figure 3.3 The interface of the GeoGebra window](image)

Many studies have demonstrated GeoGebra’s efficacy in teacher professional development for technology integration (Andresen & Misfeldt, 2010; Bu et al., 2013; Hudson, 2012; Prodromou, Lavicza, & Koren, 2015). Andresen and Misfeldt (2010) gave an account of how GeoGebra was used to get teachers in tune with the competence required of them to use technology in secondary school mathematics. Among the activities the teachers engaged in was how to use GeoGebra to design mathematics lessons based on the teachers’ knowledge of the theory of mathematics education, knowledge about concrete activities and teaching materials, and knowledge of didactic instruction and pedagogy. It was observed, for instance, that the tessellation (repeated patterns that embody the concept of the polygon) material in GeoGebra initiated the teachers to ask more open-ended questions about how they could use it to support students’ learning. At the end of the iteration of the professional development programme the teachers became enthused about helping students to use ICT facilities to create mathematics artefacts and share them with their colleagues.
The teachers in Prodromou et al.’s. (2015) study used GeoGebra-based activities to initiate student-led class discussion in lessons on constant, absolute value, linear, logarithmic, exponential, trigonometric, and quadratic functions. The authors reported that the features in GeoGebra helped the teachers to perform facilitatory roles in the lesson. For example, the students first worked in pairs to come out with their solutions before consulting with the teacher, and then validated their procedures and answers using a GeoGebra worksheet, which eventually led to whole class discussion. The CAS (computer algebra systems) feature in GeoGebra provided not only the algebraic treatment and solution, but it also expanded the tasks, which could have been difficult through pencil and paper approach (Alejandra, Fernando, & Leonardo, 2015). Though the teachers in the Prodromou et al. (2015) study admitted that using GeoGebra required time and experience, they indicated that it made them conduct lessons that engaged students in dialogue to enhance their (students) knowledge construction. Similarly, Bu et al. (2013) found that the use of GeoGebra in a professional development programme challenged teachers’ view about the nature of mathematics and student-teacher interactions and further enriched their mathematical knowledge and pedagogical choices.

Other studies have also demonstrated the multiple functionality of GeoGebra in the mathematics classroom. In the work of Bulut and Bulut (2011), pre-service teachers were able to use GeoGebra to (i) create interactive web pages for students’ mathematics learning, (ii) enact discovery-based learning where real-life examples were used to develop mathematical concepts and thinking, and (iii) create multiple representations to aid students’ construction of mathematical ideas. Lavicza, Hohenwartner, Jones, Lu, and Dawes (2010) similarly reported that teachers used GeoGebra in three pedagogical approaches. First, they used GeoGebra as a demonstration tool to engage students in discussion. The demonstration helped the teachers gain experience in using GeoGebra in the classroom. Second, as the teachers became comfortable with GeoGebra, they then allowed the pupils to interact with the teacher-created files by experimenting and playing with the objects. Third, the teachers asked the pupils to create their own files to help them develop conjectures, theories, and proofs. Meanwhile, Kul (2012) explored the changes in beliefs of six Turkish mathematics teachers after professional development mediated by GeoGebra. The professional development was orchestrated with the aim of encouraging teachers to understand the theories and practices of mathematics teaching and learning through interaction with GeoGebra-based mathematical activities rooted in a constructivist perspective. The GeoGebra-based mathematical activities were designed not only to engage teachers in
critical thinking, but to enhance small group exploration, discussion, and negotiation. The teachers’ pre- and post-test responses of the questionnaire indicated a positive change in their beliefs about mathematics concepts as well as mathematics teaching and learning (Kul, 2012).

The above studies suggest that professional development which is activity-driven (small group exploration, discussion, negotiation, and design of instructional materials by team members), technology-based (GeoGebra), and pedagogy-oriented (interactive instruction) could provide rich environment for teachers to develop in-depth understanding of theories and practices needed for effective technology integration in mathematics teaching. For instance, it was exhibited in Kul’s (2012) study that after the teachers had gone through the GeoGebra professional development training, they were able to (i) engage in mathematical tasks that helped them develop a better understanding of students’ role in a technology learning environment, and (ii) maintain their beliefs in favour of a student-centred instructional approach throughout the professional development programme.

Summary
This chapter contained four main sections: technology in mathematics education, theoretical frameworks for effective technology integration, professional development, and the potential of GeoGebra. The term ‘technology’ was operationalised to encompass any digital media that embody curricular and educational content, and that can be used to improve the quality of teaching and learning in the mathematics classroom. The literature reviewed demonstrated that technology could be used to widen the learning space where students could use it as a tool for doing mathematics (reduce routine work), practicing mathematics skills (provide prompt feedback), and developing mathematics concepts (build conceptual understanding). Not relegating the first two functions, the main concern has been how technology can be used to transform classroom practices towards the development of deep conceptual mathematical ideas and not only as a tool to ensure efficiency. This agenda involves interplay of multiple factors in which the teachers’ role is critical.

It was identified that the beliefs that teachers hold influence their dispositions, perceptions, and judgments, which predict their behaviour in the classroom. Definitional issues and
differing understandings of beliefs and its related constructs, such as knowledge and attitudes, muddy research on beliefs. Considering the focus of this thesis, teachers’ cognitive and affective beliefs were seen to lend themselves well with the pedagogical practices in mathematics education, particularly with the use of technology. Teachers’ cognitive and affective beliefs towards technology are consistent with philosophical perspectives of knowledge construction which involves negotiation of human interaction with the environment.

Creation of a mathematical setting, useful mathematical tasks, mathematical exploration, mathematical discussion, mathematical connection, and assessment of learning were identified central practices of effective mathematics pedagogy. However, it was argued that these practices are too generic and underspecified to inform teaching. Theorising core practices of each of these components within a specific domain (for example, using technology) would be useful to expand the literature on effective mathematics pedagogy.

The literature indicated that teachers were more likely to use technology in the classroom if they were provided with effective professional development, which is characterised by (i) job-related issues that represents a collective concern, (ii) participants being involved in the planning activities of the professional development, (iii) problem solving through a collaborative approach, and (iv) provision of support after the initial training. This thesis is responding to this call in the context of Ghana, where teachers are expected to use technology to enhance students’ mathematics learning. The TPACK framework provides a useful model of the development of teachers’ knowledge and belief towards the use of technology in teaching and learning. The literature supports the notion that professional development, where teachers work in a design team to develop GeoGebra-based mathematics lessons, may provide the crucial support necessary for teachers to enact effective mathematics pedagogy.
CHAPTER FOUR: METHODOLOGY AND RESEARCH DESIGN

For the purpose of reiteration, Chapter Three explored the literature relating to the use of technology in mathematics teaching. Two critical issues were identified. First, effective technology integration in mathematics teaching is connected to teachers’ beliefs, attitudes, and knowledge regarding the use of technology. Second, the nature of professional development teachers receive is a crucial determinant of how teachers incorporate technology in their pedagogical decisions. The elements of the new paradigm in professional development reflect the philosophical assumption of the interpretive research paradigm, where both the researcher and the research participants are interdependent in solving classroom related problems (Carson, Gilmore, Perry, & Gronhaug, 2001). Thus, this thesis set out to explore how professional development influenced teachers’ pedagogy and technology dispositions in mathematics teaching and learning. The following research questions were formulated to guide the study.

1. What were the teachers’ initial dispositions toward the use of technology in mathematics?
2. How did teachers enact effective mathematics pedagogy when using GeoGebra following their engagement in professional development?
3. How did the professional development impact on the teachers’ dispositions towards technology integration in mathematics?

To address these questions, this chapter presents the research paradigm in which the study is situated. The ontological, epistemological, and methodological stance of interpretivism will be discussed in relation to the research design. Following this the structure and content of the professional development adapted for the study will be reviewed, along with the sampling procedure, ethical considerations, and methods for data collection. Finally, the procedure for data analysis and trustworthiness of the research findings are presented.

Research Paradigm

According to Kuhn (1992), a paradigm refers to a set of values, laws, beliefs, assumptions, and practices shared by members of a particular community of study. Vedeler (2000) described a paradigm as a theoretical framework which embodies the philosophical
considerations underlying the choice of all decisions (for example, the sampling procedure, methods for data collection, analysis, interpretation, and presentation) in the research. Thus, the term is used to connote acceptable scientific practices: law, theory, and practices used in a particular field of study to provide an understanding of the world. It represents a framework that shapes or informs the way a body of knowledge is constructed (Braun & Clarke, 2013; Guba & Lincoln, 1994; Olsen, Lodwick, & Dunlap, 1992). A paradigm is a basic belief system based on ontological (beliefs about the nature of reality), epistemological (for instance, the beliefs about the relationship between knowledge and the researcher during the discovery), and methodological (for instance, the way of discovering the knowledge) assumptions (Carson et al., 2001). There are two main ontological and epistemological ideologies: positivism and interpretivism.

Positivist ontology holds the belief that reality is singular and objective. That is, positivist ontologists believe that the world is external and that there is a single objective reality. Irrespective of the researcher’s perspective or belief, to have a positivist ontology is to have the view that the existence of knowledge is absolute and, as such, knowledge should be explored through a controlled and structural approach where a clear research topic is identified, an appropriate hypothesis is constructed, suitable methodology is adopted, variables are manipulated, and appropriate statistical techniques are used for data analysis and interpretation (Carson, et al., 2001; Guba & Lincoln, 1994; Hudson & Ozanne, 1988).

Interpretivist ontology, on the other hand, holds that beliefs about reality are multiple, holistic, and can be constructed (Guba & Lincoln, 1994). Interpretive ontology posits that knowledge is “social artefacts, and are therefore seen as social, cultural, moral, ideological and political” (Braun & Clarke, 2013, p. 30). Thus, it can best be constructed through subjective means, where the researcher becomes inseparable from the informants, follows a nonlinear approach to data collection, and interprets the data to meet the practical needs of the study context (Carson, et al., 2001). Interpretivists prefer using non-numerical data from interviews, observations, document analysis, and discussion to understand and interpret the meanings in human behaviour, rather than establishing causal relationships as the positivists do (Hudson & Ozanne, 1988).

From the description of the two ideologies it is evident that positivist ontology is quantitatively oriented, which includes emphasis on the use of numbers, the generalisation
of the relationship between variables, values detachment, and impartiality. An interpretivist ontology is qualitatively oriented, and importance is placed on the rich description of data, gaining understanding of the local context, and subjectivity and reflexivity of data interpretation. Debate about which one is superior is unwarranted since each one has its own strengths and weaknesses and each can uniquely be used to address a particular situation (Braun & Clarke, 2013). Rather, the attention of the debate should be focussing on providing detailed illustrations of how each fits an issue under investigation, or, where necessary, how both philosophical assumptions can be merged to address an issue.

Situating the Study in a Qualitative Research Methodology

Effective technology integration is complex because of the interplay of factors such as teachers’ preparedness, the culture of the school, administrative set-up, and infrastructural availability. These factors are mutable, depending on time and the social context in which a particular technology is applied, thus making the positivists’ view of unchanging reality impracticable, especially where teachers’ dispositions have been considered pivotal in effective technology integration. It is also indicated in the literature that, to ensure effective technology integration, the professional development organised should provide opportunities for teachers to share their knowledge, skills, and experience; build their capacity; develop technology-based lessons; and, make changes in the curriculum with the aim of solving classroom problems (Koellner et al., 2011; Vescio, Ross, & Adams, 2008; Whitcomb et al., 2009). These facets of the professional development are incongruent with a positivist perspective where human behaviour is considered passive, controlled, and determined by external environment (Braun & Clarke, 2013).

Qualitative research methodology is used where the focus is on an in-depth inductive exploration of the topic; the nature of reality is relative, and the researcher is the main instrument in the investigation, leading to a subjectivist relationship between the knower and known (Braun & Clarke, 2013). The researcher of the current study believes that employing an inductive research approach to collect and analyse data concurrently is crucial in exploring how professional development resonates with teachers’ subject-matter development, beliefs, and attitudes, and classroom practices to reflect their students’ mathematics thinking and learning. A positivist ontological stance, where the researcher detaches himself from the participating teachers in the professional development, will not be
appropriate because the researcher may have tacit knowledge about the informant which could be useful in the exploration. As the researcher becomes inseparable from the participating teachers, he will be able to provide expert support for the teachers to hone their pedagogical skills of using technology in mathematics teaching. The issue regarding the power relationship between the researcher and research participant will be discussed under a different section in this chapter (trustworthiness of the research findings).

Thus, the ontological standpoint of this thesis is that reality is not singular and objective, as the positivists hold, but it can be constructed and subjectively deduced within the context of a specific society. Specifically, this research study is rooted in the philosophical assumption of the interpretive paradigm, the methodological principles of a case study, and triangulation data collection strategies. In the next section, the case study as a research design for this study is discussed.

**Research Design**

The case study approach is one of the main qualitative research methodologies (Goldin, 2008; Stake, 2000) and it enjoys “a natural advantage in research of an exploratory nature” (Gerring, 2011, p. 10). Yin (1984) defined a case study as an empirical enquiry that investigates a contemporary phenomenon within its real-life, especially when the boundaries between phenomenon and context are not clearly evident. Using a case study approach provides opportunity for examining a “specific phenomenon such as a programme, an event, a person, a process, an institution, or a social group” (Merriam, 1998, p. 7). It is used in research that focusses on generating hypotheses or conjecturing theories based on the construction of knowledge through individual perspectives about the world. The case study approach is considered appropriate for the current study in that the researcher is interested in exploring how the professional development resonates with teachers’ beliefs, attitudes, and knowledge related to pedagogical use of technology in the mathematics classroom.

Yin (2003) enumerated five key components that dictate the use of case study research design: (i) a study’s questions, (ii) its propositions, (iii) its unit(s) of analysis, (iv) the logic linking the data to the propositions, and (v) the criteria for interpreting the findings. These components and how they are applied in this study are elucidated in the following paragraphs.
A case study is a preferred research design when the interest is to provide answers to how, why, what, and where questions. Research questions preceded by how, why, what, and where not only provide important clues regarding the relevant strategy to be employed, but also initiate the process of gaining an in-depth understanding of the phenomenon under study. The research questions formulated for the current study are preceded by how and what to enable the researcher to gain an in-depth understanding of how the teachers’ interaction with GeoGebra in a professional development programme resonated with their beliefs, attitudes, knowledge, and skills related to the use of technology in the mathematics classroom (Yin, 2003, 2009).

The proposition outlines the rational/purpose and direction of the study. A case study involves a report about a person, group, or situation under investigation. Therefore, it is hard to arrive at a valid conclusion if (i) what is to be studied, (ii) who is to be involved in a study, (iii) where and when the study will take place, and (iv) which strategy will be used are not clearly outlined. The study proposition not only serves as guide, but also draws the researcher’s attention to what needs to be examined and the scope of the exploration. In this current study, due diligence was made to ensure that the rationale, content, participants (teachers), the technological tool (GeoGebra), and the time frame (12 months) for the professional development were appropriate and could yield results that would answer the research questions. For example, this research was conducted upon the critical reading and revision of the research proposal by the researcher, the two supervisors of the research, and the Dean of the University of Otago College of Education. Also, final approval before the research was carried out was sought through the Human Ethics Committee of the University of Otago (see Appendix A) upon the formal consent from the authorities of the school where the study took place (see Appendix B).

The unit of analysis is another component that typifies a case study research design. It is the major entity that is analysed in a study (Trochim, 2000). Brewer and Hunter (1989) identified six types of units that are dominantly used in educational research: persons; attributes of persons; activities and interactions; residues and artefacts of behaviour; settings, incidents and occasions; and, collectives. Any of these units could lead to case study research. In this study, the case is the individual teachers who participated in the study. The unit of analysis is the teachers’ dispositions (beliefs, attitudes, and knowledge) related to the use of technology in mathematics teaching. To gain a holistic perspective of how professional
development influenced teachers’ pedagogical use of technology, each teacher’s beliefs, attitudes, and knowledge were explored separately and at the same time treated as a collective unit of analysis as the teachers came together to interact with GeoGebra in their own teaching environment (Levin & Wadmany, 2005). Yin (2003) indicated that in a classic case study, an individual can be a case in which relevant information about him or her can be collected, with several of such individuals constituting a multiple-case study. For example, Ottenbreit-Leftwich, Glazewski, Newby, and Ertmer (2010) employed within-case and cross-case data analyses to gain insights about the relationship between teachers’ beliefs and their use of technology. They concluded in their study that personal dispositions of individual teachers are distinctive and influence their instructional practices (Ottenbreit-Leftwich et al., 2010).

The logic linking the data to the propositions is the fourth component of a case study. Drawing a variety of data from a ‘case’ is an indication of reaching a solid data analysis that reflects the subject being explored. Following the pioneering work of Donald T. Campbell (1975), Yin (2003) identified ‘pattern matching’, where “several pieces of information from the same case may be related to some theoretical proposition” (p. 26). Thus, in case study research, the use of multiple sources of data such as interviews, observation, focus group discussion, document analysis, and designing of artefacts allows triangulation of the findings (Gomm, Hammersley, & Foster, 2000). In this current study, as the teachers journeyed through the professional development programme, their beliefs, attitudes, and knowledge related to pedagogical use of technology in mathematics teaching was explored through the triangulation of the data: observation of teachers’ GeoGebra-based lessons, teachers’ self-report questionnaire, lesson plans developed by teachers, and focus group discussion.

The fifth component of a case study research is the criteria for interpreting the findings. There are no cookbook strategies for interpreting the findings of case study research. However, a good case study should identify and address rival propositions. Yin (2003) illustrated that using “different patterns are sufficiently contrasting that the findings can be interpreted in terms of comparing at least two rival propositions” (p. 27). Thus, it is crucial in case study research to observe and explain whether theories, intervention, and implementation processes, other than the one proposed in the initial stage, account for the outcome of the results. The researcher of the current study holds the notion of interpretive ontology: that knowledge acquired in a discipline can be socially constructed through
multiple perspectives of a given phenomenon. Thus, the researcher used (a) the individual teachers’ perspectives relating to pedagogical use of technology in mathematics, and (b) multiple forms of data (lesson observation, self-report, lesson plan, and recorded audio interviews and focus group discussion), collected over a period of 12 months, to provide a rich account of technology integration within the context of Ghana’s senior high school mathematics curriculum (Hamilton & Corbett-Whittier, 2013).

Again, the essence of using multiple perspectives of teachers’ use of technology and multiple sources of data over that extended period is to provide adequate evidence that assesses whether professional development, situated in the TPACK framework, supported teachers to effectively integrate technology in mathematics teaching. Pollard (1987), who is among the forerunners of case study design, used different data sources, such as classroom field notes, classroom photos, video recordings in the classroom, teachers’ interviews, teachers’ documents, and school event field notes, to produce a rich account of a student’s learning and social world. Needleman and Needleman (1996) confirmed that

> the object of qualitative reconnaissance is not to construct a statistical profile of the setting based on secondary data source, but rather to gain direct insight into the setting’s social dynamics through observing, reading documents and talking with those on the scene (p. 330).

It has been elucidated in this section as to how and why the current study was situated in a case study research design. Despite the use of a case study design being robust in providing (i) in-depth analysis of a phenomenon, (ii) credible results through the use of multiple techniques in data collection, and (iii) strong evidence that underpins causal relationships between specific interventions and specific outcomes, it has also been criticised for its difficulty to generalise to other situations (Merriam, 1998; Yin, 2009). Van Lier (2005), Miles and Huberman (1994), and Mills, Durepos, and Wiebe (2010) suggested that the findings of a case study can be applied in different contexts if the researcher (i) presents sufficient description of each case study and (ii) applies a multiple-case methodology that enables evidence to be compared. This study follows Lincoln and Guba’s (1985) model for establishing the trustworthiness of the findings obtained.
The Structure of the Professional Development

From the literature reviewed, the professional development programme constructed and used in this study involved three phases: (i) collection of preliminary data, (ii) workshop training, and (iii) enactment of GeoGebra-based mathematics lessons in actual classrooms. The diagram in Figure 4.1 shows the activities carried out at each phase.

![Diagram of professional development phases](image)

**Figure 4.1 Activities of the professional development**

For clarity of presentation, the activities have been arranged linearly, but in practice they were not. In particular, the activities in phase two and three were more cyclic and iterative than displayed here. That is, each activity was used to inform the subsequent one. However,
in situations where teachers had difficulties, remedial workshops were offered to address the preceding activity (cyclic). The teachers then used the supports offered by the expert or their peers during the remedial workshops to improve the quality of the artefacts (lessons) they were developing (iterative). Also, the after-school workshop in phase three involved one-on-one interaction with the teachers after they had enacted GeoGebra-based lessons in the actual classroom. The following paragraphs provide detailed description of each phase of the professional development.

The first phase of the professional development programme was carried out in May 2017 and lasted four weeks. It involved the recruitment of participants and collection of preliminary data from the teachers. Interviews (see Appendix C) and a self-report questionnaire (see Appendix D) were used to collect the participating teachers’ demographic information and data about their experiences of their current use of technology in the mathematics classroom. The purpose was to gain an initial understanding of the participating teachers’ beliefs, attitudes, and knowledge about the use of technology in mathematics teaching. This initial information was used to guide the professional development programme.

For example, the teachers’ preparedness to use technology was demonstrated in the number of suggestions and support they offered to get the professional development organised. During the preliminary interaction with the participants, it became obvious that within the constraints of the limited resources in the school, the teachers could only conduct technology-based mathematics instruction with a projector, a laptop computer, and a modem for internet connectivity. For example, two of the participants willingly offered their personal laptop computers to augment the two laptop computers, a projector, and a Vodafone router (for internet connectivity) that the researcher provided to support the programme. The provision by the researcher was in support for the subsequent use of technology by the teachers when the programme was over. Also, the participant teachers proposed a different room other than the school’s ICT laboratory for the professional development. The participants initiated this move to avoid frequent interruption from other users of the school’s ICT laboratory. They assisted in putting the room they proposed in order.

The teachers indicated they preferred to work with others who had the same or similar level of technology skills. They did their own pairing and groupings to suit whom they wanted to
work with. Their groupings were also informed by their instructional time and other co-
curricular activities they were responsible for in the school. They took group ability, time of
their instructional period, and co-curricular activities into consideration because they
proposed one- to two- hours weekly workshop training during the school session.

The second phase was carried out in June 2017 through to September 2017. The teachers
worked in design teams comprising two to three members. Each group chose a different day
within the week for the weekly workshop which spanned June to July 2017. Apart from the
regular schedule, the individuals visited the training centre (the room we created for the
workshop) during their free time to practice the GeoGebra tool on their own. The facilitator
was always in the training centre to provide assistance to the teachers when needed.

Ten lessons were organised for the teachers to introduce them to the basic tools in the
GeoGebra window (Table 4.1). The activities they were engaged in were central to the
mathematics curriculum they were teaching in the school. A tutorial manual for the activities
was given to each participant to enable them to practice at their own pace before the weekly
meetings. Before the teachers were introduced to GeoGebra, the researcher presented two
GeoGebra-based lessons as demonstration lessons. They were then engaged in discussion
after the demonstration lesson to pedagogically reflect on how the lesson was conducted and
the role GeoGebra potentially played in building students’ understanding of the mathematics
concepts.

After they had gone through the ten lessons, a five-day intensive training workshop was
organised for the teachers during the school holidays in August 2017. Each session lasted
for a maximum of six hours (see Appendix E for the detailed timetable). The teachers were
introduced to the principles of design team approach for developing instructional materials,
the concept of the TPACK framework, and how to use GeoGebra. The researcher facilitated
the professional development programme. Each teacher design team was tasked to identify
a topic in the high school mathematics curriculum that they could use GeoGebra to teach.
They designed and taught a GeoGebra-based mathematics lesson to their peers (teaching
rehearsals). Fisser, Voogt, Tondeur, and van Braak (2015) indicated that professional
development, where teachers are provided with opportunities to design and enact a
technology-based lesson in their own working environment, helps them to translate their
espoused TPACK into practical use.
<table>
<thead>
<tr>
<th>Week</th>
<th>Activity</th>
<th>Sub-activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>Introduction to GeoGebra</td>
<td>Familiarise the GeoGebra basic interface: Input bar, Algebra View, and Graphic View.</td>
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<td></td>
<td>interface, menu and constructing tools</td>
<td>Menu</td>
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<tr>
<td>Week 2</td>
<td>Polygons and Angles</td>
<td>Construct a triangle and measure the sum of the interior angles</td>
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<td></td>
<td></td>
<td>Construction of a regular polygon</td>
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<td>Week 3</td>
<td>Perpendicular and Parallel lines</td>
<td>Construct the midpoint of a line segment</td>
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<td></td>
<td>Construct a line perpendicular to a given line and through a given point</td>
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<tr>
<td></td>
<td></td>
<td>Construct a line parallel to a given line and through a given point</td>
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<td></td>
<td></td>
<td>Construct the perpendicular bisector of a line segment</td>
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<td></td>
<td></td>
<td>Construct the angle bisector of an angle:</td>
</tr>
<tr>
<td>Week 4</td>
<td>Drawing graphs</td>
<td>Create and modify algebraic coordinates and equations by using the Input Bar at the bottom of the GeoGebra window.</td>
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<td></td>
<td></td>
<td>Construction of trigonometric graphs (in radian/degree measure)</td>
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<td>Week 5</td>
<td>Use sliders to transform graphs</td>
<td>Creating sliders</td>
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<td></td>
<td></td>
<td>Using sliders in equations</td>
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<tr>
<td>Week 6</td>
<td>Transformation Geometry</td>
<td>Constructing a reflection in the x-axis</td>
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<td></td>
<td></td>
<td>Constructing a reflection of a point in the line y = x</td>
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<tr>
<td></td>
<td></td>
<td>Rotation of a point</td>
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<td></td>
<td></td>
<td>Translation of a point</td>
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<td></td>
<td></td>
<td>Enlargement of a point</td>
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<tr>
<td>Week 7</td>
<td>Kites and parallelograms</td>
<td>Construction of a parallelogram</td>
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<td></td>
<td>Statistics</td>
<td>Construction of a kite</td>
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<td>Week 8</td>
<td></td>
<td>Finding the mean, median and mode</td>
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<td></td>
<td></td>
<td>Drawing a histogram</td>
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<td></td>
<td>Drawing a box and whisker diagram</td>
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<td></td>
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<td>Finding the quartiles, standard deviation and variance</td>
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<td></td>
<td></td>
<td>Drawing Scatter plots and lines of best fit</td>
</tr>
<tr>
<td>Week 9</td>
<td>Calculus</td>
<td>Construct a tangent at a point to any curve of a function, f(x)</td>
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<td></td>
<td></td>
<td>Differentiation and construction of the curve of f(x)</td>
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<td></td>
<td>Approximating the total area underneath a curve on a graph (integral) using the Riemann sum method</td>
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<td></td>
<td></td>
<td>Calculating the area under curve: Finding definite integrals</td>
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<tr>
<td></td>
<td></td>
<td>Calculate the area between two curves</td>
</tr>
<tr>
<td>Week 10</td>
<td>Matrices</td>
<td>Matric operations: addition, multiplication, inverse.</td>
</tr>
</tbody>
</table>
The teachers were engaged in discussion after each episode of peer teaching. The discussion was meant to engage teachers in critical reflection so that best knowledge, classroom practices, and teaching experiences could be shared among themselves. Vescio et al. (2008) suggested that professional development programmes that provided teachers the opportunity to engage in discussion deepened their professional knowledge, which in turn enhanced students to learn meaningfully. Koellner et al. (2011) added that engaging teachers in discussion during professional development affords them the chance to contribute towards the programme and “take ownership of the issues raised in the professional development” (p. 129).

The third phase started in October 2017 and ran through to March 2018. The teachers were encouraged to use GeoGebra in their actual classrooms. The researcher observed the teachers’ classroom practices as they enacted mathematics lessons in the GeoGebra learning environment. Garet et al. (2001) shared that providing teachers with the opportunity to (i) plan and enact classroom lessons, (ii) observe expert teachers, and (iii) be observed in teaching, indicated salient features of ensuring effective professional development. Also, Anthony, Hunter, and Hunter (2016) noted that professional development, where the researcher/leader has the opportunity to observe teachers in the classroom and provide support where necessary, facilitates both teachers and researcher/leader “to become stakeholders in each other’s practices” (p. 119).

Phase three comprised after-school workshops where the researcher met the participating teachers fortnightly at the school to discuss how the GeoGebra professional development resonated with their beliefs and attitudes, subject-matter development, classroom practices, and their students’ mathematical thinking and learning. The feedback obtained from the teachers was used to provide further professional development to the participating teachers. Trigueros and Lazano (2012) remarked that providing teachers the opportunity to work in groups inside and outside the schools is an indication of effective professional development. Koellner et al. (2011) articulated that, in order to provide result-yielding professional development, the designers should incorporate activities that: (i) foster active teacher participation in the learning process, (ii) use teachers’ own classroom practices to provide basis for their learning, and (iii) provide a supportive professional community to enhance teachers’ learning. They also emphasised that providing professional development that centres on improving teachers’ subject-matter knowledge, instructional practices, and
students’ thinking and learning are key determinants of bringing about curriculum innovation. In this study, the activities of lesson enactment, observation, expert support, and discussion were included in the professional development to engage both the researcher and the participants in active learning, communication, and collaboration to bring the desired changes in classroom instructions (Meyers, Molefe, Brandt, Zhu, & Dhillon, 2016). Whitcomb et al. (2009) and Koellner et al. (2011) also identified that coaching, demonstration of lessons, discussion, self-reflection, and building on teachers’ prior knowledge are crucial activities in effective professional development.

Finally, the same self-report questionnaire as used previously (see Appendix D), interviews (see Appendix F), lesson plans and lesson episodes were used to collect data about how the professional development programme impacted on teachers’ beliefs, attitudes, and knowledge related to the use of technology in mathematics teaching. A researcher’s logbook was used to record day-to-day activities, and served as a source of reflection and data to augment the study.

**How the TPACK Framework was Applied in this Research**

In this study the TPACK framework, detailed in Chapter Three, was used in two different ways. First, it provided the basis of the content and structure of the professional development programme. The teachers’ knowledge and skills required to use GeoGebra to enact effective mathematics pedagogy were operationalised as their TPACK. The TPACK components used in this research are defined as follows:

a)  Pedagogical knowledge (PK): The knowledge and skills needed to enact the core practices of effective mathematics pedagogy discussed in Chapter Three.

b) Content knowledge (CK): The subject matter knowledge of mathematics.

c)  Technological knowledge (TK): The knowledge and skills needed to use GeoGebra, its affordances and constraints;

 d)  Pedagogical content knowledge (PCK): The knowledge and skills needed to teach specific mathematics subject matter based on the principles of effective mathematics pedagogy.

 e)  Technological pedagogical knowledge (TPK): The knowledge and skills needed to use GeoGebra to enact the core practices of effective mathematics pedagogy.
f) Technological pedagogical content knowledge (TPACK): The knowledge and skills needed to use GeoGebra to teach specific mathematics subject matter.

g) Context of the study: The context of the study involved the constraints and affordances of integrating technology in mathematics teaching in the school where the study took place. The study setting will be discussed later to reflect the current professional development programme.

Second, the TPACK instruments were used as a lens to explore how the teachers developed their knowledge and skills of integrating technology into their mathematics teaching as they went through the GeoGebra professional development programme, paying attention specifically to their technological knowledge and subject-matter development, classroom practices, and ability to adapt technology to suit existing mathematics curriculum and students’ mathematics thinking and learning.

Ethical Considerations

The adage of ‘first, do no harm’ has become the guiding principle in all types of research. This guiding principle embodies two elements: respect and responsibility. Researchers are expected to exhibit these ethical practices at all stages of the case study from the planning stage through data collection, analysis, and presentation (Hamilton & Corbett-Whittier, 2013). Researchers owe their informants maximum respect and they are also responsible for the informants’ well-being during the entire period of the study. A number of steps were taken in this study to ensure that a high standard of care regarding research were followed to the letter.

First, ethical consent was sought from the University of Otago’s Human Ethics Committee (see Appendix A) based on the prior formal approval by the authorities of the school where the study was conducted. The insightful suggestions and comments made by the two supervisors, Dean of the College of Education, and the University’s Ethics Committee helped shape the content and protocol of the entire study.

Second, two key gatekeepers - The Headmistress and the Head of Mathematics Department of the school - were involved in reaching the participants of the study. Initially, the purpose
of the study was communicated to the two parties for their verbal consent. The Information Sheet (see Appendix G), detailing the content of the professional development programme, was sent to the school through the Administrative Head’s (Assistant Headmaster of the school) email. After the school had realised the relevance of the study, they gave the researcher a formal consent letter to permit him to carry out the study in the school. It important to acknowledge potential power differentials from approaching the teachers through the headmistress and head of department. Although these personalities might have encouraged the teachers to participate in the professional development, the teachers were matured and autonomous in the final decision they took. Also, the teachers were given, in advance (a three-month notice), the Information Sheet (see Appendix G) and Consent Form (see Appendix H) to enable them to make an informed decision about participating in the study. In addition, verbal consent was also sought from the individual teachers willing to participate in the professional development programme. At the end, 11 out of the 13 teachers in the department participated in the study.

The Research Setting
The school was in the Central Region of Ghana. The school had an enrolment of 1600 students with teaching staff of 73 and non-teaching staff of 25. It had 33 classrooms for teaching, excluding science and ICT laboratories. The ICT laboratory had a projector, two desktop computers, four mini laptop computers, and a printer. The school had six programmes of study: General Science, General Agriculture, Home Economics, Business, Visual Arts, and General Arts.

Participants
The population of this study was 13 teachers in the mathematics department of the school where the study took place. Based on the reasons shared by Needleman and Needleman (1996), Stake (1994), Merriam (1998), and Yin (2003), a purposive sampling procedure was employed in selecting the teachers. Purposive sampling helps the researcher to select participants who have rich information that helps to answers the research questions. Thus, a purposive sample produces informants that help to understand the social process under
investigation. The school, the mathematics department, and the teachers were selected for the following reasons.

First, the school was selected based on familiarity, which helped the researcher to have good access to both human and infrastructural resources. The school again expressed interest and willingness to encourage the teachers in the mathematics department to participate in the study. In a case study research design, it is important the researcher think about the practicability of negotiating and maintaining the access of his/her case (Rose, Spinks, & Canhoto, 2015). The researcher’s familiarity with the study context enabled him to tap rich evidence to enhance the interpretation of the phenomenon explored.

Second, the teachers in the mathematics department were selected based on the reasons echoed by Garet et al. (2001). They highlighted that effective professional development is enhanced when the teachers involved in the programme are (i) from the same school, (ii) teach the same grade, and (iii) teach the same subject; this accordingly helps teachers to develop common understanding of instructional goals, methods, problems, and solutions. They also emphasised that these reasons sustain changes in the curriculum practice through shared culture, where new teachers who join the department later can benefit from the experience of the old teachers. The teachers in the school selected for the current study taught mathematics across all the three-year levels (for instance SHS1-age 16, SHS 2-age 17, SHS3-age 18). Also, within each academic year, the teachers in the school used a common scheme of work and mode of assessment in their instructional practices, which made it less difficult to tailor activities to meet their classroom needs regarding technology integration in mathematics teaching.

Again, the teachers were selected because it is documented that teachers play a key role in effective technology integration; providing them with a professional development programme affords them the opportunity to gain a high-quality learning experiences that brings changes in their classroom practices (Mishra & Koehler, 2006; Whitcomb et al., 2009). Also, the teachers were selected based on their readiness and willingness to participate in the study (voluntary and with informed consent).

Eleven out of the 13 teachers in the mathematics department of the school agreed to participate in the professional development. The background information about the
participant teachers is provided to help readers gain a better understanding of the interpretation of the data and the shift in their pedagogy and technology dispositions. The individual teacher’s background information was gained from two different sources of data: the personal introduction they provided during the individual semi-structured interviews and the responses they indicated in the introductory section of the self-report questionnaire (the structure of the questionnaire will be discussed later). In analysing each teacher’s background information, follow-up calls were made to rectify cases of discrepancies in the responses the participant teachers provided in these two sources of data. All the names of the participants mentioned in the thesis are pseudonyms.

**Cynthia**
Cynthia was one of two females who participated in the study. She was in the 20-25 years age group. She had five years of teaching experience, and had been teaching core and elective mathematics across all three year groups. She had basic knowledge of Microsoft Word and Excel. She had attended no professional development related to technology use. She had not used technology to teach mathematics before. Cynthia was the core mathematics examiner for the West Africa Examination Council. She was holding additional responsibility as a form mistress.

**Prince**
Prince was aged between 26-30 years and had five years of teaching experience in chemistry, integrated science, and mathematics in the school. He had never taught any of these subjects with technology. He held a Bachelor of Science in Chemistry. He had knowledge of Microsoft Word and the basics of Windows. Prince had no training in the use of technology in mathematics teaching during his education. He had also not attended any technology-related professional development.

**Mary**
Mary was in the 20-25 years age group. She was an intern teacher who was at the school for her final teaching practice as part of her teaching qualification programme. She was very familiar with Microsoft Word, PowerPoint, and Excel. She had not attended professional development before. She had used technology to teach mathematics during her on-campus teaching practice at the university.
**Gideon**

Gideon was in his first year of teaching. He was aged between 20-25 years. He had a Bachelor of Science in mathematics with no teaching qualification. He had been teaching first and second year students in both core and elective mathematics. He was well versed in Microsoft Word, PowerPoint, and Excel, but had never used technology to teach any of the mathematics topics. He had also not attended any technology-related professional development.

**Jonathan**

Jonathan was aged between 26-30 years. He had seven years of teaching experience. His first teaching appointment started in the school. He held a Bachelor of Science in Chemistry as well as a Post Graduate Diploma in Education. He taught chemistry and core mathematics across all the year levels. He had considerable knowledge of Windows, Microsoft Word, PowerPoint, and Excel. He had never taught any of his subjects with technology. He had no training on the use of technology in mathematics teaching during his education. He had also not attended any technology-related professional development. Jonathan was a chemistry examiner for the West Africa Examination Council. He was also holding a position of responsibility as a form master.

**Joshua**

Joshua was aged between 26-30 years. He started his teaching profession in the school. Joshua had seven years of teaching experience. Joshua taught both core mathematics and elective mathematics in the school. At the time of the study, Joshua was holding a position of responsibility as a school chaplain. During his university studies, he had his four months of teaching practice (which formed part of his teaching qualification programme) in the school. He did his one-year mandatory national service in the school before he was permanently employed as a mathematics tutor in the school. His prior knowledge in technology application includes the basics of Windows, Microsoft Word, and Excel, and SPSS. He held a Bachelor of Education in mathematics. He was introduced to the use of technology in mathematics teaching during his studies at the university. He had been using his personal Android smartphone to conduct mathematics lesson. He had attended two professional development sessions, one on assessment and the other one on the use of technology in mathematics.
**Sammy**
Sammy was between the age of 36-40 years, and he had been teaching for the past six years. He held a Higher National Diploma in Mechanical Engineering (Plant Maintenance option) and a Bachelor of Science in mathematics education. He had knowledge in Geometer sketchpad, Microsoft Word, Excel, and PowerPoint, but had not used technology to teach mathematics. He had been teaching core mathematics across all three year groups. He had not attended any technology-related professional development.

**Michael**
Michael was between the age of 40-45 years, and he was in his twelfth year of teaching. He taught core mathematics to two classes each in the first and second year groups. He also taught the Principles of Cost Accounting in the first and second year classes. He held a Bachelor of Commerce. He was an active member of the committee in charge of examination in the school. He mostly worked on the stenographer. He had no teaching qualifications. At the time of the study, he had enrolled in a Diploma in Basic Education programme (mathematics option). He had considerable knowledge of Microsoft Word, PowerPoint, and Excel. He had no training and had also not attended any professional development in technology use. He had not taught mathematics with technology.

**Bernard**
Bernard was a veteran with 16 years teaching experience within which he had four years study leave to study for a Bachelor of Science in mathematics and statistics. Prior to his university education, he attended a college of education to be trained as a professional teacher for the primary school (6-14 years) where he concentrated on mathematics and science teaching. He was 40-45 years and had been in the school for six years. He taught both core and elective mathematics. He occasionally taught physics when the need arose. He had considerable knowledge in Microsoft Word, Excel, and SPSS and Matlab. He had never used technology to teach mathematics and had not attended technology-related professional development. He was an elective mathematics examiner for the West Africa Examination Council, and he is currently head of the mathematics department in the school.

**Peter**
Peter was aged between 30-35 years. He held a Bachelor of Science in mathematics education. He also held a professional teacher certificate qualification (Teacher Certificate

“A”) which enabled him to teach at the primary school level (6-14 years). He taught at primary school before moving to the senior high school. He had 12 years of teaching experience. He had been in the school for one year. He taught both core and elective mathematics across all the year groups. He had enrolled in the master’s degree programme in mathematics education. He was a mathematics examiner for the West Africa Examination Council. He had knowledge in Maple 12, Microsoft Word, PowerPoint, and Excel. He was introduced to the use of technology during his university education. He had used Maple 12 to teach mathematics. Since he started practising teaching, he had not attended any technology-related professional development.

**Martey**

Martey was aged between 36-40 years, and he had taught for 12 years. He held a Bachelor of Management Studies, a Post Graduate Diploma in Education, and a Master of Science in Economics. He had been teaching core mathematics across all three year groups. He had knowledge of Microsoft Word, PowerPoint, and Excel, and Windows. He had attended professional development related to technology use. He had not taught mathematics with any technology application before.

**Methods for Data Collection**

Technology integration is complex because of its cultural sensitivity (Hofer et al., 2011; Mishra & Koehler, 2006) and so, for teachers to function effectively within this landscape, there is a growing call on the need to have well-triangulated strategies to evaluate evidence of teachers’ efficacy in infusing technology in their classroom practices. Methods such as interviews, open-ended questionnaires, self-report surveys, observations, and performance assessments have been used to evaluate teachers’ effectiveness of using technology to enrich their classroom activities (Harris et al., 2010; Koehler, Shin, & Mishra, 2011). Technology integration researchers’ views of using multiple data sources to ascertain teachers’ efficiency of using technology resonates with interpretivists’ notion of dense description of a case study through the use of varied data. It is in this direction that the current study employed semi-structured interviews, a self-report questionnaire, focus group discussions, field notes, and observations of lesson enactment to explore the teachers’ technology dispositions within the context of mathematics curriculum in Ghana.
Semi-structured interviews

Interviews are crucial in qualitative research because they help to (i) siphon unique information or interpretation of individual’s perspectives about a given phenomenon, (ii) collect a numerical aggregation of information from many informants, and (iii) explore information that is difficult to reconnoitre through observation (Stake, 2010). Merriam (1998) affirmed the use of interviews in qualitative research, when the intention is to examine in-depth case studies of a few selected individuals. The current study aimed to draw implications from the individual teachers’ perspectives relating to the pedagogical use of technology in mathematics teaching. Thus, it was crucial to conduct the interviews in a manner that allowed the teachers to comment or tell their own stories (emic approach), rather than following a rigid structure where questions are predetermined (etic approach), which can make it difficult for the researcher to gain an in-depth understanding of the issue being explored. Therefore, in this study, the researcher employed a semi-structured interview approach (Newby, 2010; Stake, 2010) where both informal conversation and an interview guide were used to allow open-ended questions (Patton, 1990). Each participating teacher was interviewed twice, before and after the professional development. Each interview lasted not more than 60 minutes (see Appendices C and D for interview guides). The interviews were audio recorded.

Focus group discussions

Focus group discussion has become one of the most popular tools in qualitative data collection, and is used for its own purpose or in conjunction with other data collection tools (Morgan, 1993). It was earlier indicated that the current research was partly explorative, where data is collected for the purpose of identifying patterns to inform the subsequent activities of the professional development, and also to relate teachers’ perspectives (beliefs, attitudes, knowledge, and skills) about technology integration to their classroom practices as they went through the programme. The focus group discussion approach provided a flexible and time-efficient strategy to collect divergent views from the teachers to pursue this agenda (Litoselliti, 2003; Morgan, 1993).

The teachers were engaged in multiple focus group discussions at the various phases of the professional development programme to enable them to not only share their professional knowledge and experience, but also to brainstorm and generate ideas, discuss problems, and
identify possible solutions regarding the pedagogical use of the GeoGebra tool in mathematics teaching. In each focus group meeting, the researcher initiated the discussion by giving basic questions related to the topic, but as the discussion unfolded the full detail of the topic unwrapped. All the focus group discussions were audio recorded.

Self-report questionnaire
Self-reporting has persuasive advantages of exploring the individual’s personal orientations because of its interpretability (language common to the inquirer and the informants), richness of information (informants are the best-qualified witnesses to their own personalities), casual force (engages respondent’s identity), motivation to report (people are more careful when they are rating themselves than someone doing on their behalf), and sheer practicability (efficient and inexpensive) (Paulhus & Vazire, 2007).

The teachers in this study assessed their own beliefs, attitudes, knowledge, and skills related to technology integration in mathematics teaching via questionnaire (McLeod, 2014; Yannakakis & Hallam, 2011). The same questionnaire was administered twice: before and after the professional development. It took the teachers a maximum of 45 minutes to complete the questionnaire. The questionnaire comprised largely of 5-point Likert scale items (1 = strongly disagree, 2 = disagree, 3 = neutral, 4 = agree, and 5 = strongly agree). The items on the questionnaire were grouped into four sections.

Section A involved background information. It comprised 12 items (open and closed items) for exploring participant’s personal details such as age, gender, teaching experience, and issues related to their previous experience about technology integration and professional development.

Section B comprised 49 items (TK = 9, CK = 6, PK = 7, PCK = 6, TCK = 5, TPK = 9, and TPACK = 7 items) adapted from Schmidt et al. (2009) to evaluate teachers’ TPACK development. The TPACK instrument had been validated and the Cronbach’s alpha reliability coefficients of the constructs determined: TK = .82, CK = .85, PK = .84, PCK = .85, TCK = .80, TPK = .86, and TPACK = .92 (Schmidt et al., 2009).

Section C had 14 items designed to assess the changes in teachers’ technological beliefs as they went through the professional development. The technological beliefs instrument was
thematised: personal beliefs (five items) and pedagogical beliefs (nine items). The researcher constructed these items to suit the purpose of the current study. The construction of the items was guided by extensive review of literature (Buabeng-Andoh, 2012; Christensen & Knezek, 2008; Knezek, Christensen, Hancock, & Shoho, 2000; Larbi-Apau and Moseley, 2012; Petko, 2012; van Braak et al, 2004; Yushau, 2006).

Lastly, Section D had 20 items on technology attitudes: affective domain attitudes (six items), cognitive domain attitudes (nine items), and behavioural domain attitudes (five items) with overall Cronbach alpha reliability coefficient of .90 (Albirini, 2006). These items were used to track the changes in teachers’ technology attitudes as they journeyed through the current professional development.

Even though the participants answered all aspects of the questionnaire, the analysis was reported in four themes to avoid overlap of some of the items in the beliefs and cognitive constructs. These themes were perceived usefulness (importance of technology in mathematics teaching and learning), affective, behavioural (intention and willingness to use technology), and self-efficacy (knowledge and ability to use technology).

**Observation of lesson enactment**

Performance-based assessment focussing on lesson planning has recently become one of the crucial strategies for assessing teachers’ knowledge and skills of effective technology integration (Fisser et al., 2015). It is indicated that teachers’ pedagogical beliefs do not always correlate with their classroom practices and therefore triangulating teachers’ external practices and artefacts, together with their self-reports, helps to gain deeper understanding and make better judgments of how they use technology in the classroom (Harris et al., 2010).

In this study, the lesson plan (artefact) and lesson enactment were used as evidence to evaluate the progress of the development of the teachers’ knowledge and use of technology in mathematics teaching. I observed and recorded key moments of the teachers’ actions in my field notes (logbook). An Android smartphone was also used to capture key moments. To understand how teachers enacted effective mathematics pedagogy in the GeoGebra learning environment, the framework of effective pedagogy proposed from the literature was used (Anthony & Walshaw, 2007, 2009; NCTM, 2007)
**Field notes**

Field notes were included in the data collection process for two key reasons: descriptive and reflective purposes. For a descriptive purpose, the field notes provided the researcher the opportunity to accurately account for day-to-day activities such as dates; time; setting; researcher/participants’ actions, behaviour, hunches, and conversation; and unexpected hitches were all recorded in the researcher’s logbook. For a reflective purpose, the field notes provided an insight for daily self-evaluation to reduce the influence of the researcher’s personal experience and theoretical ideology on the participants’ responses (Krefting, 1990). Data collected through field notes aided in triangulating the interpretations and findings of the research.

**Data Analysis**

In this section, the procedures adopted for analysing the data collected for the study are discussed. Table 4.3 summarises how the data collected were used to answer the research questions. The analysis of the data is presented in three sub-sections: interviews and focus group discussions, lesson enactment, and self-report questionnaires.

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Method of data collection</th>
<th>Instruments</th>
<th>Data analysis approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>What were the teachers’ initial dispositions toward the use of technology in mathematics?</td>
<td>interviews, focus group discussion and field notes</td>
<td>interview guide, audio recording, researcher’s logbook</td>
<td>inductive approach: transcription, quotes,</td>
</tr>
<tr>
<td>How did teachers enact effective mathematics pedagogy when using GeoGebra following their engagement in professional development?</td>
<td>lesson observation, interviews, focus group discussion, lesson plan and field notes</td>
<td>video recording interview guide, performance assessment rubric</td>
<td>deductive approach: transcription, quotes</td>
</tr>
<tr>
<td>How did the professional development impact on the teachers’ dispositions towards technology integration in mathematics?</td>
<td>pre and post self-report questionnaire, interviews, focus group discussion</td>
<td>questionnaire, interview guide, audio recording, researcher’s logbook</td>
<td>inductive approach: transcription, quotes,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>mean scores, standard deviation, paired sample t test, effect size (Hedges’ g),</td>
</tr>
</tbody>
</table>
Interviews and focus group discussions

The focus of qualitative data analysis is to discover patterns from concepts, themes, and insights through multiple data that provide meaningful explanation, understanding, or interpretation of a given situation. The pre and post interview and focus group data were analysed inductively (Thomas, 2006). The audio recorded interviews and discussions were organised and listened to repeatedly to enable the researcher to familiarise themselves with the data. Careful listening of the audio files revealed tentative themes, concepts, and patterns and they were recorded in the researcher’s logbook. At this stage the researcher was using the participants’ own words to identify possible codes. The audio files were then transcribed. The text was initially captured in Microsoft Word. To ensure the objectivity of the findings reported from the data, constant reflection of the data was achieved through the use of third column analysis. Third column analysis is the interpretation written at the right-hand column of the interview/focus group discussion transcribed. This interpretation was done repeatedly to make initial sense of codes originally jotted in the researchers’ logbook (Ingram, 2011). Comments, text highlight colour, and font colour features in Microsoft Word were used to achieve this (see Sample, Appendix I). The transcribed data were then imported into HyperRESEARCH 4.0.1 for detailed coding and analysis. The software facilitated clustering the codes into themes. As can be seen in Appendix J, 94 codes were generated from the pre-interviews and focus group discussion regarding teachers’ perspectives about the use of technology in mathematics teaching and learning.

The researcher then categorised teachers’ perspectives that emerged into themes based on the theoretical framework and research questions. Also, the researcher subsumed teachers’ perspectives that did not fall directly under any of the research questions. The categories were worked through for the purpose of refinement, coding, and recoding, and identification of contradictory views. This was done with the aid of HyperRESEARCH 4.0.1 by clouding the codes (see Appendix K). The following paragraphs provide an illustrative example of how the themes were derived through the clouding approach.

The cloud features in HyperRESEARCH 4.0.1 provided alternate diagrammatic representation of the teachers’ responses which were coded. Both the bar graph (Appendix J) and cloud features (Appendix K) afforded me the opportunity to see the most common codes which strongly reflected the teachers’ perspective about the use of technology in mathematics teaching and learning at a glance. In addition, while the bar graph provided the
numeric count of the codes, the cloud reduced the whole document into a single page which made it handy to work with. For example, in Appendix K, codes such as instructional use of technology, willingness towards technology, professional development in mathematics, and limited technology resources were most apparent. These codes became the initial themes which were worked with. The initial themes were critically examined in relation to other codes illustrated in Appendices J and K. Links between codes were identified by paying attention to the responses the teachers provided to questions for the purpose of refinement, recoding, and thematising (Braun & Clarke, 2013). For example, when Peter was asked during the pre-interview about his views about the role of technology in mathematics, he commented:

Using the technology will assist students to learn cooperatively because when a student presses [the computer] and he or she does not get the answer he will quickly go to a friend for a support through that they will be sharing ideas. … My role only becomes a facilitator rather than imposing formulas and other things on the students (Peter).

This excerpt was initially coded as technology can promote cooperative learning. However, during the refinement phase, I realised that this code may not entirely represent Peter’s notion about the roles of technology in teaching and learning. So, the code was revised and an additional one introduced: explorative instruction, socio-cognitive learning, and facilitatory instruction. This excerpt suggests using technology could initiate collaborative learning, “when a student presses [the computer] and he or she does not get the answer he will quickly go to a friend for a support through that they will be sharing ideas”. Also, students sharing (social) mathematical ideas indicates they are exploring. When students are exploring a mathematical concept then they are taking ownership of their learning and the teachers’ role will obviously become facilitatory. A cognitive task is involved for both the student seeking help and the one offering the support. The student offering the help has to make a cognitive argument to support his or her answer. The student seeking help will also judge or evaluate the response from his friend before he or she accepts it. Although each of the codes (explorative instruction, socio-cognitive learning, and facilitatory instruction) is different and they contribute to students’ mathematics learning in diverse ways, they are not mutually exclusive in terms of their role in teaching and learning. Therefore, explorative instruction, socio-cognitive learning, and facilitatory instruction were among other codes
sub-categorised under the theme “pedagogical tool”. At this stage, what I did was to use the new codes to identify similar evidence(s) of pedagogical effectiveness across data of all the participants. When I felt that the data had been sufficiently coded and redundant codes had been deleted or revised, HyperRESEARCH 4.0.1 was used to create cases of sub-themes. The sub-themes were then grouped into main themes based on the research questions and my understanding of the literature reviewed for the study. Finally, I arrived at six themes central to the teachers’ initial dispositions towards technology: (i) perceived beliefs of usefulness of technology; (ii) perceived beliefs of nature and utilisation of technology; (iii) perceived feelings towards the use of technology; (iv) perceived intentions towards the use of technology; (v) perceived self-efficacy towards technology; and (vi) perceived contextual influence related to the use of technology. Appendix L is an extract of the coding and categorisation of the first theme “perceived beliefs about the usefulness of technology in mathematics”.

**Lesson enactment**

The lesson plans and transcribed lesson episodes were imported into HyperRESEARCH 4.0.1. The data were deductively analysed using the effective pedagogy framework proposed for the study (Appendix M) and identified evidence of how the teachers enacted the core practices of each theme of effective mathematics pedagogy in a GeoGebra environment following their engagement in the professional development (see sample of detailed coding in Appendix N). The core practices that became common were regrouped and interpreted in Chapter Five under the themes: creating mathematical setting, useful mathematical tasks, mathematical discussions, mathematical connections, and assessment of students’ learning. It is important to acknowledge that some of the teachers’ actions were multiple coded for different themes because there was overlap of the actions of the teachers for different themes. For example, the excerpt from Sammy’s lesson was coded creating mathematical setting as well as useful mathematical task.

Observe the polygons shown on the GeoGebra window and record your observations in the table below. From the table in (i) write down the number of triangles in a given polygon in terms of *n*, (ii) write down the formula for the sum of interior angles of a polygon (*S*) in terms of *n* (Sammy).
The excerpt in the first place suggests creating mathematical setting because it serves as spark for hooking students to instructional task. In other words, the teacher is drawing the attention of the students as well as providing instruction for which the students are expected to carry out. The excerpt also suggests useful mathematical task because it is engaging students to perform certain mathematical proficiency such as observation, recording and deduction of mathematical concepts. This overlap perhaps illustrates the non-linearity and complexity of teachers’ practices in the classroom.

**Self-report questionnaire**

The pre and post-self-report questionnaire data were captured in Microsoft Excel and later transferred into SPSS (version 21). All the items on the questionnaire consisted of 5-point Likert scale (strongly disagree = 1, to strongly agree = 5). The rating 3.0 was a neutral point. It indicated a participant teacher neither agreed nor disagreed with the statement. For the purposes of comparison, items that were negatively worded were reversed so that lower scores reflected negative response and high scores reflected positive response. For example, a statement “I dislike using technology in teaching” was reversed during the coding as “I like using technology in teaching”. So, if a respondent chose a score of 2 for a negative statement, the score of 4 was recorded during the coding.

The mean score for all the individual items under each sub-construct was computed using SPSS (Statistical Package for the Social Sciences) and was used to determine the changes in the teachers’ dispositions towards the application of technology in mathematics teaching. The aggregated mean for each construct was computed. For example, perceived usefulness’ has eight items. SPSS was used to transform the eight items into a single variable “perceived usefulness” by taking into consideration of the mean score of all eight items for each participant teacher. For example, if a participant teacher chose 5, 5, 4, 3, 5, 4, 4, and 3 for the eight items, then his or her score for perceived usefulness would be the sum of these scores divided by eight (33/8 = 4.125). This was done for all the participant teachers, and the aggregated mean was subsequently computed. A mean score greater or equal to 4.00 was considered as the agreed threshold indicating a positive response (Kul, 2012; Owusu, 2014).

Hedges’ $g$ (bias-corrected effect size) was employed to provide a numerical representation of the magnitude of the change in teachers’ dispositions towards the use of technology in mathematics teaching and learning. The benchmark for interpreting the values of the effect
size ($d$) are: adverse effect (less than 0.0), no effect (0.0 to 0.1), small effect (0.2 to 0.4), moderate effect (0.5 to 0.7), large effect (0.8 or higher) (Lenhard & Lenhard, 2016). Paired sample $t$ tests were conducted to determine the significance of the differences in the pre- and post-measurements of the teachers’ technology dispositions at an alpha level ($\alpha$) of 5%. In cases where significant differences existed, the Bonferroni adjustment was conducted to compensate for the multiple comparisons of variances of the technology dispositions of the eleven participant teachers. Thus, the common threshold of 0.05 was adjusted to 0.0045 ($0.05/11$) to reduce the chance of false positives (Glickman, Rao, & Schultz, 2014; Perneger, 1998).

**Establishing the Trustworthiness of the Findings of the Study**

There are four well-known criteria for determining the rigour in a positivist paradigm: *internal validity*, *external validity*, *reliability*, and *objectivity*. The ontological, epistemological, and methodological assumptions underpinning the use of either a positivist or interpretivist paradigm have been explicated in this chapter. It became clear that the interpretivist paradigm is pluralistic (consisting of a variety of approaches); hypothesis and theory generative; based on socio-cultural construction of knowledge (that is there is no singular reality); responsive to close researcher-participants relationships; based on a dense description of specific phenomenon; and grounded on the fact that inquiry is value-bound (comprising the value of the investigator, the choice of the research paradigm, the choice of theory to guide the study, and contextual value). The positivists on the other hand hold opposite views to each of the interpretivists’ assumptions. Therefore, many authors (Agar, 1986; Guba, 1981; Krefting, 1990; Lincoln & Guba, 1985; Morgan, 1983) suggested that the criteria for assessing the rigour of quantitative inquiry are irrelevant and inappropriate in determining the trustworthiness of qualitative research. For instance, Agar (1986) contended that terms such as reliability and validity have quantitative connotations and do not meet the details of qualitative research. In the midst of this debate about the need to have an alternative model appropriate for testing the trustworthiness (rigour) of qualitative research without sacrificing its relevance, several models have emerged (see, for example, Kirk & Miller, 1986; Leininger, 1985; Lincoln & Guba, 1985). This study applied Guba and Lincoln’s model because (i) it has received comparatively more attention, (ii) it is conceptually well developed, and (iii) many qualitative researchers have used it in several disciplines such as education and nursing (Krefting, 1990).
Lincoln and Guba’s (1985) four criteria for ensuring trustworthiness of qualitative research, *credibility*, *transferability*, *dependability*, and *confirmability*, are parallel to the conventionalists’ criteria for testing the rigour of quantitative research, *internal validity*, *external validity*, *reliability*, and *objectivity*.

**Credibility**

Credibility refers to the “accurate description or interpretation of human experience that people who also share that experience would immediately recognize the descriptions” (Krefting, 1990, p. 216). Thus, research is said to be credible if one can ascertain the confidence in the truth of the findings for the context of the phenomenon explored (Lincoln & Guba, 1985). The credibility of the current study is ensured by following the strategies proposed by Lincoln and Guba (1985): prolonged and varied field experience, reflexivity, triangulation, member checking, structural coherence, referential adequacy, interview technique, establishing authority of researcher, and peer examination.

*Prolonged and varied field experience*: The researcher observed and documented the teachers’ perspectives and use of technology in mathematics teaching as they journeyed through the professional development programme for an extended period of 12 months. Lincoln and Guba (1985) indicated that prolonged engagement with the participants in the research milieu provides the researchers the opportunity to reduce their own biases, perceptions, philosophical, and theoretical thinking, as well as those of their informants. As much as it is crucial for the researcher to enmesh him or herself with the informants to enhance discovery of hidden information, Guba (1981) admonished researchers to be cautious of over-involvement - termed “going native” (p. 85).

*Reflexivity*: Reflexive analysis is a process of checking the influence of the inquirer’s background, perceptions, and interests, on the qualitative research process (Krefting, 1990). The researcher’s over-involvement with the participant can result in a *Hawthorne effect*; a situation where the participant will put up a false front behaviour (McCambridge, Witton, & Elbourne, 2013). In this study, there were two possible situations that could lead to fiduciary relationships between the researcher and the participant teachers. First, the researcher was a former mathematics teacher in the school and his relationship with the teachers in the school could catalyse potential exploitation of teachers’ perspectives of technology integration in
mathematics teaching. Second, the research was designed in such a way that the researcher collaboratively worked with the teachers in a professional development programme for an extended period (12 months) which could have blurred the boundary of the relationship between the researcher and the participants as the research progressed. It is important that these facts are acknowledged in the study so that possible interventions could be tailored to obviate the ethical implications of an exploitative relationship. Reflexive analysis is a strategy used to monitor the extreme researcher-participant relationship through the use of a field journal (Guba, 1981). In this study, a researcher’s logbook was used to record day-to-day activities to serve as a source of reflection to check the way the relationship affected the participants emotionally, psychologically, physically, and socially. The researcher’s logbook again served as source of data to augment the interpretation of the study. Also, the after-school workshop, where the researcher met the teachers after each lesson they taught, provided the opportunity for both the researcher and the teachers to reflect on the pedagogical use of technology in mathematics teaching.

**Triangulation:** Triangulation is the verification of information from at least two sources: different methods, different data sources, different perspectives (theories), and different investigators (Denzin, 1978; Guba, 1981; Krefting, 1990). In this study, data was triangulated using the teachers’ self-report questionnaires, the observations and video recordings of teachers’ lessons in the classroom, focus group discussions, interviews, the teachers’ lesson plans, and field notes recorded in the researcher’s logbook.

**Member checking:** Member checking is a process whereby data and interpretations are continuously checked as they occur with the informants and/or members of the varied interest (Krefting, 1990). It is ethically crucial to cross-check informants’ responses with them to ensure that their views are recorded accurately (Cutliffe & Ramcharan, 2002; Guba, 1981; Krefting, 1990). In this study, participants’ responses were cross-checked with them during the interviews and focus group discussion by repeating some of their responses for affirmation or otherwise. The focus group discussion served as a platform where individual participants had the opportunity to comment on others’ views regarding the pedagogical use of technology in mathematics teaching. During the focus group discussion and interviews, the researcher was cautious of not including insights that might be harmful to the well-being of the informant. Also, the constant feedback from the two supervisors of the thesis played a crucial role in ensuring the consistency of interpreting the data collected.
**Structural coherence:** Structural coherence is the process of testing all the data to ensure that there is no internal conflict or inconsistencies (Anney, 2014; Guba 1981). The data was collected through multiple methods and sources, so inconsistent findings and deviant cases were identified and accounted for (Guba, 1981; Krefting, 1990).

**Referential adequacy:** Referential adequacy is the process of testing the analysis and interpretation against the documents that were used during data collection before producing the final document (Anney, 2014; Guba, 1981). The data - audio and video recordings, researchers’ logbook, self-report questionnaire, and the lesson plans collected - were archived for at least five years for future reference should there be the need (Guba, 1981).

**Interview technique:** Interview technique involves the process of encouraging the respondents to answer completely, freely, and relevantly with regards the topic explored in a study (Kumar, 2014). During the interviews and the focus group discussion, questions were reframed, repeated, and expanded at different occasions to ensure the credibility of the study (Krefting, 1990).

**Establishing the authority of researcher:** It is often said that a piece of research work is as good as the inquirer. This means that the researcher’s past experience, knowledge, skills, and the training acquired have recently become determinants in assessing the credibility of a research work because the researcher is viewed as a human instrument (Lincoln & Guba, 1985). As such, the researcher is expected to have: (i) a certain degree of familiarity with the phenomenon and setting being explored; (ii) a strong interest in conceptual and theoretical knowledge and can conceptualise large amounts of qualitative data; (iii) ability to take multiple approach to explore phenomenon; and (iv) good investigative skills (Krefting, 1990). The previous academic and research background of the researcher merit this current study. Also, the researcher attended several seminars on thesis writing skills during his candidature. The two supervisors for this research had rich experience in qualitative research and their insightful suggestions and comments helped polish the interpretation of the results reported in the study.

**Peer examination:** Peer examination, according to Krefting (1990), involves discussing the research process and findings with impartial colleagues who are knowledgeable and have
experience in qualitative research. This criterion of ensuring the credibility of the study was achieved through the involvement of the two supervisors of the thesis. The critique the supervisors provided was crucial in keeping the researcher honest and reflexive. Also, the process and findings of the research were presented to colleagues in the department and at international conferences. The questions and suggestions that came during presentations were used to improve the quality of the research.

**Transferability**

For the interpretivists, the concept analogous to external validity or generalisability is transferability, which is the degree of fitness or applicability of the researcher’s interpretations between two contexts (Krefting, 1990). It is believed, in the qualitative research paradigm, that the interpretations of a case study in a particular context is transferable to another context provided there are certain essential similarities between them. The issue of transferability in qualitative research is a duo-responsibility; the original researcher has a role to play as well as the person wanting to transfer the interpretations. The onus lies on the original researcher to provide sufficient description and interpretation of the phenomenon for the readers to make their own judgement about the fitness of the findings in a different context (Lincoln & Guba, 1985). The interpretations of this study are intimately tied to the time and the context in which the study was conducted, and therefore there is no point in generalising the findings beyond the scope of the study. However, the researcher provided detailed description of the context, methodology, and results so that the readers can form their own interpretations about the particularities of the context and the transferability of the findings into their own experience (Krefting, 1990).

**Dependability**

Interpretivists refer to consistency as dependability, which embodies components of both stability and trackability required by explaining all vicissitudes that occur in the research process (Guba, 1981). An audit trail has been proposed as a strategy of ensuring both dependability and confirmability of qualitative research (Lincoln & Guba, 1985). An audit trail is a situation where an external auditor is engaged in the process of the research. Also, triangulation is advocated for eliminating the invalidities in the individual methods applied in collecting the data (Guba, 1981). That is, employing two or more methods in a single study is advantageous because the weakness of one method can be compensated by the strengths of the other method. In this study, the two supervisors provided continuous
analytical insights regarding the progress of the research report. Triangulation, repeated observations of the teachers’ lessons, and a code-recode strategy during the data analysis phase, were adopted to accomplish dependability of the current study. Also, detailed documentation of all stages of the research process was meant to ensure the dependability of the research (see Figure 4.1, Table 4.1, and Appendix E).

**Confirmability**

The parallel of objectivity, in the naturalistic perspective, is confirmability, which is not considered as investigator’s objectivity but rather ascertaining the neutrality of the data and the interpretation produced from the research (Guba, 1981). The current study closely followed Guba’s strategies of accomplishing confirmability: audit trail, triangulation, and reflexive practice.

**Summary**

This chapter elucidated the philosophical perspective, research methodology, research design, data collection and analysis approach, ethical consideration, and procedure for establishing the trustworthiness of the research findings in connection with the purpose and the research questions of the study. Juxtaposing the principles of effective professional development for technology integration in mathematics teaching with the two research paradigms, positivism and interpretivism, this study was aligned with an interdependent, subjective, interpretive, and constructivist approach of establishing knowledge. Thus, the principles of qualitative research methodology and case study research design were adapted to gain an understanding of how professional development impacted on teachers’ pedagogy, beliefs, attitudes, and knowledge related to the use of technology in mathematics teaching. The findings of the study are reported in detail in Chapter Five.
CHAPTER FIVE: RESULTS

The current study aimed to explore how teachers enacted effective mathematics pedagogy in a GeoGebra environment following their engagement in professional development. Engaging teachers to design and teach with GeoGebra in the mathematics classroom offered a unique lens from which to explore the shift in their dispositions (beliefs, attitudes, and knowledge) towards the use of technology in mathematics teaching and learning. Three research questions guided the study:

1. What were the teachers’ initial dispositions toward the use of technology in mathematics?
2. How did teachers enact effective mathematics pedagogy when using GeoGebra following their engagement in professional development?
3. How did the professional development impact on the teachers’ dispositions towards technology integration in mathematics?

The first section of this chapter reports the findings of teachers’ initial dispositions towards the use of technology in mathematics. The second section reports how the teachers enacted effective mathematics pedagogy in their classrooms following their engagement in the professional development process. The final section reports on the shift in the teachers’ technology dispositions as well as their overall impression about the professional development.

Teachers’ Initial Dispositions towards Technology

At the outset of the professional development process, teachers were interviewed and engaged in a focus group discussion. They also each completed a self-report survey. The data from the first interview and focus group discussion are examined in this section to provide insights about (i) what teachers initially believed about the usefulness of technology in mathematics; (ii) what teachers initially believed about the nature and utilisation of technology; (iii) their feelings towards technology; (iv) their intentions to use technology; (v) self-efficacy related to technology; and (vi) the contextual influence (see detailed analysis in Chapter Four and Appendices I, J, K, and L).
Perceived beliefs about the usefulness of technology in mathematics

In the interview and focus group discussions, the participant teachers had the opportunity to either construct, or reconstruct their beliefs of the importance of technology in mathematics teaching and learning. Four main themes were apparent in their responses: they perceived technology was useful for pedagogy, assessment, the development of professional competence, and their general productivity.

Pedagogical tool

With regards to the pedagogical effectiveness of technology, the participant teachers held the belief that using technology in mathematics instruction could create opportunities for students to learn mathematics meaningfully. Figure 5.1 shows the range of varied reasons they believed technology could be useful in their mathematics classroom. The frequency represents the number of times each theme was coded in the entire document related to teachers’ initial perceived pedagogical usefulness of technology in the classroom.

<table>
<thead>
<tr>
<th>Code</th>
<th>Frequency (coded)</th>
<th>Bar Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Promotes explorative instruction</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>Makes students understand mathematics concepts</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Reduces abstract teaching</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Enhances students to practice more mathematics</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Teacher becomes facilitator</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Makes mathematics teaching real</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Promotes socio-cognitive learning</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Helps students’ problem solving skills</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Enhances students interest in learning</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Makes students open in their thinking</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Catches the attention of the students</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Enhances students participation in learning</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Helps learners to be creative</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Aides teaching from known to unknown</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5. 1 Teachers’ perceived pedagogical importance of technology

Peter, Bernard, and Gideon, at the beginning of the professional development, conceded that their instructional approach had mainly been a teacher-led approach where notes were written on the chalkboard for students to copy, and procedural instructions were verbally provided for the students to emulate. They, however, believed that technology could afford them the opportunity to orchestrate a student-centred instructional approach where visuals embodying mathematical concepts could be used to initiate cooperative and explorative learning.
The traditional way of doing things will be shifted to modern. The marker/chalkboard approach where I only come and put the topic on the board and write for the students to copy will be over. Using the technology will assist the students to learn cooperatively because once the laptops are put in place in the classroom for each student. So when a student presses and he does not get the answer he will quickly turn to a friend for support through that they will be sharing ideas (Peter).

It will make the lesson more child-centred because I will demonstrate and after the demonstration, the student will play around the software. The child will be able to do it by him or herself. It will let the lesson be practical and child-centred (Bernard).

I believe it [technology] can change our instructional approach. Currently what we are doing is that the teacher goes to the class and initiates the teaching and the learning … The children will be able to see the natural aspect of mathematics because everything will be practical (Gideon).

Joshua added, during the initial interview, that mathematics lessons needed to build on students’ pre-existing experience and he believed technology could provide a means for teachers to accomplish this task.

It is better for a teacher to teach from known to unknown. If you use these technologies in the classroom, the students will be able to identify the concepts. It will also help them to build mental capacities for exploring more mathematics concepts (Joshua).

These comments suggest that the teachers were willing to embrace an alternative approach, one that they believed could engage their students, both cognitively and socially, in mathematics construction. Also, the comments seemed to suggest that the teachers were discontented with their current approach of instruction because they believed it was not yielding the desired results they wanted.

As can be seen in Figure 5.1, the affordance of technology to promote students to think openly was coded once, indicating that only one teacher explicitly shared that view. He (Sammy) indicated that:
…when you introduce a topic to students using technology, it will surprise you to see after the lesson that some of the students can come out with other ways which are a development of what you taught them ... It helps them to be creative … It will also help them to bring new ideas when you use technology to teach them. That is where the creativity comes in.

This comment indicates that he had no doubt in his mind prior to the professional development that employing technology in mathematics instruction could promote students’ knowledge creation and critical thinking in mathematics. Although the rest of the participant teachers did not explicitly talk about technology as a tool for mathematical thinking, some of their comments implicitly denoted this assertion. For example, it could be inferred from Bernard’s comment “technology assist children on how to solve problem that come their way” that through technology, teachers could assist students to restructure the mathematics concept they (students) have learned to meet new situations in mathematics. In a similar view, Joshua commented “… using this approach (technology-assisted instruction), students can relate the mathematics to the environment. Using technology can help students to do correct estimation like the distance between two locations”.

Assessment tool
Four teachers explicitly indicated that technology could be useful in assessing both teachers’ instruction and students’ learning. Their comments centred on the affordance of technology to provide feedback on students’ learning and also using it to support continuous assessment practices such as recording and computing students’ grades after they had a test.

In Joshua’s view, the affordance of the features in technology that provided prompt responses could support teachers to address unexpected mathematical questions students asked during lesson delivery. This implies that the tendency where teachers fail to respond to some critical questions that students asked could possibly be remedied through using technology. The following excerpt illustrates his assertion:

It [technology] will make your lesson introduction captivating and catchy … We can use it to google for simple basic circular shapes and show [the objects] to students to easily ascertain those shapes … It helps students to assess their own
work and come out with their shortfalls. This will help the student to have good reflection on what they have learnt and apply them during examination (Joshua).

Thus, in Joshua’s perspective, technology could be a conduit for reflection of mathematical concepts.

Bernard commented that technology could help teachers avoid rigorous computation when preparing continuous assessment because there are features in the technology which can do the calculation easily:

Even the teacher can have his or her continuous assessment sheet on the computer. It can assist you to add the figures easily. You just have to enter the figure, and it gives you the total. Unlike sitting down and doing it manually, trying to calculate everything (Bernard).

It can be learnt from Bernard’s comments that technology could be useful in administration activities where records are processed and stored in the computer.

*Professional development competence tool*

All the participant teachers pointed out that technology is a useful tool for a personal update. They indicated that it could aid them to acquire new knowledge in their subject matter and teaching skills. In Joshua’s view, for instance, technology could support teachers to update their knowledge by looking for learning materials through the internet:

The teacher will do a lot of research through the internet. The teacher can get a lot of learning materials to support their knowledge and also their teaching skills (Joshua).

We are enlightened more through technology. We are able to delve deep into things that without it, we will not be able to reach. The technology can help me increase my mathematics knowledge because the software have a link which deals with the different content of mathematics. It opens you to things that you wouldn't know. It gives you the opportunity to practice more (Sammy).
It helps the teachers to broaden their horizon. That is, they become more knowledgeable about their subject matter and other areas of teaching. With that, the teachers can easily pass it on to the students (Bernard).

General productive tool
All participant teachers believed that technology could save them time from several activities they perform in the classroom. They, for instance, indicated that using technology could reduce their workload such as copying notes on the chalkboard, preparing a lesson plan, and recording assessment. Joshua and Sammy agreed that technology could afford them the chance to cover more topics within the instructional time. However, they acknowledged it requires a fair amount of time to prepare the lesson at home:

With technology, you are able to deal with complex issue within the shortest possible time. It's a way of minimising the time of preparing a lesson plan (Martey).

Most at times, you have to draw some of the concepts and diagrams on the board. It wastes time. I think with the presence of technology; you can go to the class with a projector. You can project whatever diagram you have for the students to see. They will appreciate it better than the one we draw on the board (Jonathan).

In terms of time, it will give us the opportunity to teach a lot of concepts within a short possible time. Using ICT, notes, diagrams and other things will be clear to students than writing them on the board. Sometimes the writing on the chalkboard is not too clear, so the students end up writing a lot of mistakes. So, that aspect I think the ICT will help (Cynthia).

Bernard lamented that mathematics had become a monster, “a devil in disguise” for their students, because of the rote approach they use in learning the subject. He was convinced that the massive failure the country recorded in external examination in mathematics could be reduced if the country took practical steps towards enforcing the use of technology in mathematics instruction. Martey added that until the country took a critical look at the use of technology in mathematics instruction, the students would continue to perform poorly in mathematics examinations. In his view, the students in advanced countries outperformed Ghanaian students in mathematics because they had embraced technology in their teaching and learning:
I have been one of the advocates calling for the use of technology in the mathematics setting. I am saying this because mathematics has become something which looks like "a devil in disguise" for some of our students that we have nowadays. They see it as a very difficult task because it has become rote learning … I think that the massive failure we record in mathematics nowadays in the BECE [Basic Education Certificate Examination and the WASSCE [West African Senior Secondary Certificate Examination] could be reduced if we integrate technology into our teaching (Bernard).

We've been reading a lot from the internet … The advanced countries are a mile ahead of us in terms of lesson delivery and students' performance. It is all because of the use of technology. But in our part of the world, we are still using the olden ways. To me, not until we begin to embrace the use of technology we shall be continuing to record poor performance in the maths (Martey).

**Perceived beliefs about the nature and utilisation of technology**

The nature and utilisation of technology involves teachers’ perceptions about the kind of technology that should be used in the mathematics classroom, and how. With regards to participants’ views on the nature and potential utilisation of technology, the most dominant perception that emerged from the interviews and focus group discussion was the notion of the use of technology as a visual representation, demonstration, and practice tool (Figure 5.2).

<table>
<thead>
<tr>
<th>Code</th>
<th>Frequency (coded)</th>
<th>Bar Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visual representation</td>
<td>8</td>
<td><img src="image1" alt="Bar Graph" /></td>
</tr>
<tr>
<td>Demonstration and practice of mathematics</td>
<td>5</td>
<td><img src="image2" alt="Bar Graph" /></td>
</tr>
<tr>
<td>Download of learning materials</td>
<td>3</td>
<td><img src="image3" alt="Bar Graph" /></td>
</tr>
<tr>
<td>Online learning and personal research</td>
<td>3</td>
<td><img src="image4" alt="Bar Graph" /></td>
</tr>
<tr>
<td>A student to a computer for mathematics learning</td>
<td>2</td>
<td><img src="image5" alt="Bar Graph" /></td>
</tr>
</tbody>
</table>

Figure 5.2 Teachers perceived nature and ways to use technology

Although the teachers were not asked directly about the kind of technology they believed would be useful for their mathematics instruction, technologies such as online learning, a projector, laptops, and calculators were apparent in their narratives. Their notions about technology in the mathematics classroom comprised the use of calculator, laptop, and
projector to project mathematics concepts on the screen for student to view. For example, Prince and Jonathan believed that they could reach many students when concepts were projected on the screen for the students:

I think with ICT; we can come out with a clear diagram containing the principles of the concept we are teaching. With that, they can easily picture and understand it better than our words (Prince).

… I think with the presence of technology, you can go with the projector and project whatever diagram you have. When students see it [the mathematics concepts] they will better appreciate it to help understanding (Jonathan).

Michael’s notion of the nature of technology in mathematics instruction was about downloading software that embodies mathematical concept and showing it to the students.

Technology in itself is very essential in the teaching of maths. If there is a particular topic I want to teach, the only thing is to try to download the software from the internet. Then if I have a projector I connect it to the laptop and project whatever I have for the students to have a feel or to see it real (Michael).

Joshua, Peter, Cynthia, and Sammy on the other hand believed that one way to use technology effectively in the classroom was to provide opportunity for both teachers and students to learn how to use technology. They believe when all the students had knowledge about it and each student had a computer to practice with, then meaningful mathematics teaching and learning could be achieved:

If there are laptops for all the students to use, then that one will be more efficient because they will be doing a lot of manipulations on their own and it will go a long way to help them (Joshua).

Gideon’s comment suggested technology could be used to support a flipped classroom approach where students could explore mathematical concepts prior to their engagement with the teacher in the actual classroom.

With technology, it will help the students to find out more things before they come to the class (Gideon).
**Feelings towards technology**

Although most of the participant teachers had never used technology in their mathematics instruction prior to the professional development, they anticipated that using technology to teach mathematics would make them feel comfortable. The following excerpts also illustrate how Cynthia, Martey, and Sammy felt about the use of technology at the beginning of the professional development:

- It’s interesting. I feel very relaxing. Students seeing their teacher using technology is a plus. It is good (Cynthia).
- I feel relieved because it does not make the work too cumbersome. It makes it easier. You are not bothered to enter the classroom with a lot of textbooks (Martey).
- I feel good. I feel I am on another level. I see myself to be moving in the 21st century. I will see myself enlightened and modernised. So, it will boost my morale and confidence (Sammy).

**Intentions towards the use of technology**

As has been indicated in Chapter Four (see The Structure of the Professional Development), the teachers’ preparedness to use technology was demonstrated in the number of suggestions and support they offered to get the professional development organised. For example, during the preliminary interaction with the participants, it became obvious that within the constraints of the limited resources in the school the teachers could only conduct technology-based mathematics instruction with a projector, a laptop computer, and a modem. Two of the participants subsequently offered their personal laptop computers to augment the two laptop computers, a projector, and a Vodafone router (for internet connectivity) that the researcher provided to support the programme. The following excerpt illustrates the participant teachers’ commitment to use technology in the classroom prior to their engagement in the professional development:

- Building [using] PowerPoint is easy for me to do. But to locate certain things on the internets, I will need somebody to help me. I know [Microsoft] Word, PowerPoint and others but putting all together to create mathematics lesson, I need help. For instances, when it comes to drawing of triangle and adding certain
[features] to make meaningful mathematics instruction requires additional skills which I don’t have.

Actually anytime we hold departmental meeting we put it forward to our head of department that he should lobby with the administration so that they will provide us with a projector or a laptop at least one for the department to use … Provided the technology is available and I am given the necessary workshop or training, there wouldn't be any impediment for me to use it (Michael).

Also, the numbers of teachers who voluntarily participated in this professional development is a remarkable indication of their preparedness to use technology to teach mathematics. Out of the thirteen mathematics teachers in the school, eleven (84.6%) voluntarily participated in the professional development. The other two teachers initially expressed interest to participate in the professional development, but they later withdrew because of family reasons.

Prince and Sammy believed that it was time they learned how to use technology because failure to do so would make them lag behind.

I embrace the idea of technology development programme because in this modern century that is what is on board. So, I will be happy if it becomes part and parcel of us in our learning process. This will help the country to develop. It will help the individual to also develop (Sammy).

It is very important to learn how to use technology. We are in that world now. Putting the teaching aside, we are in this technology world that if you don't have any idea about it you will be left behind (Prince).

**Self-efficacy towards technology**

This section explored the teachers’ prior knowledge and their ability to use technology in mathematics teaching and learning. The results indicate that the participant teachers had limited knowledge or competence to use ICT hardware/software and its associated peripherals in activities related to mathematics teaching:
I have not used technology in my classroom. The know-how is one of the rationales behind it (Sammy).

I don't have the knowledge and skills to use the technology because I have not had the opportunity to be exposed to the use of technology. I have not had enough training (Martey).

In the initial interviews, the participant teachers were asked to indicate any mathematics software they were aware of, and whether they had used it in their mathematics instruction. The results in Table 5.1 shows that the participant teachers were mainly familiar with Microsoft Excel. The teachers indicated they usually used Microsoft Excel for recording and processing students’ test scores. All teachers indicated they used Microsoft Word for typing examination questions. Cynthia added in her submissions that she used her phone to check her email and find the meaning of words via Google. Peter said he used Maple to teach quadratic expression. The results in Table 5.1 suggest that the participant teachers were yet to use mathematics software such as Derive, Geometer sketchpad and GeoGebra in their classrooms. Only Peter and Gideon indicated that apart from Microsoft Excel, they had used Maple and Microsoft Encarta Math Tools respectively. Gideon indicated he used the mathematics component in Microsoft Encarta to play games.

<table>
<thead>
<tr>
<th>Example of mathematics software</th>
<th>I am aware of its existence</th>
<th>I have used it personally</th>
</tr>
</thead>
<tbody>
<tr>
<td>Derive</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Geometer sketchpad</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>Microsoft Excel</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>Microsoft Encarta Math Tools</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>GeoGebra</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>Maple 12</td>
<td>11</td>
<td>1</td>
</tr>
</tbody>
</table>

With regards to the awareness of the mandates of the curriculum, the results of the study indicated that at the onset of the professional development, two of the participant teachers were less aware of the expectation to use technology in their mathematics instruction. The comments from Joshua and Gideon in the initial interviews suggested that they were yet to
read the rationale of the curriculum which mandated the teachers to use technology to assist students in exploring mathematics concepts:

I am not sure if it is in the syllabus. I have not come across it (Joshua).

…I believe that technology can go a long way to help us to deliver our lessons well and also to help the students to learn better, I suggest that technology should be part of the mathematics curriculum (Gideon).

Among all the participants, only Joshua and Peter indicated they had used technology in mathematics instruction. Joshua was using his phone to conduct mathematics lessons. He used his phone to download mathematical concepts from YouTube which he showed to his students during his lessons. Probing further about how he used technology before this programme revealed that he did not structure his lessons with it. He used it spontaneously; that is, as and when it was needed during the lesson delivery. In the pre-interview, he described that he adopted the technology when it became necessary to explore shapes of circular objects when the lesson was ongoing:

It got to a point in the classroom where I have to explain a concept to the students. So, I used the phone to google for basic circular shapes. I showed it to the students to ascertain those shapes. Using the phone, you can go to YouTube for simple animations that can easily be used to solve problems in mathematics (Joshua).

Peter indicated he had used Maple 12 in mathematics instruction prior to the professional development. He described how he used the Maple 12 in his classrooms in the following excerpts:

The Maple 12 software is a powerful mathematical software package … I have used once it to assist students to factorise expressions. They did the manual calculation and then I asked them to compare their factors to the one computed by the Maple 12 (Peter).

The lessons he described did not reflect his belief about the use of technology in teaching and learning. Peter, in the pre-interview, said that using the technology will “assist the students to learn cooperatively … make students think for themselves … make learners feel comfortable, reduce teacher’s burden … reduce situation where teachers impose formulas
on the students … help to evaluate my lessons”. However, from the lesson he described, there was limited evidence in the way he used technology to (i) encourage cooperative learning or (ii) foster in students higher order mathematical thinking skills such as problem-solving and decision-making skills. He was only able to use it to confirm the answers after the students had factorised quadratic expression manually. This suggested a superficial application of his knowledge of technology integration (TPACK). Other environmental factors could have accounted for this. However, it is important teachers acquired the needed professional knowledge to maximise the limited resources available in the school.

**Contextual influence**

At the beginning of the professional development, the teachers were eager to use technology to teach mathematics in their school, but there were a number of factors that hindered them. Five themes became central in their discussion. The themes are represented in Figure 5.3 as a proportion of the number of times they were coded in the entire document that was analysed.

![Figure 5.3 Teachers’ initial perceived hindrances to technology use](image)
As seen in Figure 5.3, at the onset of the professional development, the participating teachers indicated that limited knowledge and skills of technology, limited technology resources, limited training opportunities, limited access to ICT laboratory, and limited administrative support were the factors that fettered them from using technology:

The ICT facilities are not available. Also, we have not been trained in how to use technology in teaching (Gideon).

I can use MS Excel, PowerPoint and Word, but I am not confident using them. I have not been practising it (Sammy).

The technology itself is not available in our locality or environment ... The ICT department has the laboratory, but most of the gadgets that they were given are now spoilt ... We were given about 50 laptops, but after some few years all the laptops are out of use … There is only one projector in the school that is used by the ICT department. It is very difficult for the mathematics department or any other department to use it … Actually, when we hold departmental meeting, we put it forward through our head of department to lobby with the administration so that they provide us with a projector or a laptop at least one for the department to use, but anytime the budget is drawn and it was given to the school administration we don't see any way forward (Michael).

Joshua added that financial motivation could inspire the teachers to prepare technology-based mathematics lesson.

Motivation is also a factor. The teacher prepares the slides in the house. Most of the work is done in the house. So, if no motivation comes from the work done in the house, I will feel reluctant to use technology in the classroom. Here, teachers buy their own laptop computers and software. Teachers go for their own training. Though you are investing in yourself, if you are given external motivation, it will put us in a better position to deliver. Liquid motivation (money) is what I am talking about (Joshua).

From the comments above, it could be argued that some of the participant teachers were reluctant to access the ICT laboratory because they felt the place was purposely for the ICT department and ICT lessons. The school received 50 laptop computers from the government’s project, “The One Laptop per Child (OLPC)” in 2012. These computers got
broken within a short time. My observations and discussion with the ICT teacher in charge of the laboratory indicated that issues such as maintenance, quality of the items, and/or ventilation of the room might have contributed to the short lifespan of the laptops. My observations also confirmed the participant teachers’ assertion that only the ICT teachers used the ICT facilities and did so to teach students basic computer skills such as Windows and the Microsoft Office suites - Word and Excel. The mathematics teachers relied on their personal modem, and they bought their own data for internet connectivity.

With regards to professional training in the use of technology, the results of the study revealed that out of the eleven teachers only four (Mary, Sammy, Joshua, and Peter) indicated they were introduced to the integration of technology in mathematics teaching during their previous education. Regarding professional development while on the job, only Joshua indicated he had attended professional development related to the use of technology.

**Summary**

This section explored the dispositions of the teachers related to the use of technology prior to their engagement in the professional development. Overall, the results of the study indicated that the participant teachers were enthusiastic about the use of technology in mathematics. They anticipated that using it in the mathematics classroom could be exciting and it would make mathematics lessons enjoyable and comfortable. They were positive towards technology-related professional development because they believed it could help conduct mathematics lessons that will engage their students. However, the participant teachers’ knowledge of the way to integrate technology into mathematics teaching was observed to be low. The majority of the teachers (nine out of eleven) indicated they had never used technology in their classroom before. Limited knowledge and skills of technology, limited technology resources, limited training opportunities, limited access to ICT laboratory, and limited administrative support seemed to be the main contextual factors preventing the teachers using technology in the school. Even though most of the teachers had never used technology in their instruction, the beliefs they shared regarding its use seemed to reflect the key principles of effective mathematics pedagogy. For example, they believed technology could promote explorative, socio-cognitive, and facilitatory learning. In the next section, how teachers enacted effective mathematics pedagogy following their engagement in the professional development is presented.
Enactment of Effective Mathematics Pedagogy

A major focus of engaging teachers in professional development is to assist them to develop classroom practices that ensure effective teaching and learning (Fullan, 2003; Guskey, 2002; Whitcomb et al., 2009). Since the impact of professional development on teachers takes time, what this section examines are the typical features and identifying nuances of the complexities of enacting effective pedagogy in a GeoGebra learning environment. This was done with the purpose of providing a response to the critique that effective mathematics pedagogy is generic and underspecified (Franke et al., 2007; Jacobs & Spangler, 2017). Lesson plans, lesson episodes, post-lesson discussion, and interviews (pre and post) were used. The study identified 31 core practices when the teachers used GeoGebra to enact mathematics lessons. These core practices are distributed over the five central themes of effective mathematics pedagogy adapted for the study: creating a mathematical setting, useful mathematical tasks, mathematical discussions, mathematical connections, and assessment of students’ learning (see Chapter Four and Appendices M and N for detailed development of themes). As discussed in Chapter Four, some of the teachers’ actions were coded for multiple themes because there was overlap in the actions of the teachers across different categories of effective pedagogy. An attempt was made to include an example of dialogue (excerpt of one-on-one post-lesson discussion) of a representative sample of the teachers’ actions to underscore their evolutionary development of enacting effective mathematics pedagogy in the GeoGebra learning environment.

Creating a mathematical setting

As can be seen in Table 5.2, nine key practices the teachers demonstrated in setting up effective environments were creation of pre-planned documents, equipment setup, shared instructional objectives, clear instruction about the task on the worksheet, linking prior knowledge to new learning, using pictures of real-life scenarios to initiate mathematical dialogue, explanation of terminologies, addressing existing misconception, and attention for individual learning needs (see sample coding in Appendix N).

With regards to pre-planned documents, all the teachers created lesson plans (see Appendix O for a sample lesson plan), students’ worksheets (see Appendix P for a sample students’ worksheet), and GeoGebra artefacts to support students to explore specific mathematics concepts. During the third phase of the professional development, the teachers were able to
transfer the knowledge acquired during phase two into the lesson planning and enactment. This indicates valuable experience which helped them to develop professional competence related to the use of GeoGebra in mathematics teaching and learning.

Table 5.2 Teachers’ enactment of mathematical setting

<table>
<thead>
<tr>
<th>Effective mathematics pedagogy</th>
<th>Description</th>
<th>Core practices</th>
</tr>
</thead>
</table>
| Creating a mathematical setting    | It involves the skills and knowledge of the teacher to set up the learning environment to hook and sustain students’ interest and attention throughout the mathematics lesson. It involves the actions the teachers take before, during, and after the lesson. | 1. Preplanned lesson documents  
2. Equipment setup  
3. Shared instructional objectives  
4. Clear instructions about the task on the worksheet  
5. Linking prior knowledge to new learning  
6. Using pictures of real-life scenarios to initiate mathematical dialogue  
7. Explanation of terminologies  
8. Addressing existing misconception  
9. Attention for individual learning needs |

A close analysis of the lesson plans revealed that the teachers prepared high-quality lesson plans that made explorative use of GeoGebra; this aligned with their targeted grade level and topics in the mathematics curriculum. All the lesson plans included appropriate instructional objectives and clear introductory activities which inducted students into the lesson. Six teachers included in their lesson plans a step-by-step instructional guide of how the artefacts could be developed in the GeoGebra window. They indicated the instructional guide was for the purpose of reference for future use either by themselves or any other teacher could use the lesson when they were absent. They also structured the evaluative exercise to align with the instructional objectives. All the teachers included in their lesson plans a closure which summarised the salient concepts they expected their students to take home. Five teachers included expected solutions/responses to the tasks the students performed in their lesson plan documents. The solutions/responses were shown on the screen to aid students to compare their answer and it also enhanced teachers’ further elaboration. Table 5.3 shows the technologies the teachers adopted to create learning spaces for their students. The teachers prepared their lesson plans and students’ worksheet with Microsoft Word. The worksheets
were printed for the students to engage them in hands-on activities. The soft copy of the worksheet in the Word document provided a focus for whole class discussion, particularly when teachers wanted to emphasise a specific task. The GeoGebra artefacts included texts, diagrams, shapes, graphs, and animations which were designed in the GeoGebra window, to provide a material for presentation and exploration of mathematical concepts.

Table 5.3 Technologies the teachers adopted for their lessons

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Lesson Taught</th>
<th>Technology used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cynthia</td>
<td>Transformations (Year 1)</td>
<td>MS Word, GeoGebra, Calculator</td>
</tr>
<tr>
<td>Prince</td>
<td>Trigonometric ratios (Year 2)</td>
<td>MS Word, GeoGebra, existing animation from GeoGebra online community</td>
</tr>
<tr>
<td>Mary</td>
<td>Mensuration (Year 3)</td>
<td>MS Word, GeoGebra, existing animation from GeoGebra online community</td>
</tr>
<tr>
<td>Gideon</td>
<td>Quadratic equation (Year 2)</td>
<td>MS Word, MS Excel, GeoGebra</td>
</tr>
<tr>
<td>Sammy</td>
<td>Polygons (Year 1); Quadratic functions (Year 2)</td>
<td>MS Word, GeoGebra, Calculator, existing animation from GeoGebra online community</td>
</tr>
<tr>
<td>Jonathan</td>
<td>Circle theorems (Year 3); Topics in chemistry (Year 1)</td>
<td>MS Word, GeoGebra, Khan Academy, YouTube</td>
</tr>
<tr>
<td>Joshua</td>
<td>Travel time graph; Equation of a circle (Year 2)</td>
<td>MS Word, MS PowerPoint, GeoGebra, YouTube</td>
</tr>
<tr>
<td>Peter</td>
<td>Circle theorems (Year 3)</td>
<td>MS Word, GeoGebra</td>
</tr>
<tr>
<td>Michael</td>
<td>Trigonometric ratios (Year 2)</td>
<td>MS Word, GeoGebra</td>
</tr>
<tr>
<td>Martey</td>
<td>Area of square and rectangles (Year 1)</td>
<td>MS Word, GeoGebra</td>
</tr>
<tr>
<td>Bernard</td>
<td>Transformations (Year 1); Mensuration (Year 3)</td>
<td>MS Word, GeoGebra, existing animation from GeoGebra online community</td>
</tr>
</tbody>
</table>

Among the teachers, only Joshua made Microsoft PowerPoint presentations where he hyperlinked his diagrams and animations from the GeoGebra window. Also, Gideon constructed some of his graphs in Microsoft Excel and imported them to Microsoft Word documents. Although the professional development mainly focussed on GeoGebra, the
teachers who had prior knowledge in other technologies were able to integrate them into GeoGebra to maximise their instruction. This showed the possibility of GeoGebra linking to other technology applications such as Microsoft Word, Excel and PowerPoint. Also, it was not surprising some of the teachers infused different technologies (calculator, existing online materials, and Microsoft Office) into GeoGebra because the activities included in the professional development were designed with some degree of freedom where teachers could have possible scenarios to explore. For example, the teachers had the option to choose their own topics from the curriculum they believed they could teach with technology after they had been introduced to the basics of GeoGebra.

As Figure 5.4 shows, Cynthia was able to set out the GeoGebra window for her lesson on rotation of objects. She expressed that often students missed the formula; they were unable to determine which quadrant the image of the object would be in when it was rotated but she believed using GeoGebra could help the students to easily figure out the quadrant without doing any computation.

The following extract from her lesson plan (Figure 5.5) suggests that setting up the GeoGebra window in advance made her pedagogically oriented by predetermining how the technology could be used in her context (small group and whole classroom teaching), the
role she would play (facilitating), and the expectation from her students (exploring a mathematics concept). It was observed that these practices became more pronounced in her second lesson than the previous one during the teaching rehearsal stage.

In her first lesson, she was able to (i) revise students’ previous idea on how to plot points on Cartesian plane; (ii) correct students’ misconception on how to draw a polygon; and (iii) make the appropriate link between students’ previous knowledge and intended learning. For the clockwise rotation, she dragged the slider forward (rightwards) at intervals of 900. Students recorded their observations each time she dragged the slider and they were able to establish with few difficulties that when a point (x, y) is rotated through 900 clockwise the image point is (y, -x), 1800 clockwise is (-x, -y), 2700 clockwise is (-y, x), and 3600 is (x, y). However, using the same slider to navigate students through an anticlockwise rotation came with some challenges. She dragged the slider to read 3600 which brought the object to its original position. From there, she dragged the slider leftwards at intervals of 900 believing that the students could pick up the reverse form of clockwise rotation easily. The students could not see the link when she asked them to record their observations for 900 anticlockwise because on the slider it was reading 2700 (clockwise). This made her tell the students that 900 anticlockwise is the same as 2700 clockwise, making the students form this conclusion prematurely. Also, each time she dragged the slider leftwards she had to ask the students to subtract the angle they see on the slider from 3600 to obtain the actual angle for anticlockwise rotation. This provided some cognitive load on the students and it impeded the flow of conceptualising anticlockwise rotation.

In her second lesson these issues were resolved upon post-lesson reflection with her peers and expert support from the researcher. She created different sliders each for clockwise and
anticlockwise rotations. This offloaded students from routine computation and students had enough opportunity to do meaningful deductions from multiple scenarios. For example, in the second lesson she included more activities that enhance students’ engagement (see Figure 5.6). She provided higher order activities which expected students, in pairs, to apply the concept they had deduced to solve questions. This illustrates an extended development of her initial TPACK and thinking regarding how students could learn with GeoGebra. In the pre-interview before her first lesson with GeoGebra, she said: “technology can help individuals to learn on his or her own with constant practice, to solve challenging problems”. This quotation illustrates her strong understanding of the pedagogical effectiveness of technology prior to her engagement in the professional development.

![Figure 5.6 Students' task on rotation on rotation of object (Dorothy)](image)

The teachers explicitly acknowledged the need to make retrospective decisions when designing technology-based lessons by identifying what worked, or did not work, in the previous lessons. Thus, iteratively revising the lesson made teachers deeply consider their repertoire of representations and flexible teaching strategies in their PCK with technology.
For example, Gideon, Prince, Cynthia, Sammy, Jonathan, and Bernard realised from the first lesson they taught to their peers that the artefacts they developed did not fully help to achieve their instructional goal. In the second lesson, they included animations from GeoGebra’s online community to support their lessons. Gideon commented that it required teachers to have “reflective practice” (a term from him) to select examples that enhance students to build the concept. He designed a lesson to facilitate students’ conception of the effects of the parameters of quadratic equation. He indicated that the values he asked his students to record during the first teaching rehearsal did not give any clue as to how to deduce the formula for the equation of line of symmetry, \( x = \frac{-b}{2a} \). He added that the subsequent lesson worked because he needed to revise the values during his “reflective practice” and provided additional hints to enable the students to arrive at the formula with less difficulty:

I was able to do the introduction well as compared to the earlier lesson that I taught. I was able to use the GeoGebra to make sure everything was accurate, the effect of the parameter on the graph. I used the window to check whether what I was preparing was correct. Because of the reflective practice that I did before this lesson, I was able to choose values that made me finished the lesson faster than the first lesson. Also, I improved the worksheet and that one also helped (Gideon).

Similarly, Bernard reflectively talked about why he revised his diagrams in the GeoGebra window to prevent distortion in exploring mathematical concepts. He said:

When I introduced the students to observe something on the GeoGebra window, I saw I have more things on the screen than I needed. I saw somebody recorded something different from what I wanted them to record. For example, when I was talking about the cylinder, the cone too was on the screen. Some of the students recorded the volume of the cone when I wanted them to record the volume of the cylinder. Meanwhile, I have not gotten to that stage. So, I have to quickly remove some of the diagrams and show only the object I want them to identify to avoid distraction.
During the post-lesson interviews, several teachers (Gideon, Cynthia, and Joshua) recounted preparing mathematics lessons in the GeoGebra window as involving, but worth doing:

I was able to use the GeoGebra to make sure everything was accurate, the effect of the parameter on the graph. I used the window to check whether what I was preparing was correct (Gideon).

Though, during my preparation, I was using much time. When I finished preparing and I came to the class, everything is on point (Cynthia).

The teachers had additional responsibilities when they were going to teach with technology. As Martey shared after his lesson on area and perimeter of a rectangle:

… in our environment, we cannot rely on the electricity, so I had to come early to print my worksheets, do the necessary arrangement before the students come in. In the normal class, I don’t think about these (Martey).

Also, in a situation where the lesson was adopted from existing materials from online, the teachers had to do alterations to match their instructional purpose. As Joshua said:

… it [the lesson] was done by somebody … I have to add certain things to it, and remove the unwanted ones from it, put names that will be familiar with the students I am going to teach … to suit my instructional objectives.

There was evidence of the teachers being able to hook their students into their lessons through provision of explicit instruction on the worksheets, articulating the objectives of the lesson, reviewing students’ existing knowledge, using of real-life scenarios addressing existing misconception, and explaining terminologies. Each teacher used at least one of these strategies during the introduction stage of their lessons. It is important to acknowledge that the teachers repeated some of the strategies as the lessons unfolded. The variation in the way the teachers used these strategies were dependent on the nature of the topic they taught, the background of their students, and their own teaching style and experience. For example, in the following excerpt, Michael articulated the expectation of the lesson as a way to draw the attention of the students to what they were going to learn. Also, at the onset of the lesson, he made the students aware of what they were expected to do in the groups:
You realise that in the worksheet, we have activities. Before activity 1, there is an introduction there, it says that [he read the introduction]

In this lesson, you will be taken through activities that will help you explore the concept of trigonometric ratios: sine (sin), cosine (cos) and tangent (tan). It is expected that at the end of the lesson you will be able to apply trigonometric ratios to calculate distance and height of a given triangle. In this task, you are going to work in groups. You will record your observation and draw conclusions based on your observation (Michael).

In Gideon’s case, he provided a real-life scenario of a quadratic equation (see Figure 5.7). As illustrated in the following excerpt, he then asked questions to elicit responses about what the students were going to learn. Gideon sequenced his questions with the hope that the students would come out with terms such as parabola, or trajectory, or quadratic curve. However, the students were not able to come out with these terms automatically; they only said the paths represent a curve. The word ‘curve’ the students mentioned gave Gideon a good starting point for his lesson. He proceeded by saying such a curve is called parabola or trajectory and it is represented mathematical equation \( y = ax^2 + bx + c \) and this equation is called a quadratic curve. It can also be seen from the excerpt that he provided positive reinforcement such as ‘okay’, ‘great’ to motivate students’ efforts. An important observation from Gideon’s introduction was that the students could not get the terminologies right at the onset of the lesson, but the pictures he provided served as a means which they could practically associate quadratic equation to.

![Figure 5.7 Illustration of real-life application quadratic function (Gideon)](image)

Gideon: I have three pictures for you [he projected it on the screen]. What can you say about each of them?
S1: The first one is someone playing a basket to score.
S2: A motor rider negotiating a sharp curve.
S3: A ruler is bent.

Gideon: Okay. How does the path of each object you’ve described look like? Can you predict any mathematical equation which represents these paths?
S4: Each path is forming a curve.

Gideon: Great. These types of curves have a special name. What is it?

Similarly, in Sammy’s class, he wanted his students to appreciate the real-life phenomenon of polygons so at the introductory stage of the lesson, he provided pictures showing the symbols of traffic regulation (for example the STOP sign and Give Way sign) as an example of a real-life application of a polygon. The shapes of the STOP and the Give Way signs embody the pentagon and triangle respectively. This initiated the students to think aloud about other real applications of a polygon. They cited examples of structures (buildings) in their community that had various shapes of polygon. Sammy recounted at the end of the lesson:

… with the animations and diagrams on the board, the students were able to see how polygon is. So that one alone made them understand that it is not something miraculous or something which does not exist. It brought reality and meaning to the learning.

From these illustrations, the technology played a complementary role in selecting multiple real-life scenarios to support students’ mathematical communication. For example, the pictures facilitated the teachers to ask questions that made the students cognitively explore the mathematical connections in real-life at the onset of their lessons.

Michael and Joshua acknowledged how, through the professional development, they were able to use technology to support differentiated learning so students with unique learning behaviour or difficulties were assisted. Michael talked about how his new instructional approach had changed the behaviour of his absentee and mathematics phobia students. He indicated that from the very first time that he used GeoGebra and the activity-based worksheet to engage students to explore the trigonometric ratios, some of them had become
enthusiastic and their attendance in class seemed to be enhanced. He recounted that students no longer dozed off during his instructions:

I have realised that students pay more attention because they are able to understand things better … Those who were not formerly coming to the class because they were afraid of mathematics are now participating fully in mathematics. Before I was introduced to technology, my teaching has been more of theory than practical. So, students do not participate fully. Some students even doze off. Now, because of the technology students are willing to come to class (Michael).

From Joshua’s perspective, contextualising technology to meet the needs of their students was key in promoting an effective learning environment for the students. At the end of the professional development, he indicated in an interview how the technology could be used to support the weaker students. It can also be learned from this excerpt that Joshua not only used the technology to address individual learning differences, but also to resolve students’ misconceptions in learning distance-time graphs:

In the classroom, I am able to develop certain things to help the weaker students to catch up with whatever the brilliant ones are doing. If it is more visual it helps the weaker students to visualise the concept than without it. This makes all of them move forward. In my lesson, they all saw the car moving and when it stopped too, they all saw it. This makes it easier for them to make meaning out of it. Initially, some of them think when the car stops, its time to stops. So the concept of the distance covered by the car in relation to time was difficult for them to grasp especially when the car is at rest or return to its original position. This time they see it physically, the clock is moving even when the car comes to rest.

This section explored how teachers enacted effective creation of a mathematical setting. The results showed evidence of knowledge and skills of the teacher to set up the learning environment to hook and sustain the students’ interest and attention in the mathematics lessons conducted with GeoGebra. From the results, it is argued that adequate preparation
marks the beginning of successful enactment of effective pedagogy. For example, the teachers needed to make advanced decisions about the content to be taught, students’ learning, and how to adopt GeoGebra. From the conversation above, the preparation did not only help them to rethink their instructional approach, but it also made it easier and faster to achieve their instructional objectives. It was not surprising the teachers acknowledged the extra efforts needed to create a productive technology-based learning environment. They needed not only to rehearse the lesson artefacts they had developed in the GeoGebra window, but they also had to develop the skills to set up the projector and laptop in the laboratory by checking the resolution to enhance students’ viewing. Also, the comments from the teachers suggested they seemed to have developed an awareness of the potential of GeoGebra in teaching and learning of mathematics. They recognised GeoGebra as a tool which provided dynamic and multiple representations of both graphic and algebraic expressions that enhanced students to visualise abstract mathematical concepts. Another possible reason for the change in the behaviour of the students in Michael’s class could be that using the worksheet might have intensified his supervision in the class. This is because using the worksheet, he needed to regularly monitor the responses the students had written on it for necessary correction and improvement. As a result, students had no option to doze off, but had to focus on the learning task. Also, the opportunity for the students to learn under this environment might have made them more curious to learn new things using this approach which could have contributed to their engagement in the class.

**Useful mathematical tasks**

The results in Table 5.4 shows five key practices the teachers demonstrated in providing useful mathematical tasks to their students (see sample coding in Appendix N).

<table>
<thead>
<tr>
<th>Effective mathematics pedagogy</th>
<th>Description</th>
<th>Core practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Useful Mathematical tasks</td>
<td>Involves how the objectives and activities included in the GeoGebra-based lesson engage students in exploring and understanding mathematical concepts, procedures, and/or relationships.</td>
<td>1. Visualising mathematical concepts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Recording</td>
</tr>
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<td></td>
<td></td>
<td>3. Calculation</td>
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<tr>
<td></td>
<td></td>
<td>4. Predicting</td>
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<tr>
<td></td>
<td></td>
<td>5. Constructing new ideas</td>
</tr>
</tbody>
</table>
None of the teachers used GeoGebra in an expressive mode where students created their own artefact to express their mathematical ideas. The limited resources in the school could not have supported this instructional approach. However, the teachers adapted GeoGebra in an exploratory mode where they predesigned the lesson artefacts to match their instructional objectives in both algebra and geometry. The lessons were mainly teacher-led where they used the demonstrations in the GeoGebra window and activities on the worksheet to engage students in mathematical task that supported them to visualise, record, calculate, predict and construct mathematical ideas. For example, nine teachers used GeoGebra to engage students in geometric thinking. The following are excerpts of activities they included in their lesson plan:

Based on your observation, write the formula connecting surface area \( (\text{SA}) \), curved surface area, circular ends area and height \( (h) \) of the cylinder and the cone (Bernard).

Using the illustration on the GeoGebra window, let students in small groups observe the values of angles subtended by chord \( AB \) as the points \( C, D \) or \( E \) are dragged and let them write down their observations (Jonathan).

Observe the polygons shown on the GeoGebra window and record your observations in the table below. From the table in (i) write down the number of triangles in a given polygon in terms of \( n \), (ii) write down the formula for the sum of interior angles of a polygon \( (S) \) in terms of \( n \) (Sammy).

In these lessons, the teachers provided guided exploratory learning where students made geometric deductions from a sequence of activities including recording, observing patterns, drawing, guessing, calculation, and conjecturing. The affordance of the animation in GeoGebra facilitated the students to identify the connection between the area of a rectangle and the curved surface of the cylinder (see Figure 5.8). The dynamic development of the cylinder from its nets in the GeoGebra window helped Bernard to assist the students to realise that the total surface area of a cylinder is the sum of the area of the two circles plus the area of the curved surface \( (2\pi r^2 + 2\pi rh) \). Bernard shared this when he was asked to comment on the lesson he taught.
The animation of how the cylinder is formed from its net catch their attention. In the actual classroom, you wouldn’t get that. Being able to see how it opens enables the students to understand what we are talking about. That picture was effective for teaching and learning (Bernard).

In the pre-interview, he lamented his inability to use enough teaching and learning materials in his instruction. He said the materials were not available and it was also difficult to make cut-outs from concrete objects such as cardboard and empty tins. He said after the professional development:

There are so many topics in our syllabus which is very difficult to get teaching material to teach. Using technology has at least bridge that gap (Bernard).

He added that using technology made the thinking of students more visible because they had the opportunity to explore the concepts in a real setting:

Now I am able to create something for the students to see. I give them the opportunity to explore by themselves. They do more of the thinking than I do. Students are able to get the concept very fast (Bernard).

These illustrations suggested some observable improvement in the way Bernard enacted effective mathematics pedagogy. He was able to gather useful mathematics resources from online and adjusted them to suit his instructional objectives. Similarly, in Joshua’s lesson, he wanted his students to draw distance-time graphs for given scenarios and subsequently

Figure 5. 8 Surface area of a cylinder (Bernard)
use the graph to calculate distance travelled, total time taken, and the average speed of the moving object. The following is an excerpt of how he conducted the lesson:

Joshua: Today we are going to look at travel graph. I expect that by the time we finish the lesson, you will be able to draw and interpret distance-time graph. To begin with, let look at the following scenarios. [He projected animation from the GeoGebra window showing distance-time graph of four different modes of transport: a car, someone who is running, someone who is walking with her dog and a cyclist (see Figure 5.9). He clicked on the start button and allowed each object to travel for one hour then he clicked on the pause. He asked the students to match each colour of the graph to the corresponding mode of transport. He further asked to state the reason for their decisions].

S1: The blue graph corresponds to the car. The red is for the cyclist, the green is for the runner and black is the one who is walking. The reason is that the car runs fastest, followed by the cyclist then the runner and the one who is walking.

Joshua: Who else had different observation? [Most of the students confirmed a similar observation].

Joshua: Okay. In the normal situation, we expect that within the same time the car should travel the longest distance than any of these modes of transport. For example, if you are coming to school this morning and you all started from the station, the one who came with the car will
definitely reach here than any of the mode transports indicated. Now, look at the graph. What distance did each mode of transport covered within an hour?

S2: The car travelled 50 km within one hour.
S3: The cyclist about 23 km
S4: Runner 6 km
S5: Walking 3 km

Joshua: Okay, I am going to show the distance for each mode of transport so that you can compare what you have estimated. [Joshua reset the animation, then at each click he asked the students to discuss their observations. He drilled the students on the distance-time relationship by asking them questions based on their observations].

From the excerpt, he provided a good start by asking students to talk about multiple travel graphs presented in GeoGebra. These made the students focus on key mathematical ideas for drawing travel graphs. For example, observing the car moving and its graph drawn simultaneously supported the students to realise that when the car is moving forward, they need to draw a line which slopes positively (\(\uparrow\)) to cover the distance and time it travels. When the car stops or is at rest, they need to draw a horizontal line (\(\cdots\)) to cover the duration. When the car returns, they need to draw a line which slopes negatively (\(\downarrow\)). Making this connection was conceptually useful to facilitate the students to draw and interpret their own graphs when different scenarios were presented to them. In contrast to the first lesson he taught during the teaching rehearsal, he provided the scenarios and then guided the students to draw the graphs. He indicated in an interview after the second lesson that the first approach made it hard for the students to get the concept, especially when the object was at rest or when it was returning to its original position. He said it was useful to ask them first to interpret graphs of different scenarios before engaging them to draw. Joshua’s assertion seemed to hold some substance because engaging the students to talk about the graph could generate divergent ideas from both students and teacher. Terminologies associated with the concept could evolve through the preliminary discussion. Joshua added in the second interview after the actual classroom teaching:

I asked them to draw the distance-time for the journey between the school and their homes. They drew interesting graphs. Now we are making the mathematics
more meaningful for the children to learn. We are bringing the world to the classroom making it more practical. The abstract nature of it is gradually going down. When I finished today’s lesson, they came to me for the sites I got the materials from. This means they want to learn more on their own.

His success in the effective use of technology was a motivation for his students to engage in mathematics learning. As can be seen in his comment, students were enthusiastic to do extended reading. For example, it was observed that students went to their teachers and wanted to know how GeoGebra operated after each lesson. They were curious about how they could use it by themselves. Jonathan made a similar observation that his students came to him with their pen drives for the animations he had downloaded for his lessons. In his words, after the professional development:

I am surprised to see them after each lesson with their drives wanted to collect the animations. They are happy because the lessons seemed more practical to them. Those who formerly don’t usually talk in class you see them talking this time. They are really getting into it (Jonathan).

From Jonathan’s submission, students’ engagement in mathematics learning had increased because students who “formerly don’t usually talk in class you see them talking this time”.

In the following interaction, for example, Sammy used GeoGebra to support students to make sense of the sum of interior angles of a polygon. He first asked the students to complete a table based on their observations from GeoGebra (see Figure 5.10).

![Figure 5. 10 Sum of interior angles polygon (Sammy)](image)
Sammy: Now take your worksheet. You are going to work in pairs to complete the Table in your worksheet. As I drag the slider, observe the polygon shown on the GeoGebra window and record your observations. [After the students had completed the table, he asked questions to consolidate the students’ understanding of the formula they had written in their worksheet].

Sammy: So suppose we have a polygon which has 13 sides. How many triangles are we going to get from it?

S9: That will be thirteen minus two. So we will get 11.

Sammy: That is good. What about a polygon with 30 sides?

Students: 28.

Sammy: Okay. Now, a certain polygon has 17 triangles in it. How many sides have that polygon? [When Sammy reversed the question, not all the students were able to give the correct solution. Some of the students gave the answer as 15 and others gave the answer as 19. It is apparent that the students who got the answer wrong had not fully developed the correct language hence they were not able to differentiate between the number of triangles in a given polygon and the number of sides of the polygon. They, therefore, applied the relation \( n - 2 \) wrongly as in the case of S10].

Sammy: Those who had 15, how did you arrive at the answer?

S10: Seventeen minus two is 15.

Sammy: Okay, we shall later see if your answer is correct from the GeoGebra window. Those who had 19, what is your explanation?

S7: We had nineteen because we had already been given the number of the triangle to be 17 so to know the number of sides of the polygon, we need to find the number that when we subtract 2 from it will give us 17. That number is 19. [To rectify students’ difficulties, Sammy projected a polygon with 19 sides (Nonadecagon) and asked the students to count the number of sides of the polygon and the number of triangles in it].

Sammy: As you can see there are 17 triangles in it and that is what I asked. There are 19 sides of the polygon. So when you have the number of triangles and want to know the numbers of sides we add two to it. But
when we have the number of sides of the polygon and we want to know the number of triangles in it we subtract two from it. So this question could have set out your equation as

\[ n - 2 = 17 \]
\[ n = 17 + 2 \]
\[ n = 19 \]

It can be seen in his concluding comment that GeoGebra filtered the dialogue and he was able to elaborate on both the question and the solution he wanted his students to arrive at. Because the polygon had already been designed in a GeoGebra window, it afforded him a quick opportunity to do remedial instruction to consolidate the concept the students had established. Drawing a polygon with 19 sides by hand would have been cumbersome and clumsy on the chalkboard. Using GeoGebra, an appealing shape of polygon with the corresponding name was generated when the slider is dragged. The promptness of the feedback from GeoGebra empowered the teacher to ask many questions anytime the slider was dragged. Also, the zoom in and out features in GeoGebra facilitated viewing and counting of the number of triangles in, and numbers of sides of, the polygon. His reflection during one-on-one interaction after the lesson suggested his strong feelings about how he was able to use GeoGebra to scaffold students’ exploration in mathematics learning. His experience in teaching with GeoGebra seemed to strengthen his belief that students learn productively when they are supported to explore mathematics concepts on their own:

When it comes to memorising the formulas, they see it as something which doesn’t exist. So it does not become part of them. So they only memorise it. If that becomes the issue, you will not see the impact of mathematics in their everyday life. I believe if I am able to let them understand the concept, they can actually see the reality using the tool [GeoGebra]. With that, they will not forget and they can apply it in everyday life. I would rather prefer that they themselves discover than me telling them. With that exploration, it will broaden their scope and they will become more knowledgeable (Sammy).

The following quotations are consistent with this claim that the teachers combining GeoGebra and worksheet in a lesson helped in facilitating the students to construct mathematical ideas socially:
It [technology] has helped me to assist students to come out with their own generalisation … I gave the students a worksheet which contains activities. This engaged the students and it makes participation 100%. Everybody is involved. Using the technology, the students are given more task to perform (Cynthia).

Mostly, when we are using technology, the lesson is more practical, everybody’s concentration is captured. (Michael)

I asked the students to work in pairs today. The worksheet helped the children to be active in the lesson. They were involved in every activity on the worksheet. (Gideon)

The reflections from Michael, Cynthia, and Gideon suggested that GeoGebra and the worksheets initiated social interactions which promoted engaged learning. Students (i) made active contribution in the learning activities; (ii) demonstrated positive attitudes toward mathematical task; and (iii) got cognitively involved in the construction of mathematical concepts. In other words, effective pedagogy was in play because the students were able to enact these activities. For illustration purposes, Figure 5.11 presents the worksheet and GeoGebra window developed and used by Joshua, during his second lesson in the second-year classroom. In this lesson, the students explored, via GeoGebra, the concept of the equation of a circle. Joshua invited students to work in groups (two or three) by asking them to complete the tasks on the worksheets based on their observations and recordings from the GeoGebra window. By doing this, the students were given the time to think and reflect on their solutions before they were invited for presentation.

Figure 5.11 Equation of a circle (Joshua)
Considering the limited resources available in the school, this illustration strengthened Joshua’s belief that appropriate use of GeoGebra could be highly motivational and help create an engaged learning environment. It also helped him to build more realistic expectations about what the integration of GeoGebra in mathematics classrooms entailed in practice. In the post-interview, he shared that “this programme has also given our students some hope in the mathematics that they are learning because the abstract nature of our teaching is gradually going down”. He added “The students brainstorm and bring out their idea. My role is just to guide them to ascertain whatever concept I want them to develop, using the tool makes it simpler”. The flexibility with which he facilitated learner discussion and collaboration in the observed lessons was evidence of his strong belief in students constructing mathematical concepts for themselves, and not being spoon fed by the teacher.

Sammy noted that his students were trying to complete the tasks on the worksheet even when the instruction time was over, and they were expected to go for recess. Thus, as students get stuck in exploring mathematical tasks, their interest and perseverance in learning mathematics were enhanced:

Some of the students are reluctant to leave the class when the lesson is over. Formerly, you will not observe this. Now they are challenged, and they feel they should complete the tasks on the worksheet before they go for a break (Sammy).

**Mathematical discussion**

Engagement of students in mathematical discussions was a common feature across the lessons the teachers enacted. The results in Table 5.5 illustrate teachers’ practices of effective mathematical discussion. These were consolidating ideas students have constructed through verbal and written response, correcting misconceptions associated with the new concept, posing mathematical questions, group/individual presentations, revoicing, and giving students autonomy to apply the concept (see sample coding in Appendix N). The following excerpts are illustrative examples of how the teachers enacted these practices and complexities associated with orchestrating effective mathematical discussion.
Table 5.5 Teachers’ enactment of mathematical discussion

<table>
<thead>
<tr>
<th>Effective mathematics pedagogy</th>
<th>Description</th>
<th>Core practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical discussion</td>
<td>It involves creation of learning environment to facilitate classroom dialogue which emphasises on mathematical argument where conclusions are reached through agreement between students and teacher.</td>
<td>1. Consolidating ideas students have constructed through verbal and written response</td>
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<td></td>
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<td>2. Correcting misconception associated with the new concept</td>
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<td>3. Posing mathematical questions</td>
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<td></td>
<td></td>
<td>4. Group/individual presentation</td>
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<td></td>
<td></td>
<td>5. Revoicing</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Reword student’s solutions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Use GeoGebra to confirm student’s solution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Use GeoGebra to elaborate concepts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6. Gives students autonomy to apply the concept</td>
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</tbody>
</table>

In one of the activities in Martey’s lesson, he guided his students in a whole class discussion to arrive at a rule for drawing different rectangles which have the same perimeter. In particular, the students were tasked to draw three different rectangles that have perimeter 20 m (Figure 5.12). The following interaction took place.

Martey: Did have any trick for doing it? I mean three different rectangles which have perimeter, 20 cm.

Group 1: We were able draw two of the rectangles: 5 and 5, 6 and 4.

Group 2: Ours have dimension: 2 and 8. We are still thinking.

Martey: Good effort. Keep thinking.

Figure 5.12 Students’ worksheet on area and perimeter of a rectangle (Martey)
Group 1: We have gotten another one, the length is 7 and breadth 3. [Martey list the dimensions the students have provided on the board.]

Martey: We now have 5 and 5, 6 and 4, 7 and 3, 8 and 2. I think you can have another one too. What do you think?

Group 1: Then 9 and 1.

Martey: When you look at this pair of numbers carefully, you will see some pattern.

S3: The sum of the length and the breadth is 10.

Martey: That is right. Why the sum of the length and breadth is 10 and not any other number?

S8: The perimeter is 20. So, when we divide 20 by 2, we have 10.

Martey: We have to divide by 2 because the perimeter is given as \(2(l + b)\). So, anytime you are given the perimeter and you want to determine the dimensions of different rectangle, you have to divide it by 2 first. Then you think about pairs of numbers their sum will give you the result you obtained after the division.

In this excerpt, both the teacher and students seemed engaged. In line three, students acknowledged the struggle they were going through, and encouraged themselves by saying “we are thinking”. Martey’s inquiring attitude in line eight invited the students to develop ideas by formulating a rule for determining the dimensions of a rectangle when its perimeter is given. He rewarded students and encouraged them to “keep thinking” when the task seemed harder for the students. The teacher asked further questions for students to explain how they got the answer (line 10). He revoiced and edited students’ responses in the closing remarks (line 12). However, the conversation would have been richer if he had provided the opportunity for students to critique the solution from each group. It was observed that the kind of argument that went on was mainly within groups rather than between groups. The groups were not subjected to public (whole class) scrutiny where divergent ideas could have evolved. The excerpt also suggests that the teacher had a predetermined solution to the question and as soon as the students were able to arrive at it, there was no need to probe them further. The teacher did not invite any alternate solution from the students. Like all of the teachers, this teacher mostly selected groups they had pre-rehearsed the solution with for presentation and whole class discussion.
The following excerpt is a one-one-interaction Michael had with his student on a task involving trigonometric ratios:

Michael: What have you written?
Student 4: \( \sin 30^\circ \)
Michael: What did you write for \( \sin 30^\circ \)?
Student 4: \( \sqrt{3} \) over …..
Michael: Let’s look at the diagram. What is \( \sin \theta \)?
Student 4: Opposite over hypotenuse.
Michael: Now let look at \( \sin 30^\circ \). Where is the opposite?
Student: Here [pointing at 1]
Michael: Good. Write it.
Student 4: [She cancelled the one she had earlier written and wrote 1/…]
Michael: Where is the hypotenuse?
Student4: Here [pointing at 2]
Michael: What is \( \sin 30^\circ \)?
Student 4: one over two.
Michael: Good. You are now talking.

Figure 5. 13 Sample solution on a task involving trigonometric ratios

In this excerpt, Michael was giving individual attention in addition to the group and the whole class discussion. From the conversation, Michael realised that his student had begun the solution wrongly by writing \( \sin 30^\circ = \sqrt{3}/2 \) (line four). It was likely the student would wrongly complete the task by writing either \( \sin 30^\circ = \sqrt{3}/1 \) or \( \sin 30^\circ = \sqrt{3}/2 \). Michael was not patient enough to wait for the student to complete the task and he interrupted. Since it was the first time the student was working independently after the group work, it was possible she needed time to assimilate what she learned from the group members. She could have changed her solution or sought for assistance from her group members. Also, Michael pitched his voice when he asked, “What have you written?” indicating he was not too pleased about the effort of the student. Mathematics is a human endeavour so students’ ideas should be praised and used as a starting point for instruction. Michael could have asked the student
to explain her solution. This might have given him the opportunity to appreciate the misconception of the student and further use it to correct the student. Though she knew that \(\sin \theta\) is opposite divided by hypotenuse (line six), conceptually she was struggling to identify correctly the opposite and hypotenuse sides from the diagram. Though Michael succeeded eventually in helping the student come up with the correct solution, it is likely that this particular student would go home without the concept properly understood. The majority of the students in his class were able to perform this task, however, Michael could have used GeoGebra to reconsolidate the students’ understanding, particularly when the orientation of the angle of interest (\(\theta\)) was changed. Also, Michael’s closing comment “…You are now talking” (in the Ghanaian context, it implies what the students was doing initially was meaningless) demonstrates limited social support for the student’s achievement. Such a comment has the tendency to hurt student’s feelings and it could disengage him or her from learning activities.

In Peter’s lesson (see Figure 5.14), he expected his students to work in groups to explore the properties of a circle. The following interaction took place after he had successfully introduced the students to the various parts of a circle.

![Figure 5. 14 Circle theorem (Peter)](image)

Peter: If you consider this part of the circle [pointing at the major segment], it is the major segment and the other part is the minor segment. Is that
okay? And so from the first property of the circle, we are saying that if you have a chord and that chord subtends an angle at the circumference. Provided all the angles are within the same segment, then it means that the values of the angles should be the same. Let’s move the angle to the other segment and see whether the values will be the same or not. Let’s move the point B [he drags point B while the students observe the changes in the angles in the minor and major segments]. So, you see within the same segment, you realised that the values remained the same until we moved it to the opposite segment. So, what we have is the first property of the circle. [One of the students made this observation when Peter was dragging the point A or B on the circle].

S1: Sir, I have seen that as you keep increasing the length of the chord, the angles at the same segment of the circle increase but they are the same. [Peter re-echoed the student’s observation and further used the GeoGebra to confirm the observation for all the students to see].

Peter: Okay, let’s see what he trying to say. He is saying that as the length of the chord increases the angle formed at the circumference by the same chord also increases. [He drags the point A to increase the length of the chord].

Peter: Look at the angle. What angle do we have there?

Students: 41 degrees.

Peter: Good. Now let drag or increase the length of the chord and see what happens to the angle. [He drags the length and continued to say that] As I increased the length of the chord the angle increases. So what he is saying is right. So that is a very nice observation. Clap for him.

Students: Applauded. [Peter then moved on to the second activity which explores the relationship between the angles formed at the centre and circumference of a circle].

Peter: Okay. Now I am going to move point B so that I alter the length of the chord. Observe and record the corresponding values of the angles at the centre and at the circumference. Is that okay? [Peter did the first drag. Paused and asked the students].

Peter: What angle do we have at the centre?
Students: 120 degrees.

Peter: What about the angle at the circumference?

Students: 60 degrees

Peter: Record that.

Peter dragged the point for a number of times and asked the students to record their observation at each time he dragged the point.

Peter: What relationship can you draw between the angle at centre and one at the circumference?

S5: When we double the angle at the circumference, we will get the angle at the centre of the circle.

Peter: Any other observation?

S6: Twice the angle at the circumference is the angle at the centre.

Unlike the first activity where he told the conclusion to the students and further used GeoGebra to confirm it, in the subsequent activities he asked questions to engage students in observation, recording their conclusions. One possible reason might have accounted for why Peter overly explained the first property to the students then asked them to deduce. In the first lesson he taught to his peers, he did that section well, only he was critiqued for not being able to explain the terminologies relevant in exploring the concepts. For example, he did not explain explicitly the differences between a chord and a diameter. It was during peer discussion after the lesson he conceptualised that a diameter is a special chord which passes through the centre of a circle. In the first teaching, he was also using the term ‘produce’ instead of ‘subtend’. So, he was usually heard saying “the chord produces an angle at the circumference” instead of “the chord subtends an angle at the circumference”. In the actual class, he took his time to explain all parts of the circle to the students. He adequately explained the terminologies. So, from his previous experience of not being able to explain the terms might have accounted for why he demonstrated the first property to the students without engaging them. Despite that he provided most of the information to the students, particularly during the first activity, where a student in the class made an interesting observation. In the first activity, Peter wanted his students to establish that angles subtended at the circumference by the same chord or arc are equal. As he dragged the chord in the GeoGebra window, a student observed that as the length of the chord increases, the angles in the same segment of the circle increase but they are the same. Peter acknowledged during the post-lesson interview that the observation the student made was not in his repertoire of
mathematics knowledge until that day. He added that such an observation would never have been made in the normal classroom because of the static nature of the chalkboard. The inference from this illustration in Peter’s class suggests the potency of technology for students’ learning. The dynamic nature of it provides opportunity for students to look at mathematical concepts from different perspectives.

Like Peter, all the teachers exhibited fluctuations in their enactment of effective mathematics pedagogy. Using technology to mediate mathematics instruction was a new area for the teachers and therefore for them to switch to this new approach probably may take time to develop. Bernard, for example, conceded:

I remember when I taught during the peer teaching, I did a lot of mistakes especially during the first minutes of my representation. Trying to leave the old way of my teaching to the new one, I sometimes forget myself. I have to do it continuously so that it will be part and parcel of me.

Even when teachers had developed rich tasks on the worksheet to engage students in groups to construct their own knowledge, they sometimes forgot, and they ended up telling the concept to students. It was observed that the teachers needed constant encouragement to make judicious use of the worksheet. Giving students the worksheet and telling them “we are going to work in groups” is insufficient to maximise the potential use of the worksheet. It was observed that in the cases where roles (such as group leader and secretary) were not properly assigned to the students, they got disengaged, especially as the level of the activities got difficult.

**Mathematical connections**

A common feature across all the lessons was that the instructional activities progressed in order of difficulty, where students used the idea they had developed in the previous activity to solve challenging task in the subsequent activity. As can be seen in Table 5.6, six key practices were apparent in the way the teachers enacted mathematical connection in their classrooms (see sample coding in Appendix N). Six teachers (Michael, Sammy, Bernard, Joshua, Gideon, and Martey) created scenarios that enabled students to link newly acquired concepts to real-life phenomena and vice versa.
Table 5. 6 Teachers enactment of mathematical connection

<table>
<thead>
<tr>
<th>Effective mathematics pedagogy</th>
<th>Description</th>
<th>Core practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Connections</td>
<td>It involves how the teacher engages the student to use unrehearsed approach or pathway to generate multiple solutions for a problem. It includes the activities that probe students thinking and expansion of mathematical knowledge to real-life situation.</td>
<td>1. Repose mathematical question</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Extending concepts learned to new contexts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Use GeoGebra to emphasis real-life phenomena</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4. Reverse thinking</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5. Multiple solutions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6. Risk taking</td>
</tr>
</tbody>
</table>

Joshua, Bernard, Sammy, Peter, and Martey were observed to use GeoGebra and other technologies such as YouTube and Khan Academy to create more learning opportunities for students to construct and redefine mathematical concepts. For example, Martey used questions that started with solutions to engage students in geometric thinking. For example; (i) Draw on the grid sheet three different rectangles which all have an area of 24 cm². (ii) Draw on the grid sheet three different rectangles which all have a perimeter of 20 cm. He generated a grid sheet from the GeoGebra window and printed it for the students as an activity worksheet. In this activity, he was able to immerse students into reversed thinking where they used the solution to look for related mathematics concepts connecting geometric drawing and algebraic symbols. This generated argument, prediction, and formulation of mathematics concepts among the students. Students took risks by making correct and incorrect computations/decisions of drawing geometric figures before they arrived at the conclusion. Martey used the animated rectangle he generated in the GeoGebra window to clarify students’ doubts and misconceptions during the whole class discussion (see Figure 5.15).

![Figure 5. 15 Area and perimeter of rectangle (Martey)](image)
Though the lesson could have been conducted without technology, where the students could use their graph books or sheets, he used the affordance of GeoGebra to simultaneously produce multiples shapes of rectangles and corresponding values of the area and perimeter. This aroused and engaged students’ attention throughout the lesson.

In another activity in Martey’s lesson, students developed connections between different mathematical ideas. In this task, students were expected to calculate how much it would cost a gardener to control the moss on the lawn (see Figure 5.16). Since not all the field is covered with lawn, the students needed to geometrically figure out and calculate the area required by applying the formula they had learned. They also needed to borrow ideas from the concept of proportion to enable them to calculate the cost of maintaining the lawn. It was observed that as students navigated through the problem, they shared ideas by explaining the multiple steps they used in solving the problem. The discussion generated from this task suggested the students were deeply engaged with the concept of area and perimeter of a rectangle.

![Figure 5.16 Students’ task on area and perimeter of a rectangle (Martey)](image)

The connections which students established between the pictures of real situations and mathematical concepts were noted as extending their knowledge in mathematics.

In different lessons, Sammy and Gideon used pictures from real life to support students to extract mathematical concepts. As discussed earlier, in Gideon’s class, he let the students predict the mathematical equation that could model the path of basketball aimed at scoring, a motorbike rider negotiating a trail, and a meter ruler which was bent. The virtual image he provided initiated a discussion which let the students appreciate the practical representation of parabolic graphs such as the quadratic function.
Martey commented during the post-lesson discussion that GeoGebra served as dynamic visual manipulative tool which provided a rich environment for students to make connections between the geometric and symbolic representations of a rectangle:

> With the lesson prepared with the tool [GeoGebra], everything was on point. The length, the breadth, the area and the perimeter are connected. So, whenever any of the vertices was dragged, there were corresponding effects on the length, the breadth, the perimeter and the area. That alone made the lesson interesting (Martey).

In Bernard’s lesson, he wanted his students to apply the concept of volume of a cylinder that they had learned to calculate the volume of the metal used to make a pipe whose height was 10 cm, internal radius 2 cm and external radius 2.4 cm (see Figure 5.17).

This task challenged the students. Initially, the students were not successful in this task; very few made limited progress. Those who attempted it performed a single calculation using the formula $\pi r^2 h$ and they got lost on their way about which of the radii to use. They could not use geometric thinking to recognise that the pipe was hollow, and they needed first to calculate the area of the cross section of the pipe (which is the area of the ring). When the teacher realised this limitation, he used GeoGebra (see Figure 5.18) to provide a pictorial
hint which prompted the students to think both geometrically and algebraically. The following interaction that took place:

Bernard: When you look at the diagram. There are two circles. So, what should we do to get the cross section of the pipe which is also the area of the ring [he pointed at the cross-section area emphasising the hollow portion is not needed]?

Students: We can subtract.

Bernard: Good. Subtract which one from which?

Students: Subtract the area of the smaller circle from the bigger circle.

Bernard: That is great. If the radius of the bigger circle is $R$ and radius of the smaller circle is $r$. So from the diagram, what is the area of the bigger circle?

Students: $\pi R^2$

Bernard: Good. What about the smaller one?

Students: $\pi r^2$

This hint served as a reinforcement for the students to identify the salient information needed to solve the problem. The interaction promoted flexible and creative application of meaningfully learned mathematical process, skills, and knowledge. Though Bernard did not elicit elaboration from the students as to why they needed to do the subtraction, they were able to go beyond the routine competencies where they only substituted values into the formula and compute it. The students were then able to visualise that for them to calculate the volume of the metal used to make the pipe, they had to calculate the cross section of the
pipe, $\pi R^2 - \pi r^2 = \pi (R^2 - r^2)$ and further multiply it by the height $(h)$ or they could calculate the volume of the bigger ($\pi R^2 h$) and the smaller ($\pi r^2 h$) cylinders separately and then determine the difference. In the post lesson interview, Bernard remarked that GeoGebra afforded opportunities for his students to connect the school mathematics to the things they see at home:

It brings the picture out and the students are able to connect what they see in the house to the new situation that we are learning. They see what we are talking about like the base area of the cylinder. They also see the height, so as we vary it, they are able to see the changes in the figure on the screen and by that, they are able to understand (Bernard).

Bernard’s reflections about GeoGebra reiterated literature that the tool combines real and virtual objects, has real-time interaction, and three-dimensional affordances, which enabled his students to cognitively appreciate the spatial complexity of the pipe involved in the question. For example, dynamically rotating the pipe in the GeoGebra window facilitated the students to concretely conceptualise the relationship between the cross section and the volume of the pipe.

**Assessment of students’ learning**

From Table 5.7, five key practices were common in the way the teachers adopted GeoGebra and the students’ worksheet to assess their students’ learning. These practices were review and correct students’ errors, use GeoGebra to provide prompt feedback, remedial teaching, shared ideas, and reflection on solutions (see sample coding in Appendix N).

Table 5.7 Teachers’ enactment of assessment of students’ learning

<table>
<thead>
<tr>
<th>Effective mathematics pedagogy</th>
<th>Description</th>
<th>Core practices</th>
</tr>
</thead>
</table>
| Assessment of students’ learning | It involves the creation of a learning environment where formative feedback and feedforward from the teacher and students are promoted to monitor the progress of students’ learning in a specific mathematical task. | 1. Review and correct student’s errors  
2. Use GeoGebra to provide prompt feedback  
3. Remedial teaching  
4. Shared ideas  
5. Reflection on the solutions |
For example, Joshua, Bernard, Sammy, and Michael used the slider and checkbox features in GeoGebra to hide and unhide the solution they wanted the students to provide. The teachers unhid the solution after the students had performed the task to enable them to compare their solutions. This helped the teachers to identify individual students or groups who were struggling with the task for remedial instructions. In Bernard’s class, for example, he drilled the students on their existing knowledge on the area of a circle by using the slider and checkbox features in GeoGebra to hide and unhide the solutions he wanted the students to come up with.

In Joshua’s class, he asked the students to match each colour of the graph to the corresponding mode of transport. He asked them to state the reasons for their decisions and to calculate the speed for each mode of transport. GeoGebra provided dynamic graphical representation and the values of the speed for each mode of transport simultaneously. He was able to use this technology to assess the existing knowledge of the students before the main task of the day was presented. Bernard used a similar approach to review his students’ knowledge on the area of a circle in his lesson on mensuration.

In Michael’s case, he used the affordance of the checkbox in GeoGebra to consolidate his students’ knowledge and skills of applying appropriate trigonometric ratio. This feature in GeoGebra supported the teachers to create multiple questions for the students within a short time. It also provided learners with immediate feedback about their solutions as well as a platform for sharing their thoughts about their solutions (see Figure 5.19).

![Figure 5.19 Affordance of the checkbox in GeoGebra (Michael)](image)

There was evidence in the data where teachers provided advance information to prevent students from making possible errors and drawing the wrong conclusions. For example, in
the excerpt below, Sammy invited the students into a conversation which led the students to explore the sum of the interior angles of a polygon. He anticipated possible errors that the students could make in determining the numbers of triangles in a polygon. So, he provided a caution to prevent the students from falling into that trap (line seven). Also, he knew some of the students could wrongly arrive on the deduction \( S = 180^\circ \times n - 2 \) instead of \( S = 180^\circ(n - 2) \) (line 11). However, he allowed the students to commit that error which he used to elaborate on the appropriate way of writing the formula.

Sammy: What can you say about the figure shown on the board?
S1: I can see two triangles in the square.
Sammy: Okay. Any other observation?
S2: You’ve divided the square into two triangles.
Sammy: What about if it is a pentagon?
S3: Three triangles
Sammy: We’ve been able to have two triangles from a square and three triangles from a pentagon. There is one condition that you need to take notice of. You can join any two points of a given polygon to form a triangle, but we don’t want the situation where the lines are drawn in the polygon to intersect.

Sammy: Now you are going to do this in groups. As I drag the slider, observe the polygons portrayed on the GeoGebra window and record your observations.

Sammy: Now let’s look at what you wrote for the sum of the interior angle of a polygon. What formula did you write for the sum of interior angles of a polygon \( S \) in terms of \( n \)?
S11: We wrote \( S = 180^\circ(n - 2) \).
Sammy: That is good. Very excellent work. But I saw some of you writing your answer as \( S = 180^\circ \times n - 2 \). It is important to remember that it the number of triangles in the polygon times the \( 180^\circ \) so \( n -2 \) should be put in a bracket.

Similarly, in Cynthia’s class, she asked the students to plot an ordered pair of points to form a polygon \( ABCDE \). After she went around to supervise students’ work, she realised some of them were not able to draw the polygon as she wanted them to. They did not join the points
in the order of the alphabetical letters, so they got a different shape. She then used GeoGebra to show them the expected polygon. She explained to students why they had the polygon wrong. The worksheet and GeoGebra provided a space for her students to communicate their thinking through verbal and/or written responses. This allowed her to use the responses from the students to sequence the mathematics instruction by providing feedback to shape the intended learning. The teachers provided systematic and repeated exercises to review students’ previous knowledge.

When you are drawing a plane figure you need to join the points in order of alphabets that you plotted. So here you should join A to B first then to C, D and then E. When you do it that you will get this shape [she point the polygon drawn on the GeoGebra window] (Cynthia).

Michael and Sammy explicitly indicated that the worksheet and GeoGebra helped to determine the extent to which their instructional objectives were achieved. From their observations, the worksheet helped the students to display their knowledge about the concept they were learning:

Whatever they see or observe they put it into writing. So, I can also read and know that this is the level of their understanding. The worksheet helped to display their knowledge about the topic (Michael).

I gave them some worksheet which they use to practice. So, when I go round, I am able to realise whether they are all involved. Based on that I can determine whether I have achieved the objectives of the lesson (Sammy).

Prior to professional development, Sammy shared that technology could assist their assessment practices where they could monitor the progress of their students’ learning. He stressed that technology had the potential of catching students’ attention. Hence, according to him, it would enable teachers to assess their students’ learning as well as their own instructional goal:

Because it catches the attention of the learners; it will help me assess the skills and abilities of the learners. It will help me tap the individual ideas in the class.
With that, I will be able to realise whether I have achieved the instructional goals and objectives (Sammy).

After the professional development, he commented:

With GeoGebra, the students can see the triangle zoomed in and out. With that effect, it makes it real for the students to understand the polygon I was teaching. It is more practical using the tool than the chalkboard. It gives us a concrete answer. Apart from that, it speeds up the teaching and learning. It makes teaching and learning friendlier (Sammy).

In all three lessons he conducted, he used GeoGebra to confirm students’ verbal responses. He asked and received written responses on the relation between the number of triangles and the number of sides of a given polygon. He walked between desks to monitor and discuss answers with students in small groups. His data (his pre and post interviews and lesson observation) implied that prior to his engagement in the professional development, he was speculative in terms of his belief about the pedagogical effectiveness of technology in mathematics teaching. In the post interviews, he reiterated most of the reasons he indicated in the pre-interview, but he was more confident and concrete about his understanding of the benefits of technology in mathematics instruction. Unlike his pre-interview, he added specific mathematics-related examples during the post-interview to buttress his submissions regarding his views about the pedagogical importance of technology in the mathematics classroom. This particular observation was noted among all the teachers after their engagement in professional development. They seemed to refine their beliefs about the value of technology in mathematics after professional development.

Bernard was observed using an Excel Spreadsheet to record students’ continuous assessment during the period of the professional development. He believed, prior to his engagement in the professional development, that technology could avoid teachers from rigorous computation when preparing continuous assessment because there were features in the technology which could do the calculation easily. His comment after the professional development suggests he had intensified the way he used Excel to monitor his students’ learning. His position of responsibility as a head of the department and his prior knowledge
in Microsoft Word and Excel, SPSS, and Matlab, seemed to be an additional motivation to use this technology for his students’ assessment:

I use Excel in assessing my students’ performance I use it to analyse whether my pupils are doing well, or they are falling behind. At first, I was using this on a minimal side. Now I am into it (Bernard).

**Summary**

This section reported evidence of teachers’ core practices in enacting effective mathematics pedagogy using GeoGebra. The list of the 31 core practices offered in this study are not exhaustive. They are meant to be illustrative of the kind of teacher actions characteristic of each of the themes of effective mathematics pedagogy: *creating a mathematical setting, useful mathematical tasks, mathematical discussion, mathematical connections, and assessment of students’ learning*. It is important to acknowledge that the actions of the teachers for each theme of effective pedagogy overlapped, which make it difficult to describe each core practice in isolation. Teachers demonstrated various ways they perceived the professional development had contributed to their pedagogy and dispositions towards mathematics teaching particularly with regards to the use of GeoGebra. The next section explores the shift in the teachers’ pedagogy and dispositions in detail.
Shift in Teachers’ Technology Dispositions

This section is the final one in the results sections of this research study. The section reports on findings related to the shift in the teachers’ dispositions towards technology integration in mathematics following their engagement in the professional development. The findings are integrated and presented under four main headings: (i) changes in perceived usefulness of technology; (ii) teachers’ affective attitudes towards technology; (iii) behavioural attitudes towards technology; and (iv) changes in the self-efficacy towards technology.

Changes in perceived usefulness of technology

The results in Table 5.8 compare the teachers’ perceived belief of the importance of technology in mathematics before and after the professional development.

Table 5. 8 Changes in perceived usefulness of technology

<table>
<thead>
<tr>
<th>Perceived Usefulness</th>
<th>Before</th>
<th>After</th>
<th>t</th>
<th>Hedges’ g</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perceived Usefulness</td>
<td>M</td>
<td>SD</td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>1. Technology can aid students to think critically when solving problem in mathematics.</td>
<td>4.27</td>
<td>0.65</td>
<td>4.64</td>
<td>0.50</td>
</tr>
<tr>
<td>2. Technology could be used to enhance independent learning.</td>
<td>3.82</td>
<td>.60</td>
<td>4.27</td>
<td>.47</td>
</tr>
<tr>
<td>3. My lesson will be more learner-centred when I use technology.</td>
<td>4.18</td>
<td>.75</td>
<td>4.55</td>
<td>.52</td>
</tr>
<tr>
<td>4. Technology could provide fast and efficient means of getting information.</td>
<td>4.45</td>
<td>.52</td>
<td>4.82</td>
<td>.40</td>
</tr>
<tr>
<td>5. Technology skills are essential for students in their future careers.</td>
<td>4.82</td>
<td>.40</td>
<td>4.55</td>
<td>.52</td>
</tr>
<tr>
<td>6. Using technology in class makes many things easier.</td>
<td>4.45</td>
<td>.52</td>
<td>4.45</td>
<td>.52</td>
</tr>
<tr>
<td>7. Using technology in class can raise student performance.</td>
<td>4.54</td>
<td>.52</td>
<td>4.36</td>
<td>.50</td>
</tr>
<tr>
<td>8. Technology has administrative importance in school.</td>
<td>4.45</td>
<td>.52</td>
<td>4.27</td>
<td>.47</td>
</tr>
<tr>
<td>Overall mean</td>
<td>4.36</td>
<td>.25</td>
<td>4.48</td>
<td>.17</td>
</tr>
</tbody>
</table>

* p < .05, **p < .0045 (Bonferroni adjustment)

The mean scores recorded for all the items both before and after measurements were greater than the agreed threshold of 4.00. This indicates that the teachers held high beliefs about the
effectiveness of technology in mathematics both at the beginning and at the end of the professional development. Overall, their notions of the effectiveness of technology in mathematics moderately improved after their engagement in the professional development model (Hedges’ $g = .54$), however, the change was not significant ($p > .05$). One possible explanation for the professional development not having a significant impact on teachers’ perceived belief of the usefulness of technology could be that they overrated themselves in the pre-measurement compared with the post measurement. It is also possible that prior to their engagement they were indeed profoundly convinced that using technology in the classroom would not only promote learner-centred teaching and learning, but would also make their students think critically in solving mathematics problems ($M > 4.0$). Situating this finding in the context of the environment they worked in, and given the fact that the majority of them (nine out of eleven) had not used technology in mathematics instruction prior to their engagement, the moderate effect size recorded is practically important in terms of their overall perception about the importance of the technology in mathematics. In the post interviews and discussions, the teachers reiterated most of the reasons they indicated prior to the professional development about the effectiveness of technology in mathematics. The only difference noted in the post responses was that they added specific mathematics-related examples to buttress their submissions regarding their views about the pedagogical importance of technology in the mathematics classroom. A possible explanation is that as they practically interacted with GeoGebra, they gained experience which helped them to firm their existing beliefs about the usefulness of the technology. Gideon, Sammy, and Bernard initially were of the view that technology could be used to reduce abstract teaching so students could visualise the mathematics concept practically. In their post responses, they buttressed this conviction with a specific example of mathematics learning where technologies were adopted. Bernard indicated that the technology could offer the opportunity for students to visualise the relationship between the nets and formula for the curved surface area of a cylinder:

This [approach of using technology] was more interesting. I was able to use the software to show them the pictures. With the cylinder, I was able to get animation which able the students to see the nets of the cylinder. Being able to see how it opens enables the students to understand what we are talking about. That picture was effective for teaching and learning (Bernard).
It was apparent in the post interviews and focus group discussions that their understanding of the true value of technology in teaching and learning was heightened. For example, the participant teachers became considerably convinced after the professional development process that technology could support them to orchestrate productive teaching and learning in mathematics where their students could learn independently. In the post-interview, Sammy elaborated that “technology can play a role for the students to discover real-life application of the mathematics concepts that they are learning”. Cynthia, Martey, and Bernard strengthened Sammy’s view, by indicating that technology could provide a new window for students to work independently, practice, and solve more non-routine mathematics questions. For instance, Cynthia indicated that using only verbal exposition instruction (word descriptions of mathematical concepts) would not sufficiently immerse the students into higher-order mathematical thinking. However, if the students were taught with technology they would be more explorative and extend their mathematics idea to life outside the school. Bernard’s comment also suggests that technology provided multiple mathematics resources which enable students to achieve their learning goal.

Technology enhances students' learning and academic performance … it makes students to practice more mathematics. It is a medium which helps the individual student to learn on his or her own through constant practice … You can use the computer to look for steps on how to solve challenging problems online… computers can provide pictures and photos for them to view the things they are supposed to work on. This will make the students have a feel for the concept they are learning. That will enhance understanding than merely using words (Cynthia).

When you leave the technology with the children, it makes them explore and practice more examples and get different sources of reference from the technological basis which help them to at least hit their learning target (Bernard).

Using technology, [the students] can easily translate whatever they learnt in school in solving problem outside the school (Martey).
Joshua and Peter shared that technology helped students to make correct estimations and relate mathematics to the environment.

Using this approach (technology-assisted instruction), students can relate the mathematics to the environment. Using technology helped students to do correct estimation like the distance between two locations (Joshua).

“Seeing they say is believing” … in the GeoGebra window, it is not only the teacher who is doing the activities. The students will also click, and they will see the angle …You will not use the protractor to measure whereby the inaccuracies will come in …this one automatically the software will measure it for you (Peter).

At the onset of the professional development, Jonathan held that students’ interest in mathematics could be spurred and they would be actively engaged in learning activities when technology was adopted as an instructional tool.

Technology will arouse their interest in the subject they are learning. Their learning becomes much easier and interested … while they sit doing nothing and just watching … their interest will be whipped up when you use technology (Jonathan).

He reiterated the same view during the second interview when he was asked whether technology had some benefits at all in the mathematics classroom.

Oh yes. Based on what we have gone through for the past one year. I think it has immense benefit to the teaching of mathematics … it makes our lessons interesting and quite easy than the normal traditional way of teaching (Jonathan).

**Teachers’ affective attitudes towards technology**

The results in Table 5.9 show that, prior to the professional development, the participant teachers held strong positive feelings towards the use of technology in mathematics (mean
≥ 4.00), and this was substantially enhanced after the professional development model (Hedges’ g = 2.08).

Table 5.9 Perceived change in the affective attitudes towards technology

<table>
<thead>
<tr>
<th>Affective</th>
<th>Before</th>
<th>After</th>
<th>t</th>
<th>Hedges’ g</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
<td>M</td>
<td>SD</td>
</tr>
<tr>
<td>1. Technology do not scare me at all.</td>
<td>4.00</td>
<td>0.45</td>
<td>4.64</td>
<td>0.50</td>
</tr>
<tr>
<td>2. Technology makes me feel comfortable.</td>
<td>4.09</td>
<td>0.70</td>
<td>4.09</td>
<td>1.14</td>
</tr>
<tr>
<td>3. I am glad there are more technology these days.</td>
<td>4.27</td>
<td>0.47</td>
<td>4.64</td>
<td>0.50</td>
</tr>
<tr>
<td>4. I like talking with others about technology.</td>
<td>4.00</td>
<td>0.89</td>
<td>4.55</td>
<td>0.52</td>
</tr>
<tr>
<td>5. Using Technology is enjoyable.</td>
<td>4.36</td>
<td>0.50</td>
<td>4.64</td>
<td>0.50</td>
</tr>
<tr>
<td>6. I like using Technology in teaching.</td>
<td>4.18</td>
<td>0.75</td>
<td>4.5</td>
<td>0.52</td>
</tr>
<tr>
<td>7. Technology do more good than harm</td>
<td>4.00</td>
<td>1.18</td>
<td>4.45</td>
<td>0.93</td>
</tr>
<tr>
<td>Overall mean</td>
<td>4.13</td>
<td>.15</td>
<td>4.50</td>
<td>.19</td>
</tr>
</tbody>
</table>

*p < .05, **p < .0045 (Bonferroni adjustment)

The overall change in the teachers’ feeling towards technology was significant at both common threshold (p < .05) and Bonferroni adjustment (p < .0045) levels. This suggests that the professional development significantly contributed to teachers’ feelings of comfort and happiness towards the use of technology in mathematics teaching and learning. For example, the participant teachers were blithesome, prior to the professional development, that there were many technologies on the market. This happiness seemed to have been sustained and enhanced through the professional development model. They also seemed very comfortable and less frightened to use technology both at the beginning and at the end of the professional development. The following excerpts also illustrate how Cynthia, Prince, Martey, and Sammy felt about the use of technology after they had gone through the professional development:

It’s interesting. I feel very relaxing. Students seeing their teacher using technology is a plus. It is good (Cynthia).
For me, I feel relaxed because it is the students who are doing most of the job and the thinking. Because it is new to them, they feel excited and concentrated. I saw all of them trying to do something from the instruction stated on the worksheet. Because I am free and not doing much talking, I can supervise them and prompt them to do the work (Prince).

Sammy again added that technology made teaching livelier and presentable.

It makes it livelier and more presentable. The beauty of it attracted me to use it so that they will get the concepts unlike going about it in the traditional way (Sammy).

The comment from Martey suggested that technology has the potential of relieving teachers from tasks that are routinely cumbersome using a pen and paper approach.

I feel relieved because it does not make the work too cumbersome. It makes it much easier. You are not bothered to enter the classroom with a lot of textbooks. The whole thing becomes simple (Martey).

In the following comment, Sammy felt overwhelmingly modernised and very proud of what he believed he could do with technology in the classroom. Thus, exposing teachers towards the use of technology through professional development reignited teachers’ positive feelings to move in line with 21st century teaching, where both teachers and students are active meaning makers of mathematics concepts.

I feel good. I feel I am on another level. I see myself to be moving in the 21st century. I see myself enlightened and modernised. I see myself to be knowing. I see myself acquiring skills. So, it even boosts my morale and confidence (Sammy).

**Behavioural attitudes toward technology**

The results in Table 5.10 indicate that prior to the professional development, the participant teachers were highly positive towards the use of technology in mathematics teaching and
learning (mean ≥ 4.00). The participant teachers’ overall behavioural attitude towards technology integration in mathematics was strengthened after their engagement in the professional development model (Hedges’ $g = .31$). They strongly pointed out that they preferred to use technology to illustrate mathematics concepts than using chalkboard illustration. They were also eager to learn how to use technology to perform mathematics activities. The overall attitudes of the participant teachers toward technology slightly improved. However, their intention to use technology in the future declined after their engagement in the professional development (Hedges’ $g = -0.63$).

Table 5. 10 Perceived changes behavioural attitudes towards technology

<table>
<thead>
<tr>
<th>Behavioural</th>
<th>Before</th>
<th>After</th>
<th>$t$</th>
<th>Hedges’ $g$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M$</td>
<td>$SD$</td>
<td>$M$</td>
<td>$SD$</td>
</tr>
<tr>
<td>1. I would rather do things by technology than chalkboard illustrations.</td>
<td>4.55</td>
<td>0.52</td>
<td>4.82</td>
<td>0.40</td>
</tr>
<tr>
<td>2. If I had the money, I would buy a technological tool such as computer.</td>
<td>4.73</td>
<td>0.47</td>
<td>4.82</td>
<td>0.40</td>
</tr>
<tr>
<td>3. I would use technology as much as possible.</td>
<td>4.73</td>
<td>0.47</td>
<td>4.73</td>
<td>0.47</td>
</tr>
<tr>
<td>4. I would like to learn more about technology.</td>
<td>4.73</td>
<td>0.47</td>
<td>4.82</td>
<td>0.40</td>
</tr>
<tr>
<td>5. I have the intention to use technology in the near future.</td>
<td>4.91</td>
<td>0.30</td>
<td>4.64</td>
<td>0.50</td>
</tr>
<tr>
<td>8. I think I would need technology in my classroom.</td>
<td>4.45</td>
<td>0.52</td>
<td>4.64</td>
<td>0.50</td>
</tr>
<tr>
<td>9. Learning about technology is not a waste of time.</td>
<td>4.73</td>
<td>0.47</td>
<td>4.64</td>
<td>0.50</td>
</tr>
<tr>
<td>Overall</td>
<td>4.69</td>
<td>.15</td>
<td>4.73</td>
<td>.09</td>
</tr>
</tbody>
</table>

*p < .05, **p < .0045 (Bonferroni adjustment)

After the professional development, all the teachers again expressed their intention to use technology in their future mathematics classroom, but some of the teachers indicated they would do so when the classrooms were resourced technologically:
My interest to use technology has gone higher. I am always willing to accept new technology to teach mathematics apart from this GeoGebra tool (Gideon).

I will like to use it more often because it will broaden my knowledge. It will give me more experience, and my confidence level will go high (Cynthia).

I intend to use it as often as I can in the classroom especially in the teaching of concept I think is too abstract to the students (Joshua).

I want to use it in my class any time there is the need to do that (Jonathan).

I wish the equipment will be available always to enable me to use the technology to teach (Martey).

Two teachers had started assuming leadership roles in coordinating technology use in teaching and learning before the professional development ended. When the teachers were asked to indicate whether they would encourage other teachers to use technology in their teaching, all of them responded affirmatively. Some teachers indicated they were willing to share with their colleagues the materials they gathered through the professional development. In an informal conversation, Sammy indicated he had introduced the idea about the use of technology to one of the economics teachers in the school, and the friend welcomed the idea. Jonathan added that he had begun inviting his teacher colleagues in the science department to embrace the idea of using technology in the science instruction. He indicated:

I have seen the benefits. Currently, in the science department, nobody employs technology in his or her teaching. I will say that I am the only teacher using it to teach. I spoke with one teacher who happens to teach with me at the same level. He also teaches chemistry. I explained to him that I have these tools and if he wants to use it I could make it available to him. The response I had from was okay, but he is yet to come. (Jonathan)
Changes in teachers’ self-efficacy towards technology

Prior to the professional development, the participant teachers rated themselves as low in TK, TCK, TPK, and TPACK compared with CK, PK, and PCK, indicating weak competence in using technology to perform mathematics activities in the class (see Table 5.11).

Table 5.11 Perceived knowledge and use of technology

<table>
<thead>
<tr>
<th>Construct</th>
<th>Example</th>
<th>Before M</th>
<th>SD</th>
<th>After M</th>
<th>SD</th>
<th>t</th>
<th>Hedges’ g</th>
</tr>
</thead>
<tbody>
<tr>
<td>TK</td>
<td>I can use technology without problems</td>
<td>2.74</td>
<td>.53</td>
<td>3.52</td>
<td>.39</td>
<td>-4.50**</td>
<td>1.63</td>
</tr>
<tr>
<td>PK</td>
<td>I know how to assess students’ performance in the classroom.</td>
<td>4.05</td>
<td>.24</td>
<td>4.12</td>
<td>.19</td>
<td>-.92</td>
<td>.31</td>
</tr>
<tr>
<td>CK</td>
<td>I have sufficient knowledge about mathematics.</td>
<td>3.88</td>
<td>.29</td>
<td>4.17</td>
<td>.31</td>
<td>-4.03**</td>
<td>.93</td>
</tr>
<tr>
<td>TCK</td>
<td>I know about technologies that I can use for understanding mathematics concepts.</td>
<td>2.91 .70</td>
<td>3.88</td>
<td>.52</td>
<td>-3.86**</td>
<td>1.53</td>
<td></td>
</tr>
<tr>
<td>TPK</td>
<td>I can choose technologies that enhance the teaching approaches for a lesson.</td>
<td>3.14</td>
<td>.80</td>
<td>4.10</td>
<td>.16</td>
<td>-3.84**</td>
<td>1.60</td>
</tr>
<tr>
<td>PCK</td>
<td>I know how to select effective teaching approaches to guide student thinking and learning in mathematics content.</td>
<td>4.03</td>
<td>.32</td>
<td>4.12</td>
<td>.24</td>
<td>-.70</td>
<td>.31</td>
</tr>
<tr>
<td>TPACK</td>
<td>I can teach lessons that appropriately combine mathematics concept, technologies and teaching approaches.</td>
<td>3.06</td>
<td>.65</td>
<td>3.88</td>
<td>.32</td>
<td>-3.67**</td>
<td>1.54</td>
</tr>
</tbody>
</table>

*p < .05, **p < .0045 (Bonferroni adjustment)

The results in Table 5.11 show tremendous improvement in the teachers’ TPACK (Hedges’ g = 1.54). All participant teachers acknowledged that the professional development had contributed hugely in regard to the confidence, skills, and knowledge they needed to use technology in mathematics instruction effectively. Martey, Jonathan, and Joshua, for example, mentioned that their lessons were getting more interesting because they now read widely for information from online sources to augment their lesson preparation:

It has tremendously impacted positively on the way I teach. For instance, instead of me depending only on textbooks. Now I can easily go online to get some stuff to prepare my lesson plan (Martey).
I think my lessons have become more interesting to students of late. I now read wide. I look for different information unlike before. Even when I am not using the technological tool in the lesson delivery, I still search for more information before I go to the class. It is making me explore more when I am preparing my lessons for the class (Jonathan).

Four teachers explicitly indicated that the professional development had stirred their confidence in teaching because they could now choose appropriate technology for classroom use. This suggests the teachers seemed to have had the reorientation to repurpose technology to suit topics in the mathematics curriculum. For example, the comment from Joshua suggests that when teachers’ pedagogical beliefs are challenged through professional development, they are able to maximise it to enhance students’ mathematics learning. During the professional development, the core practices of effective mathematics pedagogy were deconstructed and modelled for the teachers to rehearse through peer teaching under the watch of an expert (researcher/facilitator) for constructive comments, suggestions, and coaching. All the teachers seemed to acknowledge that the activity had offered them a great learning opportunity to advance their pedagogical practices to enhance students’ procedural and conceptual fluency in mathematics learning. It was observed that as the teachers progressed through the professional development, they gained confidence about how to facilitate group work supported with technology and activity-based worksheet:

This time you are challenged to look for useful information for your lessons. I have now developed the habit of going to online every day for personal study (Joshua).

The syllabus emphasises the use of technology to teach students. So, it motivated me to use the technology to fulfil that objective of the syllabus (Gideon).

Through the programme, we have realised that there is no topic in our syllabus that we cannot use technology to teach (Bernard).
Martey indicated his involvement in the professional development contributed to his curiosity in mathematics teaching. His comment suggests an advancement he has made within the community of practice of technology integration in mathematics. He has extended his interaction beyond his school community by looking at what others are doing in his subject area through online:

This programme has made me more curious about adding knowledge to the one that I have. I have been going online to look at what others have done concerning the various topics I teach. That exposure alone is adding something to my existing knowledge (Martey).

Jonathan, Cynthia, Michael, and Joshua remarked that the professional development offered them the opportunities to experiment with a variety of pedagogical strategies, and also reflected on their practice in multiple modes:

I taught circle theorem. Honestly, I have taught this lesson in some occasions and most often the students found it difficult to understand the properties. Because in the classroom, normally what happens is that we draw the diagrams on the board then we explain. So, I thought that I should use the GeoGebra so that they would better appreciate the lesson and come out with the properties themselves. So that is why I decided to choose that lesson (Jonathan).

I remember I taught this topic [Travel Graph] in a class where I was trying to let the students visualise something. But they were not getting the picture. I have to struggle using a lot of examples to get them to understand what I was teaching. I saw that nothing was working. So, I have quickly adopted to this one [use of technology] so it will make the lesson very simple (Joshua).

Bernard reflected on how his new instructional approach was working for him as well as his students. It was observed that he gave students extended reading materials which the students completed at home. He used the students’ solution from the extended reading to introduce the new concept intended for the day. He allowed students to discuss their solution in groups in the form of a quiz competition between boys and girls. He allowed a group from the boys
or the girls to present their solution and further asked them questions to justify the soundness of their presentation. He scored them to determine the winner at the end of the week. Bernard indicated he intentionally gave students challenging questions which would compel them to read other books before they come to class. With that, he indicated he felt he had previously spoon fed them. Thus, the professional development seemed to have facilitated his general pedagogy in class. In his voice:

I have learnt a lot from this programme, and it is helping me. I ask students to do more reading before they come to class. My boys and girls [quiz competition] is working well. Now, no one wants to be last. It is challenging them to work harder. Now you go to the class, and you see them working on their own solving challenging questions from their textbooks (Bernard).

Sammy and Gideon’s comments could provide possible explanations for the observation Michael made about the changes in his students’ behaviour. Sammy indicated that a technology environment “brings beauty to the mathematics lesson”. Thus, it is possible that the aesthetic nature of the lessons presented in the GeoGebra window might have caught the attention of the students and hence it prevented them from dozing during instruction. Gideon said that

Using GeoGebra brings excitement. Students were very excited to see how the graph [quadratic curve, \( y = ax^2 + bx + c \) ] was opening upwards and downwards when the parameters are altered. When the value of \( a \) became zero, the graph automatically became linear. They were so happy about it.

Similarly, Gideon shared that “With the technology, we can see the practical aspect of the subject”.

Teachers’ knowledge of mathematics content was not specifically dealt with in this professional development, however, there were instances when they acknowledged that through the programme they had gained conceptual understanding in some of the topics. It was observed that teachers could manually sketch the graph of the trigonometric function prior to their engagement in the professional development. However, they appreciated the relational understanding of the transformation of \( f(x) = \sin(x) \) when they were engaged in the
hands-on activities involving GeoGebra. As illustrated in Figure 5.20, they conceptually learned that \( f(x) = \sin (x + k) \) is the \( k \) units shift of \( f(x) = \sin (x) \) along the negative \( x \)-axis, \( f(x) = \sin (x) + k \) is the \( k \) units shift of \( f(x) = \sin (x) \) along the positive \( y \)-axis, and \( f(x) = k \sin (x) \) is the stretch of magnitude \( k \) along the \( y \)-axis with the \( x \)-axis invariant. The affordance of the input bar and graphic view features in GeoGebra might have contributed to the teachers’ understanding of the transformation of \( f(x) = \sin (x) \). These features enabled the teachers to observe the variations in the equation and graph of \( f(x) = \sin (x) \) simultaneously.

![Figure 5.20 Teachers’ conception of the transformation of trigonometric function](image)

Also, the teachers became specific about the knowledge and skills they had gained in preparing lesson plan. Jonathan and Joshua indicated that they now read widely for information to augment their lesson preparation:

I think my lessons have become more interesting to students of late. I now read wide. I look for different information unlike before. Even when I am not using the technological tool in the lesson delivery, I search for more information before I go to the class. When it comes to animation, I download a lot of them. It makes me get more information than before. It is making me explore more when I am preparing my lessons for the class (Jonathan).

Unlike first that you only have a book to go through and whatever you see is final. This time you are challenged to look for useful information for my lessons. I have now developed the habit of going to online every day for personal study (Joshua).
Bernard similarly added that there had been some dynamism in gathering information to support his teaching. He indicated his lesson deliveries had most often been a rote learning approach because of the unavailability of teaching and learning materials. He admitted that despite his long experience in mathematics teaching, he sometimes struggled to portray the concepts clearly to students. He further indicated that he currently admired his teaching because he was now able to access some online teaching and learning resources which enabled him to structure his lessons to enhance students’ exploration:

At first, the kind of preparation I do was just to read and then I present the concept the way I read it because I have no teaching material to enable me to portray the picture of the concept to students. Sometime we could see that we struggle to explain the mathematics concept. Sometimes we teach it [mathematics concept] like rotenly. It takes much time for the students to grasp. The child is not able to picture the concept… However, after I have been taken through this workshop, the activities I prepare are child-centred learning approach. I can create something for the students to see by themselves. I give them the opportunity to explore by themselves. They now do more of the thinking than before. I don’t stress myself in the classroom any more. …they get the mental picture of the concept… I love what I do now. It makes it very easy for me (Bernard).

Seven participant teachers explicitly acknowledged the relevance of the suggestions offered by the expert in the professional development. They believed that when the expert was visible in the school it offered them the opportunity to address issues regarding mathematics teaching and learning.

The influence of the expert in this professional development was very great. The comments from the expert helped us improved our lesson notes, how to use the technology in the lesson, and when to use it during the lesson delivery. … the expert is with us… because expert is visible we can approach him whenever there are issues about what we are doing, not only that any general issue in mathematics teaching we feel he can help (Bernard).
As the professional development progressed, it was observed that the teachers appreciably incorporated the suggestions made by the expert into the subsequent lessons that they designed and taught. A point in case is the lesson on trigonometric ratios; the teachers were struggling with how to insert a textbox and a checkbox that would animate and at the same time show the values of the trigonometric ratios as any vertices of the right-angled triangle were dragged. After expert support, the issue was resolved, and they were able to develop the artefact in the GeoGebra window to gradually unfold the content of trigonometric ratios to students. Thus, the expert’s support in technology-related professional development provided a desirable approach of helping teachers to upskill their knowledge of pedagogical competences of using technology to teach mathematics.

The illustrations in this section indicate an improvement in the teachers’ technology dispositions after they had gone through the professional development. This result was expected. Further probing indicated similarity and variability in the development of teachers’ technology dispositions. Although the teachers believed their attitudes towards the use of technology was enhanced, and they seemed willing to adapt technology in their mathematics classrooms, further analysis indicated some variation. Unlike the other teachers, the observation and responses from Michael, Cynthia, Mary and Peter regarding their intention to use technology in future suggested they were not fully assimilated with the use of technology in mathematics teaching after they had received the training in GeoGebra. On the one hand they appreciated the possibility of making use of the limited technology resources available in the school, whilst on the other they were sceptical and reconsidering the adequacy of the resources before they could use the technology in their instruction. The adopting teachers uniformly felt the school environment was not yet ready for innovative practices. The following excerpts illustrate why their intentions to use technology in their classrooms swung on a pendulum after the professional development:

My skills in using technology is still not full. What [the lesson] I have been using I adopted from elsewhere [online]. For me to create one on my own is a problem. Some of the members in the group too try to dominate because they have upper hand in the computing than others (Mary).

Inadequate knowledge about the tools. Using it for the first time was a little challenging. After going through several times, I think it was okay (Cynthia).
I wish the equipment will be available always to enable me to use the technology to teach (Michael).

Although Peter was optimistic about using technology in the future, he anticipate that certain factors could be a hindrance. He shared that there were two factors towards the successful implementation of technology. He termed the factors as internal and external. The internal factor, he implied, was professional competence which he believed he had gotten through the professional development. The external factor, he implied, was administrative support and electricity supply. The following is his narration after the professional development when he was asked whether he needed assistance before he could use technology in the classroom:

Not at all. Once I have gone through this professional training I have learnt a lot. Now I am quite okay with the use of the tools. But whereby we don’t have electricity supply then that one is an external, not within my constraint. I cannot do anything about it. So for those supports, it should be provided by the external authorities. But the internal one which I have learnt, that one is not a problem. It will be good if each teacher has a laptop computer to practice whatever we learn in this programme at home as well (Peter).

In an informal conversation, Cynthia, Mary and Sammy shared a similar view that each teacher having a laptop computer was crucial to engage them more in practising the GeoGebra software.

We need to have our person computer to practice more … two people share one computer … it is not enough … the time you need the computer your colleague is using it … for me it didn’t help (Cynthia).

The other seven teachers similarly acknowledged the contextual factors impeding their efforts to use technology, but seemed more resilient in adapting it in their teaching, at least at the time that they were observed during this study. They seemed comfortable using GeoGebra and showed eagerness in exploring more topics within the mathematics curriculum that could be taught with GeoGebra. For example, the comments from Bernard
and Joshua suggest that they appreciated the constraints at hand but were determined to make advances of what they had learned. Joshua stressed, “no excuse” to say that using technology in their school is impossible because it has been tried and tested in their own environment:

Now we know the basics of using GeoGebra. What we need to do as a group is to come together and design more topics using the tool. At the beginning of each term, we can plan that to support each other (Bernard).

For this professional development, the environment was used as a resource centre. The place we are complaining about was used, and it worked. So, we no excuse for not using it to teach. The idea that it cannot work has been erased (Joshua).

Bernard added:

There has been a great impact in the way I teach. My orientation and skills about the subject I am teaching have improved. My skills concerning the use of the tool have also been enhanced. At first, I wasn’t able to use the computer myself. I was having a computer but I was only using it to type word. But now I can use the tool to teach some of the topics in mathematics. The expert has had a great impact on us. Sometimes we say that technology can help improve mathematics teaching but we also look at the constraints, we don’t have this, we don’t have that. But having the expertise here has helped us this time (Bernard).

The teachers demonstrated variability in the way they operationalised their TPACK. In Gideon’s lessons, for example, he was able to use GeoGebra to generate multiple graphs that supported students to explore the parameters of quadratic curve. His knowledge in basic computer skills (Microsoft Word and Excel) at the onset of the professional development was profound compared to other teachers. His flexibility in navigating through the GeoGebra window to coordinate students’ mathematics learning improved as he progressed through the professional development. He was able to resolve his own technical issues regarding equipment setups for his lessons as the professional development progressed.
Gideon held strong beliefs about pedagogical usefulness of technology in mathematics, but like the other teachers, except Peter and Joshua, he had never used technology for mathematics instruction before the programme. He indicated he used technologies such as Encarta, Microsoft Word and Excel for his personal development. For example, he indicated he used the mathematics component in Encarta for fun (playing games). He showed improvement in developing and enacting his TPACK. The lesson plan he developed showed a sequence of activities that promoted students’ participation. For example, unlike his first lesson, the introduction of the second and third lessons was livelier. He used pictures of real-life phenomenon to hook his students to the concept of quadratic equations. This enhanced the students to make meaningful mathematical connections between symbolic, numeric, and real-life representations of quadratic equations. He also managed to get the students to deduce the relation for the line of symmetry of a quadratic curve.

However, it was observed that the integrated knowledge of PCK with technology he displayed during the lesson enactments was more teacher-led than engaging the students in exploring the concept collaboratively. Not unexpectedly, Gideon’s difficulty in orchestrating meaningful cooperative learning with GeoGebra was connected to his teaching experience. He was in the first year of his teaching career and had no mathematics education qualification. Thus, it is likely that he was at the beginning stage of expanding his knowledge in addressing students’ learning, assessment, and classroom management. These classroom practices have their own complexities combined with that of technology. Hence it is going to take him and other similar teachers (Michael and Prince) time to get the fluidity of combining their PCK with technology. Gideon reflected on some of the improvements he made in his general pedagogy. In the comments he indicated he summarised the salient concepts at the end of every lesson. Gideon said that unlike before, he now walks between desks during the instruction time to monitor the progress his students are making:

Before this [professional development] the way sometimes I start the lesson and end it wasn’t the best. Now, before I start the topic, I put some questions on the board for the students to solve. I go through with them to identify their weak areas before the main topic for the day. The question I put on the board is based on their previous knowledge (Gideon).
Cynthia, on the other hand, showed appreciable skills in managing her students to work cooperatively in exploring clockwise and anticlockwise rotation of objects. She taught this lesson twice: first to her peers and then to students in the actual classroom. She was able to (i) repurpose the technology to meet her instructional objectives; (ii) use technology to revise students’ previous idea on how to plot points on Cartesian plane; (iii) correct students’ misconception on how to draw a polygon; and (iv) make the appropriate link between students’ previous knowledge and intended learning. These illustrate her improvement in using technology to enact mathematics lessons. Like Gideon, Mary, Michael, and Prince, she had never previously used technology to mediate mathematics instruction. In the initial interview, she expressed worry about not being able to use different educational platforms to explore mathematics concepts. She indicated she could only use Google to search for meanings of words. After the professional development, she said:

I used it [GeoGebra] to exhibit concepts. I used it to help students to come out with formulae without memorising them. We looked at the two forms or types of rotation: clockwise and anticlockwise movement. We rotated a given figure for the student to monitor it on the projector. They recorded the coordinates of the vertices. We did several other rotations through a given angle. After the recording, they did some comparisons and then they came out with the generalisation. The angles we looked at were $90^\circ$, $180^\circ$, $270^\circ$ and $360^\circ$. Using the tool, they were able to get the formulae easily. The understanding came very quick. After the deduction, they were rotated other figures without memorising the formulae (Cynthia).

This illustration is in line with her initial beliefs about the use of technology in mathematics. In the initial interview, she espoused that technology could hasten mathematical exploration, enhance personal research, and support problem-solving. Unlike Gideon, she struggled in handling the tools in the GeoGebra window as well as with the skills of setting up the projector and the laptop computer. This impeded the flow of her lesson when she wanted to toggle between different windows: GeoGebra and Microsoft Word. At the end of the professional development, she seemed less confident in carrying out these activities independently. In the post-interview, she expressed the difficulties of using GeoGebra, but reassured herself that she will be okay with constant practice:
Because I wasn’t conversant, sometimes I get it very difficult to move the object around it. I was having a problem with the laptop because I was not conversant with it. I was having difficulties but with practice, it will be all right (Cynthia).

Again, it was observed that Mary, Michael, and Cynthia were less explorative and needed constant encouragement before they used the technology in the actual classroom. It was observed that their self-will to use technology was low. It was noted that they only wanted to teach the lessons they had developed during the training so they could complete the phases of the professional development. Probably, they felt less motivated or fatigued by getting to the end of the programme for reasons as such as family responsibilities and workload:

I was not able to use technology teach in the normal classroom as often as I wanted. They were barriers. Sometimes when I get to the house, I am tired to prepare a lesson on this professional development, then lesson on my actual classroom work, it was somehow difficult. Not only that one. The workload was quite burdensome (Michael).

For it [technology use] to be effective, I think we need to have ICT laboratory purposely for the teachers which is resourced with the internet. We can go there and prepare our lessons during our free time while in school. Because we are family members, when we go home our responsibilities are loaded and it is difficult to make time for technology (Cynthia).

In an informal conversation, Michael expressed that having gone through this professional development he believed the school is not fully ready for the infusion of technology into teaching and learning. He believed that for ICT to be effective it needs to start from basic school with the students so that by the time they get to high school they would be conversant with it. He noted that because it is a new tool to them here some of the students were just observing the tool and how the object is simulating without paying attention to the concepts. In his voice:

They have never seen it before so they are curious not because of the mathematics but because of the tool. Also with the current situation, massive
support is needed from the administration. I think everybody needs to be on board otherwise it would be difficult. We are not well equipped (Michael).

In the pre-interview, the teachers mainly talked about professional knowledge and technology resources as the key factors for the integration of technology in teaching and learning. None of the teachers indicated family responsibilities and workload as factors that fettered them from using technology at the onset of the professional development. This suggests that as teachers begin to use technology through professional training, they gained a better appreciation of the level of commitment required of them. Comments from Sammy and Peter seemed to suggest that teachers’ feelings about the environment and sense of satisfaction were crucial in the integrating of technology into mathematics teaching. It could also be inferred from Sammy and Cynthia that complexity and uncertainty in the use of technology in mathematics teaching could hardly be resolved by applying a single professional development. Even if they believed the professional development was effective, applying the knowledge was dependent on a prior restructuring of situations that were complex and uncertain.

Joshua was using a phone to conduct mathematics lessons prior to his engagement in the professional development. He used his phone to download mathematical concepts from YouTube which he showed to his students during his lessons. Probing further about how he used technology before this programme revealed that he did not structure his lessons with it. He used it spontaneously; that is, as and when it is needed during the lesson delivery. In the pre-interview, he described that he adopted the technology when it became necessary to explore shapes of circular objects when the lesson was ongoing. Unlike his previous practice where he only used the technology during the lesson without pre-planning, he seemed to appreciate the gradual progress he had made in displaying profound integrated TPACK at the end of the professional development. He independently developed two lessons: a distance-time graph and the equation of a circle. He was able to increase student engagement by using GeoGebra to assist students who had difficulty in visualising the dynamics involved in drawing a distance-time graph. He showed enthusiasm in exploring how he could use GeoGebra to teach topics such as calculus and statistics. He commented:
I have seen a lot of difference. Using the GeoGebra with the worksheet allows students to think for themselves, make guesses, predict, estimate and ask open-ended questions (Joshua).

Joshua indicated proudly that after the professional development he could handle the projector and any issues associated with it. He narrated this when he was asked to indicate the extent to which this programme had helped him regarding the use of technology in mathematics teaching:

The confidence level has risen. Now there is no more shivering in using it [projector]. Setting it up is not a problem. If a projector is not working, I can determine where the problem is coming from. Now, if I want to do something off-screen, I can do it (Joshua).

The evidence in the data showed some unintended findings. Michael and Joshua demonstrated how they transferred the knowledge they acquired through the professional development to other areas. Michael shared that through the GeoGebra training he was able to learn new things from the equation editor in Microsoft Word.

Although it is GeoGebra we learnt, it is linking me to other things. Sometimes you might need MS Word and Excel to do something. It is widening my technological know-how. Initially, I was using MS Word for only typing. This programme has introduced me to other mathematical approaches in MS Word, for example, equation editor (Michael).

Joshua illustrated how the professional development had helped him to transfer the knowledge to support his members during church activities.

Last week we were using a projector at church, and I was the person to operate it. Through the programme [professional development], I know how to hide and extend the screen for viewing. So, it helped me so much. I was able to teach some of my colleagues how to do it.
Another unintended finding was how Jonathan enacted his TPACK. Despite the professional development centering on a specific technology (GeoGebra) and content (mathematics), he was able to apply the knowledge he obtained to teach both mathematics and chemistry. Like many of his colleagues, Jonathan had never used technology in his teaching prior to the professional development. In the pre-interview he perceived technology as an essential tool in mathematics teaching because he believed it can promote visual representation, aid students to have a clear understanding of mathematics concepts, and reduce abstract teaching. Among all the teachers, he seemed to have evaluated his decision of integrating technology into his lessons after the professional development. He was noted to use technology in his instructions quite often during the period of this research. He indicated in the second interview that he had seen the benefits of using GeoGebra which had convinced him enough to apply other technologies such as YouTube and Khan Academy in his chemistry instructions daily. Jonathan’s reflection summarises his experience of how the professional development had impacted on him as well as his students:

After the training regarding the use of GeoGebra in teaching mathematics, I saw the benefits. So, I said to myself why don’t I use it in the chemistry that I am teaching. I went to the internet and took some materials related to the topics I want to teach during the term. Some of the materials I realised they were very good so I decided to employ them in my chemistry lessons. I revised some of the materials. The first lesson I employed it, the feedback I got from the students was great. They were very happy. It’s enhanced their understanding of whatever I was teaching. So, I saw that it was okay. I realised the lesson was more student-centred. I fell in love with it, and since then I have been using it. I have used it to teach topics like mass spectrometry, atomic orbitals, periodic chemistry, periodic properties, ionisation, electronegativity, atomic size and bonding. There are other more topics I have used the technology to teach (Jonathan).

Inferred from Jonathan’s narration is that the professional development seemed to have stimulated Jonathan both intellectually and emotionally. Intellectually, he had tremendously improved in terms of knowledge and skills required to use technology to teach concepts in both chemistry and mathematics (TPACK). Emotionally, Jonathan seemed to be happy about his new approach of scaffolding students’ learning through simulation and animation.
technology. His students seemed happy too, because the dynamism in the classroom had changed and they had responded positively towards it. For example, in one of his lessons in Chemistry, he introduced his students to an analytic experiment on a mass spectrometer. Because he did not have the apparatus to set up the experiment practically, he played a four minute video demonstrating the experiment, downloaded from Khan Academy. He was able to pace the video to meet his instructional objectives. He asked students in groups to write their observations and conclusions for whole class discussion. With his already prepared questions, he guided students to draw conclusions based on their observations and recordings. In his voice, during the post-lesson discussion:

For instance in mass spectrometer, in our normal teaching, because we don’t have the apparatus, what we do is we just draw it on the board and then we just tell them the mass spectrometer is made up of different stages….. This time, I looked for animation on the mass spectrometer from the internet. I show it to them so they could see the instrument. Through the animation, they saw the sample injected into the vaporisation chamber. They could see the ionised sample moving……So I didn’t worry myself given them a lot of notes. I asked them to describe the phenomenon based on what they have seen (Jonathan).

Aside from the ICT laboratory, he had created a space in the science laboratory where he frequently conducted his technology-based lessons in both mathematics and chemistry. He indicated that “the ICT laboratory is usually occupied and sometimes it prevents me when I need the place for my lessons, so I have to find another place to conduct my lessons”. This illustrates his profound willingness to use technology teaching and learning despite the constraints. Also, in the post-interview, he indicated how his interest in the use of technology had developed. Further, he expressed interest to do master’s programme in science education where he wanted to explore more on the pedagogical usefulness on technology:

I have gotten into it so much. It is easier to plan your lessons that way. Students can have a feel of what we are teaching and gain proper understanding … In future it is likely I will explore more into it in my next education which is master’s degree level (Jonathan).
It was also observed that Jonathan had started assuming a leadership role in coordinating technology use in teaching and learning before the professional development ended. Although all the teachers responded affirmatively when they were asked to indicate whether they would encourage other teachers to use technology in their teaching. Jonathan began this campaign before this professional development ended. In a conversation, he indicated he had begun sharing the lessons he had gathered with other teachers in the science department. Jonathan added that he had begun inviting his colleague teachers in the science department to embrace the idea of using technology in science instruction. In his words:

I have seen the benefits. Currently, in the science department, nobody employs technology in his or her teaching. I will say that I am the only teacher using it to teach. I spoke with one teacher who happens to teach with me at the same level. He also teaches chemistry. I explained to him that I have these tools and if he wants to use it I could make it available to him. The response I had from him was okay, but he is yet to come (Jonathan).

Summary
This section provided a concrete picture of how participant teachers developed their technology dispositions through professional development that engaged them to work in groups to design and enact GeoGebra-based mathematics. The findings indicated that this approach of professional development holds promise in evolving participant teachers’ beliefs, attitudes, and knowledge, as shown through the self-report survey, semi-structured interviews, lesson plans, and lesson observations. Further analysis of these data pointed out that teaching experience is a function of how teachers practically demonstrated their espoused TPACK development. Overall, the teachers seemed to have developed an appreciable level of TPACK whereby they used GeoGebra to facilitate the student-centred approach of specific mathematics content to students. Optimistically, teachers’ long-life experiences with technology will yield positive teaching and learning of mathematics. However, personal commitment, organisational needs, and resources are required to enable and sustain teachers’ activeness and consistency of using technology in teaching and learning of mathematics.
CHAPTER SIX: DISCUSSION AND CONCLUSIONS

Many researchers operate under the assumption that students could reach mathematical proficiency when teachers orchestrate effective pedagogical practices including providing a supportive learning environment, mathematical exploration and discussion, making mathematical connections, and providing productive learning feedback (Attard & Curry, 2012; Calder et al., 2006; Geiger et al., 2012). For example, if students are engaged in a useful learning task where divergent views are tolerated then it is assumed they are mathematising. Theoretical arguments for this assumption are strong. Vygotsky’s work has some connection with the enactment of these practices where sociocultural and interactional settings are a complementary function of students’ mathematics learning (Herbel-Eisenmann et al., 2017). Although many research studies support this assumption (Anthony & Walshaw, 2009; Marks, 2000; Martin & Speer, 2009), what the core practices of effective mathematics pedagogy consist of are often generic and underspecified (Franke et al., 2007; Jacobs & Spangler, 2017). The current study was designed with the aim of revealing the typical features and identifying nuances of effective pedagogy in a GeoGebra learning environment. Also, engaging teachers to design and teach with GeoGebra in the mathematics content area offered a unique lens from which to explore the shift in their pedagogy and dispositions towards the use of technology in mathematics. Three research questions were set to guide the study.

1. What were the teachers’ initial dispositions toward the use of technology in mathematics?
2. How did teachers enact effective mathematics pedagogy when using GeoGebra following their engagement in professional development?
3. How did the professional development impact on the teachers’ dispositions towards technology integration in mathematics?

In this concluding chapter, the key findings are presented thematically based on the research questions and further discussed in relation to ICT in Ghana’s education system, the theoretical framework, and the research methodology adopted for the study. Limitations of the study will be discussed. Finally, recommendations and suggestions for further research will be presented.
Teachers’ Initial Technology Dispositions

Many authors have described teachers’ beliefs as a driving force of their classroom actions (Ertmer, 2005; Hermans et al., 2008; Ottenbreit-Leftwich, 2010). For example, Ottenbreit-Leftwich (2010) shared that the fundamental beliefs teachers bring into teaching have direct influence on the way they would implement technology in their pedagogical decisions. In the current study, the participant teachers’ initial dispositions towards technology in teaching and learning were explored to gain insights into their current practices of technology in the mathematics classroom. Also, their initial technology dispositions informed the activities of the professional development model organised for them. The analysis of the initial data revealed six central themes related to teachers’ dispositions towards the use of technology in teaching and learning: (i) perceived beliefs of usefulness of technology; (ii) perceived beliefs of nature and utilisation of technology; (iii) perceived feelings towards the use of technology; (iv) perceived intentions towards the use technology; (v) perceived self-efficacy regarding technology; and (vi) perceived contextual influence related to the use of technology.

Perceived beliefs of the usefulness of technology

Perceived usefulness is about the extent to which people believe that using a particular approach, tool, or system can enhance their productivity (Stols & Kriek, 2011). At the outset of the professional development, the participant teachers considered technology to be useful in mathematics teaching and learning. They identified four areas they believed technology to be helpful: pedagogy, assessment, development of professional competence, and general productivity.

Pedagogically, the teachers expressed varied reasons they believed technology could be helpful in exploratory, facilitatory, and participatory instruction (see Figure 5.1 for further details). At the beginning of the professional development, the participant teachers were inclined to state that technology enabled them to access mathematical contents that could be used to engage students in both cooperative and explorative learning. For example, they shared that using technology could afford students the opportunity to “relate the mathematics to the environment” (Joshua). Coincidentally, these beliefs these teachers shared at the onset of the professional development were in line with the Ghana’s mathematics curriculum, which expects them to use technologies such as computers, calculators, and spreadsheets, to support students to investigate real-life mathematical problems (MOESS, 2007). Although
the study did not ask questions related to their beliefs about the teaching and learning of mathematics, the teachers’ assertions reflected socio-constructivist teaching and learning, where space is created for students to cooperatively explore mathematical concepts and apply them to real-life situations. This finding is consistent with earlier studies that found that the use of technology was associated with constructivist approaches to teaching and learning, where teachers’ and students’ interaction were enhanced. For example, Calder et al. (2006), Geiger et al. (2012), and Attard and Curry (2012) argued that technology opened a space for mathematical discourse where students interactively negotiated and constructed mathematical concepts.

With regards to technology for assessment purposes, the initial results of this study indicated that technology could be useful in providing feedback on students’ learning. They shared that technology could be used to address unexpected mathematical questions students asked during lesson using online queries. Reimer and Moyer (2005) argued that the affordance of technology to provide immediate and specific feedback on learning tasks supports students to correct their own errors and misconceptions.

Again, the participant teachers conveyed the value belief that technology was capable of updating their professional competence. All the participant teachers indicated that technology could help them to acquire new knowledge in their subject matter and teaching skills. They held the view that technology could enlighten them more by delving into things which would be difficult to achieve without technology. Several studies have reported on teachers’ value beliefs about the potential benefits of technology in the teachers’ knowledge of classroom practices (Levin & Wadmany, 2005; Ottenbreit-Leftwich et al., 2010; Ruthven et al., 2005). For example, Ottenbreit-Leftwich et al. (2010) indicated that teachers shared the belief that technology was useful in improving their classroom practices.

With regards general productivity, the teachers recognised that technology could save them time and reduce their workload in terms of tasks such as copying notes on the chalkboard, preparing lesson plans, and recording assessments. The teachers believed technology could be used to reduce rigorous computation when preparing continuous assessment. Joseph and Emma had the view that technology could afford them the chance to cover more topics within the instructional time. It can be noted that these perceptions are consistent with the functionality of technology as a tool for doing mathematics (reduce routine work) and as a
tool for practicing mathematics skills (provide prompt feedback) (Drijvers et al., 2010). Jupri et al. (2015) shared that technology could assist students to carry out routine procedures such as the expansion of algebraic expression, handling large data, factorising polynomial function with higher degree functions, and drawing graphs.

**Perceived nature and utilisation of technology**

The teachers described what technology should be used in the mathematics classroom. In their collective view, technology could be used to visually represent, demonstrate, and practice mathematics. Although the teachers were not asked directly about the kinds of technology they believed would be useful for their mathematics instruction, technologies such as online learning, the use of projectors, laptops, and calculators, were apparent in their narrations. They believed they could reach many students when concepts were projected on the screen for the students. Some of the teachers (for example, Michael) had the notion that using technology meant downloading software that embodied mathematical concept and showing it to the students.

The participating teachers also believed that one way to use technology effectively in the classroom was to provide opportunities for both teachers and students to learn how to use technology. They believed that when all the students had knowledge about it, and each student had a computer to practice with, then meaningful mathematics teaching and learning could be achieved. For example, the comment from Gideon suggested technology could be used to support a flipped classroom approach where students could explore mathematical concepts prior to their engagement with the teacher in the actual classroom. Gideon’s assertion was consistent with Roschelle et al. (2017), who articulated that technology provides prompt feedback and students might not need to wait for a teacher before progressing to the next stage of mathematical task or skill. Also, technology has the capacity of creating space which offers students the opportunity to return to the skills practiced in an earlier session on a regular schedule (Roschelle et al., 2017). This enables students to personalise their learning both at school and at home (Light & Pierson, 2014; Zengin, 2017). For example, Amaral and Shank (2010) found that online learning allowed their students to spend more time on tasks outside the classroom which increased their success and course retention.
**Perceived feelings and intention towards the use of technology**

Empirical studies have found that teachers’ perceived beliefs about the usefulness of technology predicts their intention to adopt technology into their classroom practices (Stols & Kriek, 2011). Overall, the results of the current study indicated that the participant teachers, at the onset of the professional development, were enthusiastic about the use of technology in mathematics. Although the majority of them had never used technology in their mathematics classrooms, they anticipated that using it could be exciting and would make mathematics lessons enjoyable and comfortable. They were positive towards technology-related professional development because they believed it could help them to conduct mathematics lessons that would engage their students. Teachers’ intentions to use technology are influenced by the value they place on technology, in other words, their perceived usefulness of the technology or its ability to ease their classroom activities. However, teachers will not have the confidence to use technology in their practice if they do not have the general technology proficiency (Ertmer, 2005; Stols & Kriek, 2011).

**Perceived self-efficacy with technology**

Mishra and Koehler (2006) suggested that teachers with the requisite knowledge and skills in technology are able to effect positive pedagogical change in their classrooms. The data analysed in this study indicated that the participant teachers had limited knowledge or competence to use ICT hardware/software and its associated peripherals in activities related to mathematics teaching prior to their engagement in the professional development. Among the eleven teachers, only two indicated at the onset of the professional development that they had used technology in their mathematics classroom. Probing further, one of the teachers who had used technology indicated he used it to support students to confirm their answers after they had factorised quadratic expressions. The other teacher indicated that he usually used his phone to download mathematics concepts as the need arose during his instructions in the classroom. Although some contextual factors might have contributed to the way they enacted their lessons with technology, using it to confirm the answers after the students had factorised quadratic expression indicated a low knowledge and skills in technology application in the classroom (Niess et al., 2009). The limited knowledge of the teachers in technology integration reported in the study was not surprising, because many of the teachers participated in the study had their mathematics teaching qualification when technology had not been fully incorporated in the curricula which was used to train them. Only four teachers indicated they had been introduced to the integration of technology in mathematics teaching
during their previous education. Regarding professional development while on the job, only Joshua indicated he had attended professional development related to the use of technology. Although it is little over a decade since Ghana introduced technology as an instructional tool in mathematics (MOESS, 2007), this study surprisingly reported that some of the teachers (for example, Joshua and Gideon) were less aware of this mandate. Those who used technology were usually doing so for recording and computing of students’ test scores using Microsoft Excel. This implies that despite the commitment of both government and other parties relevant in education regarding provision of ICT infrastructure and professional training, many teachers in the country are yet to get in tune with the agenda of technology integration in the classroom. Previous studies have similarly shown that Ghanaian teachers have limited pedagogical knowledge in adopting technology into their classroom practices (Agyei & Voogt, 2011; Mereku & Mereku, 2015).

*Perceived contextual influence*

The school environment is a critical variable that explains successful implementation of technology in the classroom. At the beginning of the professional development, the teachers were eager to use technology to teach mathematics in their school, but there were a number of factors that hindered them. Apart from the limited knowledge and skills that have been discussed in the previous section, they were also faced issues such as limited technology resources, limited training opportunities, limited access to ICT laboratories, and limited administrative support. The study reported that technology resources were a second-order factor that fettered teachers to use technology in the mathematics classroom. Although the school had an ICT laboratory, all the teachers felt the facilities in it were not motivating enough to support effective mathematics teaching and learning. The school received 50 laptop computers from the Government of Ghana’s initiative project of one laptop computer per child. At the time of this report, the school had only one projector and four laptops. The remaining laptop computers had broken down. These laptops could not stand up to the pressure from 1600 students and 73 teachers in the school. The study reported that ventilation of the ICT laboratory, maintenance culture, and technical support could have contributed to the short lifespan of these laptops.

Many other studies have reported technology resources as a crux of effective technology integration. In the study of Ertmer (2005), technology resources were rated the first-order factor that explained the frequency of technology use in the classroom. Petko (2012)
identified the availability of technology resources as a second crucial factor that explained the variance of the intensity of classroom technology use. Buabeng-Andoh (2012) explored teachers’ skills, perceptions, and practice of ICT in teaching and learning in the senior high schools in Ghana. He concluded that the key factors that accounted for teachers’ low use of technology in the classroom were lack of access to technological resources, inadequate ICT pedagogical training, and insufficient administrative support. A year earlier, a similar situation was reported by Agyei and Voogt (2011) as they reported that Ghanaian mathematics teachers were unable to adopt technology in their pedagogical decisions because they not only lacked the knowledge and professional training, but technological resources as well. Mereku and Mereku lamented the same situation in 2015. This shows that despite the efforts the government of Ghana is putting in, all is not well regarding technology integration in education.

Surprisingly, the study reported that some of the participant teachers were reluctant to access the ICT laboratory because they felt the place was purposely for the ICT department and ICT lessons. These mathematics teachers usually relied on their personal modem, and they bought their own data for internet connectivity whenever they wanted to access information online. The study showed that the ICT teachers used the computers that were functional mainly for teaching students basic computer skills such as those used for Windows and the Microsoft Office suite - Word and Excel.

The preliminary results in this study confirmed the complex multiple interplay of factors that influence the implementation of technology in teaching and learning (Petko, 2012). Petko identified that teachers’ technology competence, availability of technology resources, perceived usefulness of technology, affiliation for constructivist teaching style, and responsibility, explain the variance of the frequency of technology use in the classroom. The participants in the current study reiterated these factors. In addition, the teachers’ reported that they received limited support from the school leadership, which fettered them from adopting technology in their pedagogical decisions. This suggests that, in these teachers’ opinion, the role of school leadership is critical in promoting the implementation of technology in the classroom. This finding reiterates one of the strategic plans in the Ghana’s ICT in education policy (Ministry of Education, 2008). The document emphasised that the inclusion of research and constant monitoring and evaluation of the ICT implementation are highly useful because these will cast new light not only on pedagogical, socio-cultural,
logistical, and technical issues, but also on the role of school leadership in effective integration of ICT in teaching and learning.

**Enactment of Effective Mathematics Pedagogy**

In this study the teachers were introduced to how GeoGebra could be used in mathematics instruction through a professional development model in which they worked in small groups to design and enact technology-based mathematics lessons. The following section discusses how the teachers enacted effective mathematics pedagogy when they used GeoGebra. The study identified five central themes related to effective mathematics pedagogy in the lessons the teachers enacted with GeoGebra. These were: creating a mathematical setting, useful mathematical tasks, mathematical discussions, mathematical connections, and assessment of students’ learning.

**Creating a mathematical setting**

The first aspect of effective mathematics pedagogy pertains to setting up an environment to enhance students’ mathematics learning. The results of the study revealed nine key practices which the teachers demonstrated in setting up an effective environment for their lessons. These nine key practices the teachers demonstrated in setting up effective environments were creation of pre-planned documents (a lesson plan, students’ worksheet, and GeoGebra artefact), equipment setup, shared instructional objectives, instructions about the task on the worksheet, linking prior knowledge to new learning, using pictures of real-life scenarios to initiate mathematical dialogue, explanation of terminologies, addressing existing misconceptions, and attention of individual learning needs. As the teachers acknowledged, it was their first time they were incorporating GeoGebra into their practices, hence they needed extra effort in terms of time, commitment, knowledge, and skills, to set the students up for learning. For example, they needed to learn the skills of setting up a projector and laptop such that the screen resolution would be appropriate to enhance students’ viewing.

With careful thought and planning, the teachers needed to design a lesson plan, students’ worksheet, and GeoGebra artefact to enhance smooth sequencing of their lesson. As they began to design their lessons in the GeoGebra environment, they explored and adopted more instructional strategies. These practices aligned with the literature on effective mathematics
pedagogy where space is created for students to share mathematical ideas. The teachers demonstrated an *ethic of care* (Anthony & Walshaw, 2007, 2009) because as they designed their lessons in the GeoGebra environment, they became more pedagogically oriented by contextualising the new tool to enhance small group and whole classroom discussion. Ingram (2013) and Attard (2011) shared that teachers need to create a classroom community of practice where positive relationships can be built through interactions and dialogue.

Also, the teachers inducted the students to their lessons through the provision of explicit instructions about the tasks via the students’ worksheet. They also articulated the objectives of their lesson, reviewed students’ existing knowledge, used real-life scenarios to initiate communication, addressed students’ existing misconceptions, and explained terminologies associated with the new concepts. Each teacher adopted at least one of these instructional strategies to induct the students into the learning. The nature of the topic they taught, the background of their students, and the teaching style and experience of the teachers accounted for variations in the way they enacted these instructional strategies. The technology played a complementary role in teachers selecting multiple real-life scenarios to hook students into the mathematics lessons. For example, pictures facilitated the teachers to ask questions that made the students explore the mathematical connections in real-life at the onset of their lessons.

Also, evidence from the study indicated the teachers were able to use GeoGebra to create a responsive space to support (i) students with different learning abilities (high and low performing students); (ii) students who had mathematics phobia; (iii) students with prior misconceptions; and (iv) students’ participation. The animations the teachers created using GeoGebra facilitated and addressed individual learning behaviour. For example, two teachers (Cynthia and Michael) noted that GeoGebra and the worksheet-based activities facilitated in improving students’ participation and reducing mathematics phobia. The participant teachers observed that there had been an improvement in teacher-student cordiality in mathematics teaching and learning. These findings support earlier reports in the literature. Arora and Pany (2018) concluded that when teachers’ TPACK level was enhanced through deliberate efforts such as training or workshops, they could rethink pedagogical strategies that ameliorated students’ fear in mathematics learning. The authors added that teachers with adequate TPACK could select innovative approaches such as using technology to demonstrate concrete and virtual manipulatives, using a flipped classroom approach,
dynamic mathematics software, and animated content demonstration in the mathematics classroom. In Hudson’s (2012) study, the teachers adopted GeoGebra to reinforce slow learners and students who had learning difficulties to learn through hands-on activities. Ruthven et al. (2005) found that the use of worksheets provided structure for instructional and learning activities which in turn freed the teachers to play a more facilitative and less directive role in during lesson delivery.

**Useful mathematical tasks**

The usefulness of the task is how the objectives and activities included in the lesson support students to grow both procedure and conceptual fluency in mathematics (Artigue, 2002). Anthony and Walshaw (2009) talked about “thinking” and “communicating with tools” (p. 23) as an important approach for students to make sense of mathematics. In the current study, the teachers adapted GeoGebra in an exploratory approach where students made algebraic and geometric generalisations from a sequence of activities including visualising, recording, calculating, predicting, and constructing new ideas. Students were challenged to make inferences based on the information they had recorded on the worksheet. For example, in Bernard’s class, the affordance of the animation in GeoGebra facilitated the students to identify and conceptualise the connection between the area of a rectangle and the curved surface of the cylinder. This offered the students a crucial step to deduce the algebraic formula for the surface area of the cylinder. As the students began to think with GeoGebra in Joshua’s class, it facilitated them to grasp the key mathematical ideas required for drawing distance-time graphs when new scenarios were presented to them. In Joseph’s class, the opportunity for students to simultaneously observe a racing car moving and its graph reinforced their understanding of the dynamics in drawing a distance-time graph. Cynthia used the slider and rotation tools in GeoGebra to enhance her students’ ability to predict mathematical phenomenon with reasonable explanation. She allowed the students to predict the quadrant where the image of the object might be in after rotation. This offered support for students to generate their own interpretation, promoted shared knowledge building, and enhanced student curiosity. In Jonathan’s class, students came to him with their pen drives for the animations he used for his lessons. This suggests that students’ interest to do extended work in mathematics was stirred when the teachers engaged in GeoGebra learning activities. These findings are consistent with Martin and Speer’s (2009), Ingram’s (2013), and Attard’s (2011) reflections of effective pedagogy. Martin and Speer (2009) held that selecting **good problems or useful mathematical tasks** (Anthony & Walshaw, 2009; NCTM, 2007) for
classroom use enhances students’ productive disposition where they develop an inclination towards the usefulness of mathematics (National Research Council, 2001).

GeoGebra seemed to open multiple pathways for students to look at mathematical concepts from different perspectives. A case in point was a student in Peter’s class. Peter wanted his students to establish that angles subtended at the circumference by the same chord or arc are equal. This is a conventional concept in the textbook which Peter wanted his students to explore with GeoGebra. However, as he dragged the chord in the GeoGebra window, a student observed that as the length of the chord increases, the angles in the same segment of the circle increased but were the same. This conclusion was not initially in the repertoire of Peter’s mathematics knowledge, as he admitted during the one-on-one post lesson discussion. The student’s observation prompted him to further explore its authenticity with the class. This extended the engagement of the students in the lesson because they felt this new concept was generated by their own peer and they were eager to accept it or refute it upon further investigation. This finding is in line with the conclusion of Sacristán et al. (2009), who explored the influence and shaping of digital technologies in students’ mathematics learning. Sacristán et al. indicated that students usually identify diverse ways to construct or develop mathematical thinking and problem-solving competencies when they are offered opportunities to learn with technology. These authors highlighted that technology has the affordance of providing multiple representations of mathematical concepts which can cater for individual learning needs.

**Mathematical discussion**

One of the important indicators of effective mathematics pedagogy is the ability of the teacher to facilitate classroom dialogue which focuses on mathematical argument (Anthony & Walshaw, 2009; Martin & Speer, 2009; NCTM, 2007). The engagement of students in mathematical discussions was a common feature across the lessons the teachers enacted in this study. As the professional development progressed, the teachers demonstrated improvements in the way they used GeoGebra and worksheets to engage students to communicate their mathematical thinking orally and in writing. As modelled in the professional development, the teachers were able to provide opportunities for both groups and individual students to present and explain their solution to the whole class. The teachers demonstrated an inquiring attitude which invited the students to develop mathematical ideas.
They re-voiced and edited students’ responses to consolidate understanding of the concepts. They provided attention to group and individual learning.

Despite this improvement, there were certain aspects of effective communication the teachers struggled to implement throughout the professional development. The kind of discussions the teachers engaged the students in did not generate the in-depth mathematical discourse most literature has articulated (Franke et al., 2007; Jacobs & Spangler, 2017; Stein et al., 2008). There was scant evidence in the lessons of nine of the eleven teachers of times when students were encouraged to take and defend positions against varied views or solutions from other members. In those lessons, the groups or individuals were not subjected to adequate public scrutiny where students were engaged in open debates about the solution the group or individual students had presented. In most cases, the teachers had a predetermined solution/answer to the question and discussion ended abruptly when students arrived at that solution. These teachers hardly probed students for alternate solutions. The teachers mostly invited groups with which they had pre-rehearsed the solution to lead whole class discussion. Although this enhanced accurate presentation of mathematical facts and ideas, it prevented the students from the awareness of other possible misconceptions or alternative solutions related to the concepts they were learning. It could also restrict the teacher from explaining concepts in varied ways to the students.

In all the lessons, the students were expected to construct their mathematical knowledge through the sequence of activities presented in the worksheet and using GeoGebra, but the teachers sometimes ended up telling the concept to the students. They sometimes forgot to guide the students through the activities outlined on the worksheet. It was observed that the teachers needed constant encouragement to make judicious use of the worksheet as well as GeoGebra. The use of GeoGebra and the worksheet had the potential to initiate communication among the students but the teacher has the critical responsibility in strengthening this conversation by asking questions that emphasise mathematical reasoning. It was noted that the teachers developed this skill more slowly than expected during the various sessions of the professional development.

One possible explanation is that teachers might have an agentic position towards improving practice, however, the challenges involve doing away with their old practice. As Bernard reflected, “Trying to leave the old way of my teaching to the new one, I sometimes forget
myself. I have to do it continuously so that it will be part and parcel of me”. This underlines the situated nature of teachers’ existing practice as an important factor towards successful use of technology in mathematics teaching and learning. The reflection from Bernard could explain the finding reported by So and Kim (2009), who reported that teachers may develop the knowledge and skills towards the use of technology, but they found it difficult to put them into practice.

In the studies of Valtonen et al. (2006) and Tømte et al. (2015), it was found that teachers tended to choose familiar teacher-centred instructional approaches over student-centred approaches in designing their lessons after they had been engaged in professional course focussing on the development of digital competence in pedagogy. For example, the teachers of Tømte et al.’s (2015) study expressed difficulty in adjusting themselves to the use of a learning management system (LMS). Thus, facilitating teachers to develop the skills of balancing content, pedagogy, and technology knowledge seemed insufficient for teachers to engage students in in-depth mathematical communication via technology. Bernard’s experience “… I have to [use the technology] continuously so that it will be part and parcel of me” reiterates the notion of Niess et al. (2009) that it requires a competence pathway to develop the professional competence in the use of technology to teach mathematics meaningfully.

**Mathematical connections**

Another common feature across all the lessons was that the instructional activities progressed in order of difficulty; students used an idea they had developed in the previous activity to solve challenging tasks in the subsequent activity. Six teachers created scenarios that enabled students to link newly acquired concepts to real-life phenomena and vice versa. The connections which students established between the pictures of real situations and mathematical concepts extended their knowledge in mathematics. For example, the students of Sammy and Gideon were able to extract mathematical concepts from pictures of real-life phenomena. In Gideon’s class, students predicted quadratic equations for the path of a basketball aimed at scoring, a motorbike rider negotiating a curve, and a metre ruler which was bent. These images of concrete representations initiated discussion which let the students appreciate the representation of parabolic graphs such as the quadratic function. Similarly, Bernard was able to provide tasks that supported the students to go beyond the routine competencies where they only substituted values into a formula and computed it. He
was able to challenge students to visualise, both geometrically and algebraically, the volume of metal used to make a pipe/tube. This finding is consistent with Hiebert and Grouws’ (2007) notion of effective mathematics pedagogy. The authors espoused that students’ mathematical proficiency is facilitated through challenging tasks that offer opportunities for students to make connections among ideas, facts, and mathematical procedures. Thus, an important aspect of effective pedagogy is the provision of an important mathematical task that offers opportunities for students to extend their mathematical knowledge, thinking and skills of problem-solving (Anthony & Walshaw, 2007; Martin & Speer, 2009; NCTM, 2007).

**Assessment of students’ learning**

According to Anthony and Walshaw (2009), effective teachers use a range of assessment practices to explore students’ reasoning and understanding of mathematical concepts. Evidence from the current study shows that the teachers provided clear instructional objectives in their lesson plans which they used to monitor the progress of their students’ learning. The worksheet and GeoGebra played an integral part in the way the teachers assessed their students’ learning. The questions the teachers included in the worksheet were structured to offer systematic and repeated exercises where students’ previous knowledge was reviewed and then built on. For example, Michael and Sammy explicitly indicated that the worksheet and GeoGebra helped them to determine the extent to which their instructional objectives were achieved. According to them, the worksheet helped the students to display their knowledge about the concepts they were learning. This suggests that the worksheet provided a space for students to communicate their thinking through verbal and/or written responses. As teachers walked between desks, they used the responses the students had provided on their worksheet to sequence the instruction by providing feedback to shape the intended learning.

While the worksheet provided a pen and paper approach of assessing students’ learning, the teachers used GeoGebra to offer immediate feedback which generated discussion. Joshua, Bernard, Sammy, and Michael used the slider and checkbox features in GeoGebra to hide and unhide the solution they wanted their students to provide. The teachers un hid the solution after the students had performed the task to enable them to compare their solutions. This helped the teachers to identify the individual students, or the groups, who were struggling with the task, for remedial instructions. In Joshua’s class, he asked the students to match
each colour of the graph to the corresponding mode of transport. He asked them to state the reason for their decisions and to calculate the speed for each mode of transport. GeoGebra provided a dynamic graphical representation and the values of the speed for each mode of transport simultaneously. This affordance of technology helped Joshua to assess the existing knowledge of the students before the main lesson. Bernard used a similar approach to review his students’ knowledge on the area of a circle in his lesson on mensuration. In Michael’s case, he used the affordance of the checkbox in GeoGebra to consolidate students’ knowledge and skills of applying an appropriate trigonometric ratio in a particular task.

Evidence in the data demonstrated instances where students appreciated and accepted the challenging tasks presented to them. Also, there were many instances where teachers provided positive reinforcement to motivate students to press on until they got the solution to the question. Teachers provided advance information to prevent students from possible errors and drawing the wrong conclusions. For example, in Cynthia’s class, she asked the students to plot an ordered pair of points to form a polygon. After she went around to supervise students’ work, she realised some of them were not able to draw the polygon as she wanted. They did not join the points in order of the alphabet, so they got a different shape. She then used GeoGebra to provide multiple scenarios of how the task could be performed accurately. Similarly, Sammy invited the students into a conversation which led the students to explore the sum of interior angles of a polygon. He anticipated possible errors that the students could make in determining the numbers of triangles in a polygon. So, he provided a caution to prevent the students from falling into that a trap.

**Shift in teachers’ technology dispositions**

This study focussed on the teacher factor by exploring how professional development impacted on teachers’ technology dispositions (beliefs, attitudes, and knowledge) and classroom practices in mathematics.

**Teachers’ technology beliefs**

Prior to the professional development, teachers’ view about the usefulness of technology reflected a constructivist approach of teaching and learning. Although the teachers reiterated most of the pedagogical usefulness of technology they stated in the pre-interviews, the
striking difference in their submissions in the post interviews was that they were succinct and stated specific examples to buttress their points. That indicates that the professional development contributed to the consolidation of their constructivist views about the use of technology. It was evidenced in the study that, as teachers engaged in professional development to interact with technology, they internalised their pedagogical belief to reflect the best practices of student-centred mathematics teaching and learning. For instance, the teachers appeared more persuaded after the professional development about the potential benefits of technology in promoting students’ knowledge creation, critical thinking, problem-solving skills, achievement, engagement, and collaboration in mathematics learning. It was evidenced in the study that, as teachers engaged in professional development to interact with technology, they internalised their pedagogical belief to reflect the best practices of student-centred mathematics teaching and learning. For instance, the teachers appeared more persuaded after the professional development about the potential benefits of technology in promoting students’ knowledge creation, critical thinking, problem-solving skills, achievement, engagement, and collaboration in mathematics learning. It was illustrated in the comments provided by Gideon and Sammy that technology eased teachers from the struggle they go through when explaining geometric concepts to students. They indicated that the geometric figures drawn on the chalkboard are static, and fail to portray the underlining principles of the concept to the students. With technology, the teachers believed they could produce dynamic geometric figures that embodied multiple representations of a concept in mathematics. Similarly, Cynthia, Martey, and Bernard suggested that technology could provide a new window for students to work independently, practice, and solve more non-routine mathematics questions. Cynthia, for instance, held that using only a verbal exposition instruction would not sufficiently immerse the students into higher-order mathematical thinking, but technology would do.

All the participant teachers in the post interviews strengthened their arguments that a meaningful way to teach and learn mathematics is by providing opportunities for individuals to work either independently or in small groups to construct mathematics based on their existing learning experience. They seemed to believe that technology has the potential to enhance conceptual fluency of mathematics teaching and learning. They conveyed the pedagogical belief that technology could be useful in mathematising where through visual images students could be engaged in a meaningful dialogue to explore mathematics concepts. The teachers again espoused that technology could afford them the opportunity to teach from known to unknown. The participant teachers’ pedagogical beliefs of technology reflected the proposition of Ertmer and Ottenbreit-Leftwich (2010) that “teaching is not effective without the appropriate use of information and communication technologies (ICT) resources to facilitate student learning” (p. 255).
The findings in this study were consistent with that of Brinkerhoff (2006) who reported that the teachers who participated in the professional development felt that the programme significantly changed their technology beliefs. The teachers in Brinkerhoff’s study strongly held that a teacher’s job includes teaching students how to use technology. Similarly, other studies have reported teachers’ value beliefs of technology in promoting deeper conceptual learning in students. For instance, all eight teachers in Ottenbreit-Leftwich et al’s. (2010) study believed that technology had the potential of enhancing students’ comprehension and higher order thinking skills. Authors such Almeqdadi (2000), Hoong and Khoh (2003), Niess (2005), and Hannafin et al. (2008) have reported that technology provides opportunities for the students to gain mathematical problem-solving and higher-order thinking skills.

The current study showed that the participant teachers espoused student-centred technology beliefs. Apart from this belief, they expressed several beliefs related to the importance of technology. The teachers emphasised that technology was useful in monitoring the progress of their students’ mathematics learning. Three participant teachers (Cynthia, Bernard, and Sammy), in particular, drew attention to how technology was useful in monitoring students’ learning progress through continuous assessment. They indicated technology had the potential of freeing the teacher to a facilitatory role, thereby getting the opportunity to double check students’ learning progress during instruction.

The participant teachers accentuated that students’ interest and engagement in mathematics could be spurred in a technology learning environment. This finding is consistent with that of Ottenbreit-Leftwich et al. (2010). The authors indicated that teachers were enthused to use technology when they believed it could engage and motivate students in the classroom. In other studies, Neurath and Stephens (2006) reported that technology had the potential for not only improving students’ mathematics achievement, but making students feel interested in learning mathematics.

This thesis reported that all the participant teachers believed that technology had the potential of helping teachers to meet their instructional objectives. For instance, Sammy recounted that using technology enabled him to accomplish his instructional objectives. He said because he was not talking too much, he had reserved energy to supervise students individually or in small groups to ensure that the students achieved the learning goal.
Similarly, Ottenbreit-Leftwich et al. (2010) reported that the teachers in their study used technology because the teachers believed it could help them to meet their curricular goals.

The teachers reiterated in the post-interviews that technology could save instructional time. They indicated that using technology could help the student to develop the mathematics concepts quickly and free the teachers from writing notes on the chalkboard, in a lesson plan, and recording continuous assessment manually. Unlike the pre interviews, two teachers drew attention to the fact that it required considerable time to prepare the lesson at home using technology, but they admitted that once that was done, it made the lesson delivery simple and more straightforward compared with the conventional approach where the teacher would spend time drawing geometric figures on the chalkboard manually. Kopcha (2012) reported that when access to technology and training were improved for his participants, subsequently they consistently indicated that using technology required time especially in planning, teaching, and classroom management. The participants in my study seemed to disagree with Kopcha’s participants on the issues of instructional time and classroom management. All the teachers in the current study usually grouped the students into pairs to work on activities outlined on the students’ worksheet they had prepared. This perhaps might have shortened the time the teachers spent on helping the students to develop a concept.

According to Ertmer (2005), teachers’ pedagogical beliefs precede their technology beliefs. Many have argued that teachers whose personal beliefs align with a constructivist approach of teaching are most likely to use technology in mathematics instruction (Ertmer, 2005; Ertmer & Ottenbreit-Leftwich, 2010; Petko, 2012; Sandholtz et al., 1997). It has been demonstrated in the current study that the professional development model had relatively little impact on the experienced teachers compared to the less experienced teachers, particularly regarding their pedagogical beliefs related to the use of technology in mathematics. The less experienced teachers seemed to respond more favourably to technology-related professional development than the more experienced teachers. Changing experienced teachers’ beliefs and attitudes is hard, and it requires time and resources to gradually assimilate curriculum innovation that has positive outcomes (Ertmer, 2005). Ertmer remarked that “some beliefs about the nature of teaching are formed over many years of experience….If that is true, then core beliefs about teaching will influence new information about teaching is processed including ideas related to teaching with technology” (p. 31). Similarly, in the study of Prestridge (2012), 73 teachers from different cultural
backgrounds participated in an online professional training programme. All the teachers in her study had ten or more years of teaching experience. The pre and post measurements indicated a decrease in the teachers’ beliefs and intentions towards student-centred learning. She reported the participant teachers were less convinced about the appropriateness of a student-centred style of teaching compared to a teacher-led instruction approach.

Teachers’ previous experience and current practices seemed to be a motivational factor for the successful integration of technology in mathematics. To the greatest extent, the activities of the professional development model seemed to have provoked the previous pedagogical beliefs of the less experienced teachers. Teachers within this bracket might have had some exposure to technology integration through their university education. Therefore, they seized this opportunity to (i) advance their existing experience in technology integration; and (ii) build their pedagogical assumptions in mathematics teaching and learning. On the other hand, the experienced teachers might hold on to their existing beliefs of appropriate teaching styles because of their additional responsibilities. It was observed that most of these teachers had responsibilities for the core activities of school management and administration. These activities might have impeded them from adopting the new information for its intended purposes. They saw the new information relevant to their classroom practices. However, they saw it as an additional task to their already burdened responsibility. Therefore, most of them would prefer to repurpose the new information to meet the school management and administration tasks they performed rather than using it in the classroom. For instance, it was observed that the teachers used the knowledge they acquired through professional development to perform other activities such as the preparation of a teaching timetable, an examination timetable, and students’ assessment records.

Evidence from this study indicated that teachers perceived the professional development related to the use of technology to be continual and not a one-time practice. All the teachers recommended in their submissions during the post-interviews the need to have continual training to consolidate the knowledge and skills they had acquired. This finding is consistent with Kim et al. (2013), who indicated that it was less likely that a one-time effort to change teachers’ belief systems could transform teachers radically. Kim et al. summed that up:

> Fundamental changes do not happen quickly or automatically. In order for the sustained growth and positive changes in teaching practices to occur,
incremental supports should be provided, and the supports should be ones that satisfy progressive needs for change. First of all, any incremental step or change should be sensitive to the current needs of teachers. For that, both formative and summative evaluations should be conducted (p. 83).

*Teachers’ technology attitudes*

The teachers in the study indicated that the professional development model played a role in shaping their affective attitudes towards technology. The teachers’ positive feelings towards the use of technology in mathematics grew stronger by the end of the professional development. Teachers who were initially uncomfortable and anxious about the use of technology in mathematics teaching indicated that their confidence and interest had grown as a result of the professional development. Similarly, Myers and Halpin (2002) reported that professional development training played a role in reducing teachers’ anxiety, related to the use of technology.

After the professional development, some of the teachers in this study indicated they wanted to use technology in mathematics instruction because of the beauty it brought to the lessons. For instance, Sammy narrated during the post interview that using technology made teaching lively and presentable. He added that its beauty attracted him to use it in the classroom. In a similar report, Heitink et al. (2016) explored teachers’ professional reasoning about the pedagogical use of technology in the classroom. The authors identified that teachers’ reasons for including technology in their pedagogical decisions included attractiveness, efficiency, and effectiveness. The authors reported that “Most teachers said they used technology to make the learning process more attractive and motivating for students” (p. 76). Larbi-Apau and Moseley (2012) explored computer attitudes of multidisciplinary teaching faculty in higher education in Ghana. They identified that affective attitude was the highest contributor to teachers’ technology integration, followed by perceived usefulness, behaviour, and perceived behavioural control attitudes. Teachers’ affective attitudes towards technology is a critical function in technology integration. It was observed in this study that teachers might respond favourably to curriculum innovation when they felt their old practices were no longer productive. All the teachers in this study responded positively towards professional technology development, because they felt that their usual instructional approach of using words only to explain concepts seemed less helpful to their students. Teachers would use
technology in the classroom when they perceived it had pedagogical relevance (Ertmer et al., 2012).

The self-report survey showed a slight increment in the participant teachers’ positive attitudes towards technology integration in mathematics. The participant teachers strongly thought that technology could motivate their students to learn more. The participant teachers also agreed that using technology could help them retrieve information very fast and efficiently. These findings are consistent with that of Albirini (2006). The teachers in Albirini’s study considered technology as an educational tool brought improvement to schools and classrooms. Brinkerhoff (2006) also found that the professional development contributed to the growth of teachers’ attitudes towards technology use in teaching and learning. He reported that teachers’ decisions to include technology in their instructional planning became more pronounced after their engagement in professional development.

At the onset of the professional development, all the teachers demonstrated strong intentions to learning how to use technology to perform mathematics activities. The professional development model contributed to promoting the teachers’ behavioural attitudes towards technology integration in mathematics. Many of the teachers reported that their intention to use technology was strong, and grew stronger, as a result of their engagement in the professional development model. All the teachers expressed a willingness to encourage other teachers to use technology in their teaching. Two of the teachers started leadership roles in coordinating technology use before the professional development ended. Myers and Halpin (2002), similarly, reported that professional development explained the improvement of their participants’ positive attitudes towards computer adoption, and willingness to use computers in the future. However, Rienties et al. (2013) found that the online professional development they engaged the teachers in did not enhance the teachers’ intention to transmit the knowledge acquired into practice. The authors reported that the participants were less convinced about the appropriateness of using technology to orchestrate student-centred instructional approach away from their teacher-centred instructional approach. Harris (2005) cautioned educators, developers, and researchers about pedagogical dogmatism, where a teacher is considered a failure if he or she is unable to repurpose technology to meet a student-centred style of teaching. Harris went further to add that technology integration on a large scale has been a perceived failure because the attention has been on “how the students would use the tools to obtain information” rather than “how the students’ content learning
would be assisted with the tool use” (2005, p. 116). It is essential that technology is used as a medium to enhance best instructional practices and support students with distinct learning needs to achieve curriculum goals (Harris, 2005; Ottenbreit-Leftwich et al., 2010).

Though the current study reported that the teachers indicated their intention to use technology in the classroom after the professional development, it was observed that the less experienced teachers seemed more confident and willing to do so than the experienced teachers. The experienced teachers expressed their willingness to use technology in the future, but they indicated it is dependent on the adequacy of technology resources in school. Many studies have reported that experienced teachers respond slowly to curriculum innovation because of their already established beliefs and attitudes (Bebell et al., 2004; Buabeng-Andoh, 2012; van Braak et al, 2004). Most of the experienced teachers in this study had their university education when technology integration in mathematics was not part of the curriculum. It was through this professional development that most of these teachers practically experienced how technology is used in mathematics teaching. Petrogiannis (2010) studied the relation between teachers’ preparedness to use technology and their prior experience with computers. Petrogiannis found that teachers with some computer knowledge seemed more prepared and ready for technology integration than the non-experienced teachers who had no prior knowledge of computers. Similarly, Myers and Halpin (2002) engaged teachers in professional development. Their preliminary results indicated that teachers who had no previous computer workshop experience felt more intimidated by computers than their counterparts who had computer workshop experience.

At the outset of the professional development, the teachers perceived several factors that hindered them from using technology to teach mathematics. These factors included limited knowledge of technology integration, inadequate technology resources (hardware and software), limited training opportunities, limited access to an ICT laboratory, and less administrative support. The teachers in the study acknowledged that the professional development increased their confidence and knowledge to make use of the limited resources available. The teachers’ mindset that the ICT laboratory was meant for the ICT department and for ICT lessons only was disabused at the end of the professional development. Notwithstanding, the teachers after professional development reiterated the need for administrative support, adequate technology resources, and financial motivation. The professional development seemed to have supported the teachers to obtain a better
appreciation of the constraints of using technology in their own school. This finding is consistent with the results of Soebari (2012). She reported that teachers in her study were keen to translate the knowledge and skills they acquired through the professional development, but indicated their intention could be hampered by the contextual factors such as limited support from school leadership, and inadequate resources and facilities.

Admiraal et al. (2017) classified teachers into five main groups based on the teachers’ beliefs about learner-centred teaching and attitudes towards technology. The five teacher types the authors identified were (i) learner-centred teachers with technology; (ii) teachers critical of technology use in school; (iii) teachers uncomfortable with technology; (iv) teachers uneasy with learner-centred teaching; and (v) teachers critical of a clear-cut stance. Overall, the participant teachers’ positive beliefs of, and attitudes towards technology characterised the first two groups of teachers identified by Admiraal and his colleagues. Admiraal et al. went further to suggest strategies for supporting teachers in each group. They indicated teachers in the first group (learner-centred teachers with technology) needed to be supported to take initiatives with other colleagues and/or stakeholders to ensure effective implementation of technology in teaching and learning. Two teachers in the current study had begun a leadership role in coordinating technology integration in teaching before this professional development ended. For the second group (teachers who were critical of technology use), they should be supported with technology infrastructural needs. A technology awareness programme and peer teaching could be helpful to such teachers. Teachers’ beliefs and attitudes are hard to modify (Ertmer, 2005). Kim et al. (2013) proposed observation, practice, reflection, social, and cultural support as important strategies to promote changes in teachers’ beliefs and attitudes.

**Teachers’ technology knowledge**

Findings triangulated from the self-report survey, interviews, lesson plans, and lesson observations provided a concrete picture of how the participant teachers developed their TPACK through the professional development.

Effective mathematics teachers have the trademark of integrated knowledge in both content and pedagogy. They have the expertise to structure and facilitate learning opportunities for students to learn mathematics meaningfully. The study reported that the participant teachers’ knowledge in pedagogy (PK), subject matter of mathematics (CK), and the integrated
knowledge between these constructs (PCK) were high prior to their engagement in the professional development. This was not surprising because their prior expertise focussed in these areas. Notwithstanding, the professional development seemed to have contributed to strengthening the participant teachers’ knowledge in organising specific mathematics content meaningfully to students. Among these constructs, the survey report indicated a large effect size for CK and a small effect size for PK and PCK. With regard to content knowledge (CK), the participant teachers indicated they had gained substantial knowledge related to mathematics concepts, facts, theories and procedures through the professional development (CK).

The GeoGebra hands-on activities could explain the expansion of the participant teachers’ knowledge in the mathematics content. The activities involved in the GeoGebra training challenged the teachers to use this new tool to structure specific mathematics concepts in a way that facilitated students’ learning. This engaged the teachers to concurrently draw upon several knowledge factors including students’ learning and misconceptions, extensive reading on the concept, and how the concept could be simplified for students to understand. This initiated the teachers into relearning the concept and how their students learn. For instance, all the participant teachers acknowledged that the activities of the professional development had consolidated some of the mathematical concepts they knew. For example, the comment from Michael suggests that the professional development had helped the teachers to gain more command in the topics they teach. It was observed in my study that participant teachers consolidated their understanding of the transformation of trigonometric function, \( f(x) = \sin(x) \) when they were engaged in the hands-on activities involving GeoGebra. The affordance of the input bar and graphic view features in GeoGebra enabled the teachers to observe the variations in the equation and graph of \( f(x) = \sin(x) \) simultaneously. This helped them to conceptually understand that \( f(x) = \sin(x + k) \) is the \( k \) units shift of \( f(x) = \sin(x) \) along the negative \( x \)-axis, \( f(x) = \sin(x) + k \) is the \( k \) units shift of \( f(x) = \sin(x) \) along the positive \( y \)-axis, and \( f(x) = k\sin(x) \) is the stretch of magnitude \( k \) along the \( y \)-axis with the \( x \)-axis invariant.

This finding supports the existing body of literature (Doering et al., 2009; Koh & Chai, 2014; Kul, 2012) indicating that professional development expands teachers’ understanding of the concepts related to their subject area. For example, Doering et al. (2009) reported that all the
teachers who participated in their study indicated an increase in the knowledge of the content of geography.

The preservice teachers in the study reported by Mouza et al. (2014) showed a substantial effect size improvement in all the TPACK constructs except CK. Inservice teachers develop their TPACK differently from preservice teachers because of the difference in the teaching experience and beliefs (Yeh et al., 2017). One possible explanation that accounted for the substantial increase in the teachers’ knowledge in mathematics content in the current study is that out of the eleven participants who participated in the study, five (45.5%) of them held academic qualifications in subject areas (chemistry, commerce, and business) other than a teaching qualification in mathematics. It was observed that many of these teachers gained broader procedural and conceptual understanding of some of the mathematics concepts adapted for the study. Prince, for example, conceded that he was unaware prior to the professional development that the volume of three cones makes the volume of a cylinder of the same radius and height. He expressed his joy for relationally learning the relationship between these two concepts through the professional development.

Pedagogically, the participant teachers in the current study acknowledged that the activities of the professional development had contributed to expanding their knowledge in addressing students’ mathematics learning, assessment, classroom management, and lesson plan preparation. The teachers believed that they had gained considerable knowledge of using varied instructional approaches that fostered both cooperative and independent learning among students. They also pointed out that through the professional development they were able to orchestrate an instructional approach that facilitated higher order thinking skills among students. The lesson observations confirmed improvement in the instructional approaches that made students responsible for their learning. The results of the study indicated improvement in the teachers’ knowledge to repurpose the objectives and contents of the mathematics curriculum to meet the learning needs of their students. The professional development provided an opportunity for some of the teachers to learn the holistic approach of handling some of the topics in the senior high mathematics curriculum. For instance, Michael acknowledged he learned that the volume of a cylinder and a cone could be integrated into a single lesson. He indicated he usually taught the two concepts separately. These findings are in line with the study of Doering et al. (2009) who found that professional development exposed teachers to additional pedagogical approaches. Similarly, Mouza et al.
(2014) found that professional development had the potential of influencing teachers’ pedagogical knowledge positively. David, a participant in Doering et al.’s. (2009) study narrated after their engagement in the professional development that he realised he was not doing things strictly right and the professional development reoriented him to teach with critical “geography lens”:

This is where I saw the positive change for me. I truly believe I wasn’t doing geography before this but that I was having students complete worksheets. The environment and professional development helped me think about teaching with a geography lens (David).

David’s comment is similar to most of the participants in my study, in that professional development provided an opportunity for them to relearn some of the concepts in mathematics. For example, in my study, Martey indicated that the professional development had enhanced his curiosity in searching for more information relevant to the topics in the mathematics syllabus.

In this study, participant teachers showed considerable improvement in the technology-related constructs (TK, TCK, TPK, and TPACK) compared to the non-technology related constructs (PK, CK, and PCK). Among the technology-related constructs, the participant teachers recorded the most significant improvement in the technology knowledge (TK) domain. This finding is consistent with the study of Doering et al. (2009) who reported that the teachers recorded the highest positive change within the technology knowledge domain than the other TPACK constructs. When teachers are engaged in technology professional development, they begin to make a tighter connection between knowledge of technology, pedagogy, and mathematics content and this eventually influences their overall TPACK. Chai, Koh, and Tsai (2010) found that the percentage of variance explained by TK increased as teachers made stronger connections between TK, PK, CK, and TPACK after professional development. Thus, the fundamental knowledge in technology, content, and pedagogy is essential because a positive shift in any of these may lead to gains in TPACK (Doering et al., 2009; Mishra & Koehler, 2006). Other studies have accentuated that teachers need a high level of proficiency in technological skills to effectively use technology in their instructions (Littrell et al., 2005; Wang & Chen, 2006; Zhao et al., 2002). Conversely, Prestridge (2012) concluded in her study that teachers did not need have a high level of competency with ICT.
before they had the confidence to use it in the classroom. She recommended that research on the relationship between ICT competence and confidence would be important for effective ICT use in the classroom, as well as professional development that focusses on pedagogical use of ICT in the classroom.

Prior to the professional development, the participant teachers were mainly familiar with Microsoft Word, Excel, and PowerPoint. The professional development focussed on GeoGebra technology. However, responses after the professional development suggested that the participant teachers had gained awareness of technologies such as Derive, Geometer’s Sketchpad, Inspiration, Macromedia Authorware, Green Globe, Graphmatica, Khan Academy, Wolfram Alpha, and teAchnology as well as GeoGebra. Also, the results of the study suggest that the professional development had enhanced teachers’ understanding of legal, ethical, cultural, and societal issues related to technology. The teachers became critically cautious of how to share electronic materials at the end of the professional development. They had also become more aware of the mandate of the curriculum requiring them to use technology in mathematics teaching.

The study recorded positive change regarding teachers’ knowledge of technology and mathematics content (TCK). The professional development seems to have enhanced the participant teachers’ knowledge to use technology to depict specific mathematics content within the syllabus. Particularly, their confidence, knowledge, and skills in using GeoGebra to illustrate multiple representations of mathematics concepts was enhanced through this professional development. Also, the results of the study indicated a large increment in the participant teachers’ knowledge to infuse technology into their pedagogical decisions in the classroom (TPK). The participant teachers again acknowledged that through the professional development they had gained considerable knowledge and skill to apply technology to facilitate higher order thinking skills, including problem-solving, critical thinking, decision making knowledge, and creative thinking in mathematics (TPACK). The findings of this study were consistent with Kafyuililo et al. (2016) who reported that teachers showed increased knowledge and skills in using technology in science lessons after they had been engaged in a professional development programme. The authors indicated that through the programme teachers were able to learn and use technologies such as animation, PowerPoint, and videos to orchestrate more interactive instruction in physics and chemistry. Similarly, the teachers in Soebari’s (2012) study indicated their knowledge of subject matter and
classroom practices increased after they participated in professional development. It is reiterated in the study of the Office for Standards in Education (2004) that professional development training positively impacted on the teachers’ subject knowledge, teaching practice, students’ standards, and curriculum planning skills.

The participant teachers acknowledged that they had become more resourceful since they had engaged in the professional development. They indicated that the professional development had added value to the way they prepared their lesson plans. They now looked for online resources to augment the information in the textbooks and syllabi. Some of the teachers pointed out that it is more straightforward for them to prepare lesson plans using technological tools than pen and paper. They also indicated that they now paid more attention to the conclusion and evaluation aspects of the lesson plan than they previously did. For example, Gideon noted he now takes his time at the end of the lesson to summarise the salient ideas of the topic for the students. The study also showed that the professional development provided an opportunity for the participant teachers to upskill their knowledge and confidence in handling some of the mathematics topics in the senior high curriculum. These findings resonate with many authors who have articulated that the underlying principles of professional development are to advance teachers’ knowledge and skills in using the technology to improve pedagogies and classroom practice (Ertmer, 2005, Mishra & Koehler, 2006; Ottenbreit-Leftwich et al., 2010). Similarly, the teachers in the study of Ruthven et al. (2005) identified the internet as a wellspring of teaching and learning materials where the teachers selected materials to support their lesson preparation and enactment. Anthony and Walshaw (2007) held that teachers who were sensitive to the many learning opportunities or resources were likely to optimise those opportunities to extend students’ mathematics learning through multiple representations of mathematical ideas.

Webb et al. (2005) suggested that an effective professional development programme should have the potential to (i) improve teachers’ pedagogical knowledge and skills; (ii) modify teachers’ classroom practices and enhancing quality teaching; (iii) develop leadership capacity within the school; and (iv) build professional learning communities. Based on the focus of the current research study, the findings discussed in this chapter mainly align with the first two indicators of successful professional development identified by Webb et al. (2005).
However, the activities the teachers engaged in during the professional development reflected some aspects of the last two indicators. With regards to developing leadership capacity within the school, the teachers in this study worked in small groups of two or three in their own school for a continuous period of 12 months. They developed technology-based mathematics lessons, rehearsed among themselves, and subsequently taught the lessons to students in the actual classroom under the supervision of the researcher. Each group was a small learning community where they coordinated their own activities supported with exemplary lesson materials and lesson demonstrations. This promoted the “notion of distributed cognition” (Putnam & Borko, 2000, p. 8) where the participant teachers shared responsibilities, ideas, experiences, and materials both within and across groups. These activities contributed to the improvements in the participant teachers’ classroom practices as well as the use of technology in mathematics teaching. In an earlier report, Putnam and Borko (2000) reiterated that “experiences situated in the teachers' own classrooms may be better suited to facilitating teachers' enactment of specific instructional practices” (p. 7).

Hudson (2012) confirmed in her study that teachers who participated in school-based professional development were more inclined to use technology in mathematics classrooms than those who did not. Supovitz and Tuner (2000) added that there was a significant and robust relationship between professional development and teachers’ classroom practices and classroom cultures. They further found that professional development that engaged teachers to share pedagogical experience for a minimum period of two weeks was likely to bring a big change in teachers’ classroom practices and cultures.

Although the teachers showed an appreciation of the benefits they gained from the programme, some challenges were identified during the professional development. It took some teachers an extended time to acquaint themselves with some of the tools in GeoGebra: move icon, checkbox, and text formatting. Some teachers needed basic skills in computer windows to navigate through the various tools in GeoGebra successfully. They also indicated that working in groups required patience and tolerance to reach decisions.

**Transformation of teachers’ TPACK**

Over the past two decades, discussions regarding PCK and teaching with technology have gained considerable attention in the literature related to technology integration in education. Niess et al. (2009) identified and described five developmental levels (recognising, accepting, adapting, exploring, and advancing) of teachers’ TPACK when they are engaged
in technology-oriented activities. The level of proficiency in using technology to facilitate meaningful mathematics teaching and learning increases along this continuum. Key conclusions from the analysis of the data collected during the study were that teachers’ involvement in professional development activities, such as the design and enactment of GeoGebra-based mathematics lessons, seemed promising in facilitating their competences in integrating technology in mathematics. However, there were some similarities and variations in the development of the teachers’ TPACK following their engagement in the professional development.

The teachers entered the programme at an accepting level. All the teachers held strong beliefs and positive attitudes towards the use of technology at the onset of the professional development programme. Their prior experience with technology was mostly for administrative activities such as using Microsoft Word for typing examination questions, using Microsoft Excel for recording students’ continuous assessment, checking email, or finding the meaning of words via Google. Others (Gideon) used the mathematics component in Encarta to play games. Two of the teachers indicated they had used technologies such as Maple 12, and videos from YouTube for mathematics lessons. However, the descriptions they provided during the pre-interview about how they used these tools characterised the accepting level of TPACK. For example, Peter indicated he used it to support students to confirm their answers. Joshua, on the other hand, said he used it to aid visual representation of mathematical concepts. These indicate superficial application of TPACK (Niess et al., 2009).

Considering the TPACK development pathway, each teacher made progress in the development of the knowledge and skills of using technology in mathematics teaching. This result was expected because of the nature of the activities included in the professional development. Results reported in this study showed that when teachers’ pedagogical beliefs were challenged through professional development, they were able to maximise those beliefs to enhance students’ mathematics learning. During the professional development, the core practices of effective mathematics pedagogy were deconstructed and modelled for the teachers to rehearse through peer and classroom teaching under the watch of an expert (researcher/facilitator) for constructive comments, suggestions, and coaching. At the end of the professional development, all the participant teachers acknowledged that the programme
offered them a great learning opportunity to advance their pedagogical practices to enhance students’ procedural and conceptual fluency in mathematics learning.

Consequently, it was observed that as the teachers progressed through the professional development, they gained confidence about how to facilitate group work through mathematics lessons supported with technology and activity-based worksheets. In this regard, the improvement in the participant teachers’ classroom practices was like those of teachers in the constructivist-based professional development programme conducted by Myers and Halpin (2002). Antoniou and Kyriakides (2013) also reported that the professional development the teachers engaged in improved the classroom practices of the teachers as well as the students’ learning and achievement. Also, the findings in the current study resonate with those of many authors who have articulated that the underlying principles of professional development are to advance teachers’ knowledge and skills to improve pedagogies and classroom practice (Ertmer, 2005; Mishra & Koehler, 2006; Ottenbreit-Leftwich et al., 2010). Similarly, the teachers in the study of Ruthven et al. (2005) identified the internet as a wellspring of teaching and learning materials where the teachers selected materials to support their lesson preparation and enactment. Anthony and Walshaw (2007) held that teachers who were sensitive to the many learning opportunities or resources were likely to optimise those opportunities to extend students’ mathematics learning through multiple representations of mathematical ideas.

Despite the improvement in the way they enacted the principles of effective mathematics pedagogy with technology, critical analysis of their lesson episodes and interviews revealed some variability. The TPACK Michael, Mary, Prince, Gideon, and Cynthia demonstrated characterised the adapting level. Teachers demonstrated the ability to select appropriate technologies (GeoGebra, Microsoft Word, and Microsoft Excel) to engage students in collaborative learning in constructing mathematical concepts with teacher-directed support. For example, they were able to use GeoGebra to generate multiple graphs, text, diagrams, and animated objects that supported students to explore mathematics concepts. As demonstrated for them during the professional development, the lesson plans they developed showed sequence of activities that promoted students’ participation (Niess et al, 2009).

However, their prior experience in teaching and knowledge of basic computer skills seemed to obstruct the fluidity of enacting their PCK with technology. For example, Gideon made
some progress through the three lessons that he taught with GeoGebra. Unlike his first lesson, he was able to include pictures of real-life phenomenon to hook his students to the concept of quadratic equations. This facilitated the students to make meaningful mathematical connections between symbolic and real-life representations of quadratic equations. His prior experience in basic computer skills accounted for the flexibility in navigating through the GeoGebra window to coordinate students’ mathematics learning. He was able to resolve his own technical issues regarding the equipment setup for his lessons as the professional development progressed. However, it was observed that the integrated knowledge of PCK with technology he displayed during the lesson enactment was more teacher-led than engaging the students in exploring the concept collaboratively. Gideon’s difficulty in orchestrating meaningful cooperative learning with GeoGebra was connected to his teaching experience. He was in his first year of a teaching career and had no mathematics education qualification. Thus, it is likely that he was at the beginning stage of expanding his knowledge in addressing students’ learning, assessment, and classroom management.

Cynthia, on the other hand, showed appreciable skills in managing her students to work cooperatively in exploring the clockwise and anticlockwise rotation of objects. However, she struggled in handling the tools in the GeoGebra window, as well as the skills in setting up the projector and the laptop computer. This impeded the flow of her lesson when she wanted to toggle between different windows: GeoGebra and Microsoft Word. She seemed less confident in carrying out these activities independently at the end of the professional development. This is understandable as it was through this professional development that she practically experienced how technology was used in mathematics teaching. Myers and Halpin (2002) engaged teachers in professional development. Their preliminary results indicate that teachers who had no previous computer workshop experience felt more intimidated by computers than their counterparts who had computer workshop experience. Petrogiannis (2010) studied the relation between teachers’ preparedness to use technology and their prior experience with computer. Petrogiannis found that teachers with a minimum level of computer knowledge seemed prepared and ready for technology integration, rather than the non-experience teachers who had no prior knowledge of computers.

Overall, the adapting teachers reported their knowledge and confidence in using technology had been enhanced, but probing them further revealed that extra effort in terms of further
professional development and provisions of adequate technology facilities would be key to further improvement. The responses they provided regarding their intention to use technology in future suggested they had not fully assimilated the use of technology in mathematics teaching after they had received the professional training. It was observed that their self-will to use technology was low. It was noted that they only wanted to teach the lessons they had developed during the training so they could complete the phases of the professional development. It was observed that they needed constant encouragement before they used the technology in the actual classroom. They also expressed concerns about family responsibilities and workload in school as crucial factors towards the use of technology mathematics teaching. Consistent with earlier research, Kul (2012) reported that teachers, after gaining experience with GeoGebra-based activities, felt they had developed new ideas about teaching and learning of mathematics, but expressed concern about the time required to implement innovative teaching strategies in the classroom.

Joshua, Bernard, Sammy, Peter, and Martey were observed to use GeoGebra and other technologies such as YouTube and Khan Academy to create more learning opportunities for students to construct and redefine mathematical concepts. These included activities in their lessons that immersed students into reversed thinking where they (students) used the solution to look for related mathematics concepts connecting geometric drawing and algebraic symbols, which is evidence of teachers at the exploring level (Niess et al., 2009) and effective mathematics pedagogy (Anthony & Walshaw, 2007; NCTM, 2007). For example, Martey used questions that start with solutions to engage students in geometric thinking (i.e., (i) Draw on the grid sheet three different rectangles which all have an area of 24 cm$^2$. (ii) Draw on the grid sheet three different rectangles which all have a perimeter of 20cm). This activity engaged students to take risks and reformulate different approaches and thoughts before they arrived at the conclusions. Using varied instructional strategies to elicit responses from students is demonstration of effective enactment of TPACK (Mishra, & Koehler, 2006; Niess et al, 2009). The degree to which the exploring teachers demonstrated a student-centred approach of instruction and suitability of the application of technology was more visible compared to adapting teachers (Niess et al, 2009). According to Niess et al, the exploring teachers go beyond what is modelled for them during the professional development. Joshua and Martey were able to immerse students into reversed thinking. Anthony and Walshaw (2007) and NCTM (2007) both emphasised the engagement of
students in open-ended tasks helps them to extend their mathematical proficiency. For example, engaging students in classroom dialogue (Anthony & Walshaw, 2007; Martin & Speer, 2009) has the likelihood of promoting students’ proficiency in adaptive reasoning where they could extend their mathematical knowledge to unfamiliar situation in their settings (National Research Council, 2001).

In Joshua’s class, there were instances he demonstrated the exploring level of TPACK. He was able to increase student engagement by using GeoGebra to assist students who had difficulty in visualising the dynamics involved in drawing distance-time graphs. Using different colours to represent the graphs of different modes of transportation and further asking students to interpret the journey of each mode of transport, according to him, facilitated both high and low achieving students to grasp the abstract aspect of drawing distance-time graphs. He emphasised that in the instances when he had taught the same lesson without technology, the students, particularly the low achieving students, struggled to make sense of the concept of drawing a positively sloped line (\( \cdot \cdot \cdot \)) for a moving car, horizontal line (\( \cdot \cdot \cdot \)) when the car was at rest, and negatively sloped line (\( \cdot \cdot \cdot \)) when the car was having a return journey. At the end of the professional development, he seemed appreciative of his effort of using technology to redress particular learning needs of students. Thus, he seemed to have gained an increased understanding of using technology to provide opportunities for students to interact with multiple representations. These findings reiterated that of Saralar et al. (2018) who indicated that “some cognitive tasks are undertaken by students while interacting with the multiple representations, which allows them to construct deeper understanding” (p. 17). Franke et al. (2007) noted that mathematics learning arises from students’ participation and interactions within social space of the classroom that provide autonomy for students to share mathematical ideas.

Further evidence of the performance of the exploring teachers was that, despite limited resources being available in the school, they seemed optimistic about adopting technology in mathematics instruction in the future. They seemed confident in handling the projector and technical issues associated with it. They demonstrated pronounced confidence in setting up their equipment for mathematics instructions. They also appreciated the constraints at hand and were determined to make advances on what they had learned. They seemed comfortable using GeoGebra and showed eagerness in exploring more topics within the mathematics curriculum that could be taught with GeoGebra. For example, Joshua stressed
that teachers have “no excuse” to say that using technology in their school is impossible because it has been tried and tested in their own environment. Notwithstanding their optimism in using technology in the future, they also anticipated that certain factors, such as administrative support and electricity supply, could shackle their moves to use technology in the classroom. This finding is consistent with the results of Soebari (2012) as she reported the teachers in her study were keen to translate the knowledge and skills they acquired through the professional development, but they indicated their intention could be hampered by the contextual factors such as limited support from school leadership, and inadequate resources and facilities.

The data gathered in the study identified one teacher (Jonathan) at the advancing level at the end of the professional development. Jonathan had never used technology in his teaching prior to the professional development. His perceived knowledge and use of technology at the beginning of the professional development could be considered as being at the accepting level. In the pre-interview he perceived technology as an essential tool in mathematics teaching because he believed it could promote visual representation, aid students to have a clear understanding of mathematics concepts, and it could reduce abstract teaching. He became able to use GeoGebra to demonstrate these beliefs profoundly as he progressed through the programme.

In addition, Jonathan was able to transfer the knowledge he gained through the professional development into another discipline (chemistry). The improvement in his TPACK was related to his decision to adopt technology in his classroom work (Niess et al., 2009). By the close of the professional development programme, it seemed he had evaluated his decision to use technology and he had begun using it frequently in the classroom. He indicated in the second interview that he had seen the benefits of using GeoGebra, which had convinced him enough to apply other technologies such as YouTube and Khan Academy in his lessons. He indicated he had begun sharing some of his technology-based lessons with other teachers in the science department. He added that he had begun inviting his teacher colleagues in the science department to embrace the idea of using technology in science instruction. Assuming a leadership role in coordinating the use of technology in teaching and learning is an indication of profound application of TPACK. Although all the teachers in the study accepted the idea of championing technology integration, he had begun putting it into practice before the programme ended. He further expressed interest to do a master’s degree
programme in science education where he wanted to explore more of the pedagogical usefulness of technology. Thus, with the right support, teachers could go beyond the beliefs they professed about the use of technology in mathematics teaching and learning.

Other evidence consistent with Jonathan being at the advancing level was his commitment to facing the challenges in using technology in the school. Because the ICT laboratory is occupied most often for ICT lessons, he had created a space in the science laboratory where he used a laptop and a projector for his technology-based lessons. This illustrated his profound willingness to use technology in teaching and learning despite the constraints. This result is consistent with Niess’ (2007) assertion that advancing teachers identify the possible barriers of adopting technology and devise an extensive approach to maximise the resources available in their school. At the advancing level of TPACK, the teachers ought to be active, consistent, and novel in using technologies to create teaching and learning spaces where students could (i) explore every aspect of mathematics; (ii) expand mathematics concepts; and (iii) use internet to investigate mathematical problems (Niess et al., 2009).

The study showed that teachers who had a lot of teaching experience were able to demonstrate more flexibility and confidence in using technology in a more student-centred approach, than the less experienced teachers. The extant research finding confirmed this. For example, Niess et al. (2006) reported that teachers who had less teaching experience struggles to make strong connections between technology, content, and pedagogy practically. Pierson (2001) also found that teachers with low pedagogical knowledge most often failed to make strong linkages between pedagogy and technology, even if they had a high knowledge of technology. Orlando and Attard (2016) observed that teachers’ knowledge of mathematics content, as well as pedagogy, are crucial determinants of effective integration of digital technology in teaching and learning of mathematics. Orlando and Attard indicated that teachers might have had a clear intention to use technology to enhance teaching and learning, but their level of mathematics and pedagogy knowledge could reduce their success. Similarly, Ingram et al. (2016) identified that the experience and confidence levels of teachers were crucial determinant in technology adoption in mathematics.

For teachers to successfully integrate technology into their pedagogical decisions, the recent report by Niederhauser et al. (2018) reiterated that the kind of activities incorporated into
technology-related initiatives are vital factors. The authors highlighted that professional intervention grounded in inquiry and constructivist principles of learning could offer a potential change in teachers’ use of technology. The professional development chosen for this current study had elements of constructivist principles of learning. The participant teachers were engaged in a GeoGebra technology environment to explore the authentic use of this new tool in mathematics teaching. They worked in groups to develop and teach GeoGebra supported lessons, which they first taught to their peers, discussed, revised, and subsequently taught the same lessons to students in the actual classroom. These activities offered the participant teachers the opportunity to rehearse and transition the knowledge of technology and pedagogy into practice where they personally and dynamically consolidated their existing PCK with technology within instructional contexts. The participant teachers indicated that the activities included in the professional development created enormous awareness, confidence, and the needed pedagogical insights for using technology to teach mathematics. They also affirmed that working in groups offered them the chance to have pedagogical reflections of using technology effectively in mathematics instructions. It was exhibited in Kul’s (2012) study that after the teachers had gone through the GeoGebra professional development training rooted in constructivist principles, they were able to engage in mathematical tasks that helped them develop better understanding of the students’ role in technology learning environment, and maintain their beliefs in favour of student-led instructional approach. Sacristán et al. (2010) contended that when teachers used digital technology in planning their lessons, it offered them the opportunity to examine and explore diverse ways of teaching mathematical concepts and problem solving to their students.

It can be hypothesised that teachers’ professional knowledge and technology resources are important in the implementation of technology, but it takes self-will and determination to combine these elements to maximise student learning. On the other hand, these elements seemed to be highly influenced by the kind of professional training the teachers were exposed to. This study has provided a concrete picture of how participant teachers developed and enacted their TPACK through professional development that engaged them to work in groups to design and teach GeoGebra-based mathematics lessons supported by demonstration lessons and peer-reflection. The findings indicate that this approach of professional development holds promise in developing the behaviour of most of the teachers regarding the best practices in mathematics classrooms (Cochran-Smith, 2005; Supovitz &
Tuner, 2000). Teachers need to be given sufficient “time to interact with and help each other as they explore new technologies and new pedagogies” (Ertmer, 2005, p. 35).

**Implications of the Findings**

This research study does not only contributes to our understanding of how the professional development programme influenced participant teachers’ pedagogy and technology dispositions, but the findings have many implications for mathematics education practices and TPACK research.

**Contributions to effective mathematics pedagogy**

The literature reviewed for this study shows the strength of the growing knowledge base in effective mathematics pedagogy. However, there is contention about the constituents of effective mathematics pedagogy, particularly when it comes to the use of technology in teaching and learning. This study joins this debate by providing insights into how teachers enacted effective mathematics pedagogy using GeoGebra. This study offers two key contributions to the literature of effective mathematics pedagogy. First, drawing on the earlier works of Anthony and Walshaw (2009) and NCTM (2007), this study condensed the principles of effective mathematics pedagogy into five central themes: creating mathematical settings, useful mathematical tasks, mathematical discussions, mathematical connections, and assessment of students’ learning. Second, it theorised 31 core practices across these themes. For the purpose of reiteration, Table 6.1 provides a summary of the descriptions and the core practices of each theme. The study does not claim the exhaustiveness of these core practices. Rather, it provides a starting point of addressing the generic and underspecified description of effective mathematics pedagogy in the literature, particularly with regards to the use of GeoGebra in the mathematics classroom. It is important to acknowledge once again that both the central themes and the core practices identified overlap indicating the non-linearity of teachers’ actions in the classroom.
<table>
<thead>
<tr>
<th>Effective mathematics pedagogy</th>
<th>Description</th>
<th>Core practices</th>
</tr>
</thead>
</table>
| Creating mathematical settings | Involves the skills and knowledge of the teacher to set up the learning environment to hook and sustain the students’ interest and attention throughout the mathematics lesson. It involves the actions the teachers take before, during, and after the lesson. | 1. Preplanned lesson documents  
2. Equipment setup  
3. Shared instructional objectives  
4. Clear instruction about the task on the worksheet  
5. Linking prior knowledge to new learning  
6. Using pictures of real-life scenarios to initiate mathematical dialogue  
7. Explanation of terminologies  
8. Addressing existing misconceptions  
9. Attention for individual learning needs |
| Useful mathematical tasks | Involves how the objectives and activities included in the GeoGebra-based lesson engage students in exploring and understanding mathematical concepts, procedures, and/or relationships. | 1. Visualising mathematical concepts  
2. Recording  
3. Calculating  
4. Predicting  
5. Constructing new ideas |
| Mathematical discussions | Involves creation of a learning environment to facilitate classroom dialogue which emphasises mathematical argument where conclusions are reached through agreement between students and teacher. | 1. Consolidating ideas students have constructed through verbal and written responses  
2. Correcting misconceptions associated with the new concept  
3. Posing mathematical questions  
4. Group/individual presentations  
5. Revoicing  
   - Reword students’ solutions  
   - Use GeoGebra to confirm students’ solution  
   - Use GeoGebra to elaborate concepts  
6. Gives students autonomy to apply the concept |
| Mathematical Connections | Involves how the teacher engages students to use unrehearsed approaches or pathways to generate multiple solutions for a problem. It includes the activities that probe students' thinking and expansion of mathematical knowledge to real-life situation. | 1. Repose mathematical question  
2. Extend concepts learned to new contexts  
3. Use GeoGebra to emphasis real-life phenomena  
4. Reverse thinking  
5. Multiple solutions  
6. Risk taking |
| Assessment of students’ learning | Involves the creation of a learning environment where formative feedback and feedforward from the teacher and students are promoted to monitor the progress of students’ learning in a specific mathematical task. | 1. Review and correct students’ errors  
2. Use GeoGebra to provide prompt feedback  
3. Remedial teaching  
4. Shared ideas  
5. Reflection on the solutions |
Contextualising these core practices of effective mathematics pedagogy, there are a number of implications that can be drawn in relation to ICT in education policies and the mathematics curriculum for Ghanaian schools. The recent ICT policy document reviewed by Ghana’s Ministry of Education reiterated, among other factors, the criticality of teachers’ knowledge, skills, and attitudes in effective use of technology in enhancing student-centred teaching, collaboration, creativity, and higher order thinking skills (Ministry of Education, 2015). Reflecting on the current review and the mandate of the country’s mathematics curriculum, the core practices identified offer a reference point for teachers to rethink the pedagogical decisions for infusing technology into mathematics instructions.

The teachers who participated in this study share similar characteristics of most of the teachers across the country in terms of teaching qualifications for mathematics in the senior high school. Therefore, the evidence drawn from them could be replicated by many teachers in the country. Also, these core practices were generated from teachers who worked under limited ICT resources, therefore, it would be easier adopting these practices in schools that are well-resourced as well as schools that share similar characteristics of where the study took place.

With regards to professional development for mathematics teachers in the country, the literature reviewed indicated that most of the programmes were generic and offered less professional competence for teachers to use technology in mathematics classroom. Therefore, untangling the confounding perceptions regarding the stand alone technology professional development programme (Wang & Chen, 2006), these core practices offer a potential pedagogical guideline for supporting teachers to enact effective mathematics pedagogy with technology. The study hypothesised that engaging teachers to pedagogically explore the strategies of using GeoGebra for classroom use could enhance the level of their beliefs, attitudes, and knowledge, where they could select appropriate technology to foster students’ mathematical thinking and exploration.

**Contribution to the TPACK framework**

This research study joins in the debate that effective professional development for technology integration should be rooted in a theoretical framework that involves practical activities in specific technology, pedagogy, and content. Though Mishra and Koehler (2006) acknowledged the relevance of context (the school environment, prior experiences, beliefs,
and attitudes of teachers) in the effective implementation of technology, this component is less explored in most of the TPACK research studies (Chai, Koh, & Tsai, 2013). The current study fills that gap in the literature by providing concrete evidence of how the teachers transformed their espoused knowledge, beliefs, and attitudes into practice through the influence of professional development. TPACK frameworks provided unique approaches for understanding the complex interplay among the teachers’ knowledge, beliefs, and attitudes related to the use of technology in mathematics. For example, TPACK was effective in distinguishing patterns of responses among participant teachers and in identifying relationships between broad categories of their beliefs and classroom use of technology.

Studies in the literature are inconsistent about which of the TPACK domains mostly contribute to the teachers’ TPACK self-efficacy. The current study could not address this gap through a robust statistical technique because of the limited number of the participants involved in the study. However, this study suggests that this inconsistency is due to the sensitivity of the context to the development of teachers’ TPACK. For example, it was obvious in this study that the increase in the individual knowledge domains (TK, CK, and PK) and their intersections (TCK, PCK, and TPK) compositely contributed to the teachers’ ability to demonstrate their espoused TPACK in the classroom. The findings from this study seemed to suggest that the improvement in the teachers’ perceptions and demonstration of TPACK were mainly explained by the development in their TK. However, teachers with more teaching experience seemed to demonstrate their TPACK more profoundly compared with less experienced teachers.

Again, the research study updates us on the applicability of Niess et al.’s (2009) model in explaining the development of teachers’ TPACK pathways. The model provided a useful understanding of how teachers progressed through the combined knowledge of technology, pedagogy, and content. Considering the Niess et al. model, the findings reported in this work have a number of implications for the practices of mathematics education. First, the study found that participant teachers were at the lowest level (recognising) of TPACK development prior to their engagement in the professional development. They had a strong desire and value belief for the implementation of the technology in their practices, but due to the limited technology resources, knowledge, and skills, they were not able to apply the technology meaningfully. This suggests that 10 years after introducing technology into the educational system in Ghana, the methodology courses offered to prospective teachers at the
various universities and colleges in Ghana have not adequately prepared them to use the tools for teaching and learning.

Contribution to research methodology
There is evidence suggesting an in-depth interpretivist research provides a rich understanding of how teachers adopt technology in teaching and learning (Ertmer, 2005; Kim et al., 2013). The study adopted multiple methods and sources of data to provide a holistic picture of teachers’ learning to teach mathematics with technology. Data collected through a self-report survey, semi-structured interviews, focus group discussions, lesson plans, and lesson observations is a response to previous literature indicating the need to expand the approach of exploring teachers’ TPACK. Also, the third column and clustered approach analysis adopted in studying the qualitative data is the first of its kind in TPACK literature. The column analysis approach was reported in an affect and identity study by Ingram (2011). To ensure the objectivity of the findings reported from the data in my study, constant reflection of the data was achieved using the third column analysis. The third column analysis is the interpretation written in the right-hand column of the interview/focus group discussion form transcribed or in the Word document of the lesson plans. This interpretation was done repeatedly throughout the study. The researcher’s logbook and Comment feature in Microsoft Word was used to achieve that. When the transcribed data was finally transferred into the HyperRESEARCH software (version 4.01) for detailed coding and analysis, the affordance of the Description feature in the software was used to continue the third column analysis. The third column analysis helped to do multiple coding that captured teachers’ feelings of comfort and discomfort, excitement, reactions, values, and beliefs about the use of technology in mathematics teaching and learning. The cluster feature in HyperRESEARCH 4.01 provided a quick glance which helped me to do the inductive and deductive coding.

Also, the study was categorised into three phases. Each phase offered the opportunity for the participant teachers to construct and reconstruct their world views about technology integration in mathematics, through semi-structured interviews, focus group discussions, a self-report survey, lesson plans and/or lesson observations. This provided a combined breadth and depth of findings that would not have been achieved using a single method of data collection or data collected in a single shot (one-time data collection). For example, the teachers were engaged in multiple focus group discussions at the various stages of the
professional development to enable them to share knowledge and experiences. It also supported them to brainstorm and generate ideas, discuss problems, and identify possible solutions regarding pedagogical use of the technology in mathematics teaching. Capturing the significant ideas or issues from the participants at each phase helped to improve the subsequent activities of the professional development. Also engaging the teachers in group discussions promoted autonomy and reduced power differentials. This allowed them to express their views about technology integration freely. The lesson plan preparations and enactment offered opportunities for the teachers to operationalise their espoused improvement in their TPACK.

**Contribution to the principles of effective professional development for teachers**

TPACK has attracted worldwide attention as a framework to guide research and professional development focussing on teachers learning to use technology in the classroom. However, there is a paucity of TPACK research studies within Sub-Saharan African countries. In Ghana, Agyei and Voogt (2012) adopted TPACK as a framework for pre-service teachers’ professional development to integrate technology in mathematics teaching. In Tanzania, it has been used for both pre-service (Kafyulilo et al., 2014; Kafyulilo, Fisser, Pieters, & Voogt, 2015) and in-service (Kafyulilo et al., 2016) teachers’ professional development. Thus, the current study broadens our understanding and further confirms the usefulness of the TPACK framework within the African continent, particularly Ghana. The literature reviewed for this study included only one professional development research study in Ghana, conducted by Agyei (2012). He focussed on the preparation of pre-service teachers for ICT integration in mathematics instruction. Other studies that have been conducted in the country on technology in mathematics education have mainly centred on the exploration of teachers’, students’, and other stakeholders’ perspectives regarding technology integration through survey and/or interview reports. The current study addresses the paucity of research studies that focus on professional development as an interventional approach to assist in-service mathematics teachers to gain professional competence regarding the effective use of technology in mathematics teaching and learning. The study showed that though the teachers had positive beliefs and attitudes toward the use of technology in mathematics instruction, they were less aware of the mandate of the curriculum expecting them to use technology in mathematics instruction. Also, many of the in-service teachers in this study had limited awareness of existing mathematics software for teaching and learning. This implies that
much more needs to be done in terms of regular updates of teachers’ technology competence through professional development.

The research study also contributes to the literature in terms of understanding the various approaches for developing and measuring teachers’ TPACK alongside their technology beliefs and attitudes. First, the study provided a detailed account of the activities of the professional development that enabled teachers to advance their knowledge, beliefs, and attitudes towards the use of technology in the mathematics classroom. The activities include the introduction to GeoGebra, demonstration lessons, group work, teaching rehearsals, classroom teaching, post-lesson discussion, and expert support. The findings indicate this approach of professional development holds promise for advancing mathematics teachers’ professional competence related to use of technology. For example, the study reported that when teachers’ existing pedagogical beliefs were modelled with innovative approaches (mathematics instructions supported with GeoGebra and activity-based worksheet), and presented to them for replication, it challenged them to reorient their beliefs and thinking to enhance effective mathematics instruction. The study found that the lesson demonstrations conducted by the researcher stimulated teachers’ interest and pedagogical reflection about the use of technology to teach mathematics. The group and post-lesson discussion offered teachers the opportunity to share the best knowledge, classroom practices, and teaching experiences among themselves. The post-lesson discussions and the feedback from the expert offered a useful guide for the teachers to iterate the GeoGebra-based lessons they developed to the next level.

**Proposed principles for technology professional development**

The initial assumptions for this research study aligned with the TPACK framework, which emphasises the integrated knowledge teachers need to have in terms of technology, pedagogy, and content for effective implementation of technology in mathematics teaching and learning. The data from my participants and the literature reviewed for this study have highlighted the complexities for effectively engaging teachers in professional development related to the use of technology in mathematics teaching. Evidence from the data deepened our understanding of the contextual influences (particularly, school environment and teachers’ prior experiences - beliefs, attitudes, and knowledge towards technology) on the development of the mathematics teachers’ TPACK.
It was not the initial focus of this study to propose effective principles for professional development. However, my understanding of the literature reviewed for the study, contextual factors of the school where the study took place, and the potential of the activities implemented in this professional development has culminated into principles that could be useful for future implementers of professional development. The principles proposed are listed below. Some of the principles indicated below reiterate literature from different perspectives of in-service teachers with varied teaching experience in mathematics. Another uniqueness of these principles is that they were created from a theoretical perspective (TPACK), rooted in the core practices of effective pedagogy and mathematics curriculum. Also, they were framed from the teachers’ perceptions of knowledge, beliefs, and attitudes towards technology, as well as demonstrated evidence of their knowledge and ability through hands-on activities in a GeoGebra learning environment for a continuous period of 12 months. The emergence of “expert visibility” (the second item) which the participant teachers perceived as crucial in shaping their professional competence (knowledge, beliefs, and attitudes) in the use of technology in mathematics instruction has not been reported in the literature. The participants of this study acknowledged the visibility of the expert (which in this case was the researcher) as a key factor in enhancing their confidence in using technology in mathematics teaching. When the expert is frequently visible in the school environment, it offers opportunities for the teachers to resolve issues related to the new instructional approach which they are engaged in, as well as other general pedagogical issues in mathematics.

1. When the activities of professional development are framed based on teachers’ existing beliefs and their prior experiences, it challenges them to shift their beliefs towards effective classroom practices.

2. Having expert(s) available and visible throughout the professional development programme offers frequent opportunities for teachers to resolve pedagogical issues. Regular availability of the expert reignites teachers’ pedagogy and dispositions towards the new instructional approach. It offers teachers the opportunity to have prompt and constructive feedback to enhancing iteration of their lesson preparation and enactment.

3. Extended and continual professional training enhances regular updates of the professional competencies of the teachers. The study reported that all the teachers in the study recommended that they need additional training to consolidate their knowledge in the new pedagogical practice they had acquired.
4. The opportunity for teachers to make inputs into the activities of professional development enhances the ownership of the products that come out of it.
5. Situating professional development in a pedagogical framework engages teachers thinking towards the best practices in mathematics classroom.
6. Teachers’ knowledge and skills in specific technologies are crucial for effective implementation of technology in mathematics teaching and learning.
7. The technology used to mediate professional development should be adaptable in the school environment to enhance continuity. It should embody mathematics content and pedagogy. It should be flexible to use, readily accessible, and affordable, to attract teachers to expand their knowledge after the programme is over.
8. Group work activities offer useful opportunities for teachers to share the best knowledge and experiences of their professional practices.
9. Exemplary materials and demonstration lessons stimulate teachers’ interest. It further offers them the opportunity to engage in deep pedagogical reflection and replication.
10. Post lesson discussion offers a platform for peer critique and potential suggestions to improve the lessons developed and enacted by the teachers.
11. Teaching rehearsals offer opportunities for the teachers to practically simulate how they will enact the new instructional approach. The suggestions offered by peers and expert(s) are used to improve the quality of the lessons the teachers develop.
12. School-based professional training offers opportunities for teachers who have the same culture and curriculum goals to share diverse views on how to achieve their goals. It also challenges them to maximise the limited technology resources available in their school.
13. Having sufficient resources (hardware, software, and technology support) is crucial in effective professional development. This study reported that one teacher per laptop computer would be useful for extended practice both at school and home.
14. Ensuring cordiality and respect with the participant teachers in a professional development programme is important to maintain the trust each person has in one another.
Limitations of the Study

The effectiveness of a professional development programme is difficult to fully gauge when evidence is obtained from only teachers engaged in the programme (Penuel et al., 2007). Although the study focussed on how to support teachers to gain professional competence in the use of technology in mathematics teaching in a particular school, the findings reported may have been richer if additional data had been triangulated from students and administrative heads of the school. For example, additional information from students and administrative heads about the teachers’ classroom practices could have strengthened the delivery of the professional development organised for the teachers. According to Penuel et al. (2007), “when teachers are asked about specific practices and the frequency with which they engage in them, there is often good agreement between teacher self-report and observations or regularly collected teacher logs” (p. 926). Thus, teachers are often “aware of and biased favorably toward endorsing” a particular instructional approach introduced to them (p. 926).

Another limitation is associated with the breadth of the professional development. It is often difficult to determine the success or failure of a professional development model when evidence is drawn from a specific domain, such as mathematics, or a specific instructional approach, or set of curriculum materials (Penuel et al., 2007). In terms of breadth, the current professional development was conducted in a single school and in a mathematics discipline. Different schools have different cultures, and thus the efficacy of the professional development model proposed in this study could have been more adequately evaluated if it was carried out in multiple schools. The interview data augmented the evidence of the core practices of effective mathematics pedagogy identified through lesson episodes and lesson plans. However, the interview data relied on general questions about changes of the classroom practices of the teachers following their engagement in the professional development (see Appendix F). The core practices identified could have been richer if specific questions had been asked related to each theme of the effective mathematics pedagogy. For example, *How did you use GeoGebra to enact assessment in your classroom?* or *How did you use GeoGebra to enact effective mathematical discussion in your classroom?* These kind of questions could have generated an interesting discourse related to teachers’ enactment of effective mathematics pedagogy. Also, this study explored the link between teaching practice and professional development to establish the core practices of effective
mathematics pedagogy. The results of this investigation point toward a strong relationship between professional development and the kinds of teaching practices enacted by the teachers. However, a more comprehensive professional development programme would have further established how the changes in the teachers’ classroom practices influence students’ achievement (Supovitz & Tuner, 2000).

A further limitation of this study was my role as a developer, observer, and leader of this professional development programme. Although the issues outlined in the ethics of this study were followed to the letter, it is important to acknowledge the element of selectivity in some of the decisions I took. The selection of the ten basic lessons (see Table 4.1) for the introductory activity of GeoGebra were based on my evaluation of the teachers’ preliminary dispositions towards the use of technology and the content of the mathematics curriculum for the academic year. The activities were meant to support them to identify various constructing, geometric, algebraic, graphic, and statistical tools in the GeoGebra window. However, I realised that the introductory activity could have been more focussed if we had concentrated on a particular tool because not all the teachers successfully mastered the skills in each tool, as they indicated during the post-interviews. Notwithstanding, this brought variations in the lessons that they selected and designed for classroom use. Also, another area of selectivity was the analysis and interpretation of the data. For example, the themes generated from the data were based on my own interpretation and understanding of the literature I reviewed for the study.

The inherent limitation in a case study approach could influence the findings reported in the study. A small number of participants were selected for the study for the purpose of gathering detailed information about the shift in teachers’ pedagogy and technology dispositions following their engagement in professional development mediated with GeoGebra. The study hypothesised that the teachers’ increased knowledge in technology contributed to the enactment of effective mathematics pedagogy. However, this assertion was not reached through a rigorous statistical approach because of the small number of participants included in the study. Thus, the study is narrow in terms of the number of participants involved and that makes generalisations from the findings problematic (Williamson-Leadley, 2015).

Considering the duration of the study (12 months), the changes in the teachers’ classroom practices and the use of technology reported in the data could be temporary. However, they
give us hope that the professional development has at least reawakened the minds and behaviour of most of the teachers regarding the best practices in mathematics classrooms which in turn may affect student learning (Cochran-Smith, 2005; Supovitz & Tuner, 2000). Teachers need to be given sufficient “time to interact with and help each other as they explore new technologies and new pedagogies” (Ertmer, 2005, p. 35).

**Recommendations**

1. The initial results of the study confirmed the limited knowledge and skills the teachers in this Ghanaian school had in the use of technology in mathematics teaching. This school could be considered a reasonable representation of schools within Ghana. This implies concerted efforts are needed to address teaching with technology throughout the country. I argue that the COEs and the universities that train teachers need to face-lift their curriculum. Teachers ought to be trained to appreciate the contextual difficulties of infusing technology into their mathematics instructions. It is hard to meet all the needs of technological resources in the school, but this research study advocates that when teachers are competently trained, they would be able to maximise the limited resources available in their schools. Also, the Ghana Education service, donor partners, and the subject associations should rally around to provide professional development which is school-based, with pedagogy oriented to promote the use of technology in mathematics teaching and learning. Funding is needed to ensure successful organisation of professional development programmes.

2. The lack of ICT facilities still poses a major threat to the successful implementation of technology in the classroom. Although teachers preferred to work in groups when they are designing GeoGebra-based mathematics lessons, they believed such an approach would be more effective if each teacher had their own computer. The government and other stakeholders in education should commit more resources to ensure uniform distribution of technology resources in the school.

3. Teachers who have limited prior experience with a computer’s operating system (e.g., Windows) struggled to navigate smoothly in the GeoGebra window. Therefore, future leaders of GeoGebra in professional development for teachers should take this
into consideration by ensuring that teachers who enter GeoGebra professional development have basic skills in using computers.

4. The study reported that teachers applied the knowledge they acquired through technology-related professional development differently. For example, it was observed that teachers who had additional responsibilities related to school management and administration applied the new knowledge in these areas to offload their tasks. This implies that carefully planned activities of professional learning and support are necessary for effective use of technology in the mathematics classroom. Teachers could be grouped based on their responsibilities in the school and professional development organised for them accordingly.

**Directions for Future Research**

The findings of this study leave us additional questions to answer. Example of such questions are stated as follows:

1. This study proposed a model for exploring how teachers enacted effective mathematics pedagogy in a GeoGebra learning environment. The model identified five constructs (creating a mathematical setting, useful mathematical tasks, mathematical discussion, mathematical connections and assessment of students’ learning) and reflective practice as being central to effective mathematics pedagogy. My experience in exploring all these core practices in a single study showed it is a daunting task because of the complexity and non-linearity nature of teachers’ actions in the classroom. Therefore, future research may concentrate on a particular practice of effective mathematics pedagogy to enable detailed exploration.

2. The teachers in this study indicated that conducting mathematics lessons in a GeoGebra environment changed their instructional approach, which they believed enhanced their students’ learning and achievement in mathematics. This study could not collect data (achievement test scores or interviews) from the students to confirm this assertion. Hence, further research could look at a professional development approach where data from both teachers and students are collected to establish any correlation between changes in the teachers’ instructional approach and achievement in the students’ learning. Such study should be longitudinal, and it should ask
questions from the perspective of both the teachers and students about which actions of the teacher-student interaction promoted effective instruction and learning.

3. Mobile technology and social media applications are growing. They are handy and ubiquitous in most geographical areas. Therefore, to build on this study, inquiry into professional development mediated with these technologies could offer useful insights into new approaches of altering teachers’ pedagogy and dispositions digitally, particularly in schools which are under-resourced in terms of technology facilities.

4. The study identified the efficacy of expert visibility in a school-based professional development. This invites further qualitative and/or quantitative research for confirmation.

5. To gain a holistic understanding of the relationships between technology and teachers’ classroom practices in a specific discipline, for example, mathematics, in a particular school, data triangulated from classroom teachers, students, and administrative heads in the school would offer useful insights into effective pedagogical use of technology.

Final Thoughts

The results of this study and the recommendations are likely to create much-needed awareness and attention among mathematics teachers to improve their pedagogical skills and to make mathematics learning enjoyable. The results of the study could provide mathematics teachers in Ghana and countries of similar contexts the needed information on technology integration in the teaching and learning of mathematics. The results of this study may again provide some hope for the government and Ministry of Education on the agenda they are pursuing regarding the use of ICT in teaching and learning. Further, the findings of this research add to the existing body of knowledge on the importance of integrating technology in teaching and learning. The findings provide a new discourse in technology integration where TPACK and the central themes in effective mathematics pedagogy have been integratively explored to gain insights into how teachers develop their pedagogy and dispositions towards the use of technology. Thus, the findings reported in the study could inform policymakers, stakeholders, teachers, and curriculum developers about the relevant
information needed for supporting teachers to build sufficient competencies in technology integration.

Finally, this research provides a promising approach to professional development that builds teachers’ dispositions towards the use of technology in mathematics teaching and learning. There were clear indications that teachers engaged in design team professional development were able to develop and orchestrate GeoGebra-based mathematics lessons which engaged students to observe patterns, record and conjecture ideas, take risks, argue mathematical facts, make connections between a concept and its related real-life phenomenon, and draw conclusions independently and/or collaboratively. These findings are evidence of effective mathematics pedagogy.
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doi.org/10.1007/BF03173218


APPENDICES

Appendix A: Approval Letter from the Human Ethics Committee, University of Otago

2 March 2017

Dr C Linsell
College of Education
Division of Humanities
145 Union Street East

Dear Dr Linsell,

I am again writing to you concerning your proposal entitled ‘Ghanaian in-service mathematics teachers' beliefs, attitudes, knowledge, and skills related to the use of technology in mathematics teaching.’, Ethics Committee reference number 17/014.

Thank you for your e-mail of 28th February 2017 with response attached addressing the issues raised by the Committee.

On the basis of this response, I am pleased to confirm that the proposal now has full ethical approval to proceed.

Approval is for up to three years from the date of this letter. If this project has not been completed within three years from the date of this letter, re-approval must be requested. If the nature, consent, location, procedures or personnel of your approved application change, please advise me in writing.

The Human Ethics Committee asks for a Final Report to be provided upon completion of the study. The Final Report template can be found on the Human Ethics Web Page

http://www.otago.ac.nz/council/committees/committees/HumanEthicsCommittees.html

Yours sincerely,

[Signature]

Gary Witte
Manager, Academic Committees
Tel: 479 8258
Email: gary.witte@otago.ac.nz

c.c. Dr K Pratt  College of Education
Appendix B: Consent Letter from the School

GES/ASHS/PF/IB/16
21st February, 2017

Dear Gary Witte,

LETTER OF CONSENT FOR THE PROPOSED RESEARCH STUDY TO BE CARRIED OUT

I, [Name], Headmistress, Senior High School together with the School Management Committee and the Mathematics Department of the school, write to give our formal consent to Dr. Chris Linsell, Dr. Naomi Ingram, and Mr. Isaac Benning of University of Otago, New Zealand to carry out the research study entitled “Ghanaian in-service mathematics teachers’ beliefs, attitudes, knowledge, and skills related to the use of technology in mathematics teaching” in the above-mentioned school.

The school is committed to improving teaching and learning of recommended subjects, including mathematics, which plays a crucial role for admission into tertiary institutions in Ghana. We have realized that the proposed research study is on professional development that focuses on promoting teachers’ pedagogical use of technology in mathematics teaching. The school is therefore ready to encourage the teachers in the mathematics department to participate fully in the study.

Also, the principal researcher, Mr. Isaac Benning previously worked with us in the school for five years and knows the culture here; as such his presence in the school will not obstruct the academic activities.

It is our hope that this letter would serve the needed purpose, by paving way for him to pursue his research work in your esteemed institution.

Please, for any further information concerning the matter, do not hesitate to contact me for assistance.

Yours sincerely,

[Signature]

(HEADMISTRESS)
Appendix C: Interview Guide for Teachers before the PD

Thank you for showing an interest in this project. This interview is exploring you views about the use of technology in mathematics teaching. In the event that the line of questioning develops in such a way that you feel hesitant or uncomfortable you may decline to answer any particular question(s) and/or may withdraw from the project without any disadvantage of any kind. You are also reminded that any issue(s) you raised during this interview will only be used for the purpose of this study. Please try to be candid as much as possible in the responses to the following questions.

1. Have you used technology in your mathematics teaching before?
2. How did you use the technology in mathematics teaching?
3. What motivated you to use technology in your mathematics teaching?
4. What prevented you from using technology in mathematics teaching?
5. Does the use of professional development influence your knowledge in technology integration in mathematics teaching?
6. In what way could the use of professional development influence your classroom practices?
7. Do you think technology has some benefits at all in mathematics classroom?
8. Would you encourage other teachers to use technology in their teaching? Why or Why not?
9. Do you believe it is important to learn how to use technology?
10. Describe how you feel about using technology in the classroom?
11. Do you believe the use of technology could change your instructional approach? How?
12. How often do you use technology to teach mathematics?
13. Do you need assistance before you can use technology in the classroom?
Appendix D: Self-report Questionnaire

This questionnaire is meant to collect information about your knowledge, skills, beliefs and attitudes of integrating technology in mathematics teaching. The information provided in this questionnaire will be used for reference only. All information will be treated with high confidentiality anonymity.

SECTION A: Personal Information

Indicate the appropriate box by a tick (√)

1. Sex:     Male ☐     Female ☐


3. Teaching experience: less than a year ☐ one-five years ☐ six-ten years ☐ Ten years and above ☐

4. I have attended technology related professional development before: Yes ☐ No ☐

5. I have been using technology to teach mathematics: Yes ☐ No ☐

6. If yes which technology (name as many as you have used)

…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………

7. I have been using technology to teach mathematics since (please state it in years)…….

8. I know the basics of computer window: Yes ☐ No ☐

9. I have learnt MS word before this course: Yes ☐ No ☐

10. I have learnt MS excel before this course: Yes ☐ No ☐

11. I have learnt MS PowerPoint before this course: Yes ☐ No ☐

12. Other technology application(s) I have learnt:

…………………………………………………………………………………………
…………………………………………………………………………………………

SECTION B: Technology integration knowledge and skills

Indicate by circling whether you are Strongly Disagree (SD) = 1, Disagree (D) = 2, Not Sure (NS) = 3, Agree (A) = 4, or strongly agree (SA) = 5 to each of the following statements.

<table>
<thead>
<tr>
<th>Technological Knowledge</th>
<th>SD</th>
<th>D</th>
<th>NS</th>
<th>A</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 I can use technology without problems.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2 I know how to solve my own technical problems.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Statement</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>---</td>
<td>---------------------------------------------------------------------------</td>
<td>---</td>
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<td>---</td>
<td>---</td>
</tr>
<tr>
<td>3</td>
<td>I can learn technology easily.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>I have the technical skills, I need to use technology.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>I have sufficient opportunity to work with different technologies at the school.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>I keep up with my important new technology.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>I know about a lot of different technology.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>I can use technology to solve real life problem.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>I understand the legal, ethical, cultural, and societal issues related to technology.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

**Pedagogical knowledge**

<table>
<thead>
<tr>
<th></th>
<th>Statement</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>I know how to assess students’ performance in the classroom.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>11</td>
<td>I can adapt my teaching based on what students currently understand or do not understand.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>I can adapt my teaching style to different learners.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>13</td>
<td>I can assess student learning in multiple ways.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>14</td>
<td>I can use a wide range of teaching approach in a classroom setting (e.g. collaborative learning, direct instruction, inquiry learning, problem/project based learning, and problem pose learning).</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>15</td>
<td>I am familiar with common student understanding and misconception.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>16</td>
<td>I know how to organise and maintain classroom management.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

**Content knowledge in mathematics**

<table>
<thead>
<tr>
<th></th>
<th>Statement</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>I have sufficient knowledge about mathematics.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>18</td>
<td>I have various strategies of developing my understanding about Mathematics.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>19</td>
<td>I know about a lot of different approaches of solving mathematics problems.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>20</td>
<td>I have sufficient knowledge about structure of knowledge in mathematics.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>21</td>
<td>I know concepts, facts, theories and procedures within mathematics.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>22</td>
<td>I appreciate that mathematics concepts are valid and reliable.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

**Technology content knowledge**

<table>
<thead>
<tr>
<th></th>
<th>Statement</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>I know about technologies that I can use for understanding mathematics concepts.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>24</td>
<td>I know how to use specific software and website to enhance my mathematics content.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>25</td>
<td>I can find and evaluate the resources that I need for my mathematics learning.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>26</td>
<td>I can use technology for presenting particular mathematics concept.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>27</td>
<td>I can use technology tools and resources for managing and communicating information about mathematics.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
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</table>

**Technology and pedagogical knowledge**

266
<table>
<thead>
<tr>
<th></th>
<th>Description</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>I can choose technologies that enhance the teaching approaches for a lesson.</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>29</td>
<td>I can choose technologies that enhance students’ learning for a lesson.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>30</td>
<td>I am thinking critically about how to use technology in my classroom.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>31</td>
<td>I can adapt the use of the technologies that I am learning about to different teaching activities.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>My teacher education programme has caused me to think more deeply about how technology could influence the teaching approaches I use in my classroom.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>I can use technology resources to facilitate higher order thinking skills, including problem solving, critical thinking, decision making knowledge, and creative thinking.</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>34</td>
<td>I can use technology tools and information resources to increase productivity.</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>35</td>
<td>I can infuse technology to strategies of teaching.</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>36</td>
<td>I can use technology for more collaboration and communication among.</td>
<td></td>
<td></td>
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</tbody>
</table>

**Pedagogical content knowledge**

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>37</td>
<td>I know how to select effective teaching approaches to guide student thinking and learning in mathematics content.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>I know the purposes and objectives for mathematics content.</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>39</td>
<td>I have the curriculum knowledge about mathematics.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>40</td>
<td>I know instructional strategies that are suitable for mathematics concepts.</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>41</td>
<td>I have prior knowledge of students’ mathematics learning.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>I know how and what to assess about students’ mathematics learning.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Technology pedagogical content knowledge**

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>I can teach lessons that appropriately combine mathematics concept, technologies and teaching approaches.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>I can select technologies to use in my classroom that enhance what I teach, how I teach and what students learn.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>I can use strategies that combine mathematics concept, technology and teaching approach that I learned about in this professional development in my classroom.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>I can provide leadership in helping others to coordinate the use of the mathematics content, technology and teaching approaches at any school and/or district.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>I can choose technologies that enhance the learning of mathematics concepts for a lesson.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>I can evaluate and select new information resources and technological innovations based on their appropriateness to specific tasks in mathematics.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>49</td>
<td>I can use mathematics content, specific technological tools to support learning and research.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
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</table>

**SECTION C: Technological Beliefs**

<table>
<thead>
<tr>
<th>Personal Belief</th>
<th>SD</th>
<th>D</th>
<th>NS</th>
<th>A</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Technology skills are essential for students in their future careers</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2 Using technology in class makes many things easier</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3 Using technology in class can raise student performance</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>4 Technology has administrative importance in school*</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5 Technology is useful for parents of the students*</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pedagogical belief</th>
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<th>D</th>
<th>NS</th>
<th>A</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 Technology will not change my teaching approaches*.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>7 Using technology in class will make my students learn independently*.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>8 Technology will help me reach more students in class than ever*.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>9 My role in the class will not be affected when I use technology in a lesson*.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>10 The major role of a teacher is to transmit knowledge to students even if technology is used in a lesson*.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>11 Learning occurs primarily through drill and practice*.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>12 I can use technology to make my students feel important in a lesson*.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>13 My lesson will be more learner-centred when I use technology*.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>14 Technology can aid students to think critically when solving problem in mathematics*.</td>
<td>1</td>
<td>2</td>
<td>3</td>
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<td>5</td>
</tr>
</tbody>
</table>

*Added by the researcher

**SECTION D: Technological Attitudes**

<table>
<thead>
<tr>
<th>Affective Domain Attitude</th>
<th>SD</th>
<th>D</th>
<th>NS</th>
<th>A</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Technology do not scare me at all.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2 Technology make me feel uncomfortable.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3 I am glad there are more Technology these days.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>4 I do not like talking with others about Technology.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5 Using Technology is enjoyable.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>6 I dislike using Technology in teaching.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

268
<table>
<thead>
<tr>
<th>Cognitive domain attitude</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7. Technology save time and effort.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>8. Schools would be a better place without technology.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>9. Students must use Technology in all subject matters.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>10. Learning about Technology is a waste of time.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>11. Technology would motivate students to do more study.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>12. Technology are a fast and efficient means of getting information.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>13. I do not think I would ever need Technology in my classroom.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>14. Technology can enhance students’ learning.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>15. Technology do more harm than good.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Behavioural domain attitude</th>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>16. I would rather do things by hand than with a Technology.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>17. If I had the money, I would buy a technological tool such as computer.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>18. I would avoid using Technology as much as possible.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>19. I would like to learn more about Technology.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>20. I have no intention to use Technology in the near future.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
### Appendix E: Timetable for the Workshop Training during School Holidays

<table>
<thead>
<tr>
<th>Day</th>
<th>Time</th>
<th>Content of the PD</th>
<th>Actors/ facilitator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day one 7/8/2017</td>
<td>9.00 am-11am</td>
<td>Presentation and discussion on teacher design teams (TDT)</td>
<td>Researcher/participants</td>
</tr>
</tbody>
</table>
|              | 12.30 pm -3.00 pm | 1. Demonstration lesson in mathematics using GeoGebra.  
|              |               | 2. Discussions and reflections on the lesson in relation to TPACK framework.     | Researcher/participants |
| Day two 8/8/2017 | 9.00 am –12.00pm | Designing GeoGebra-based lesson mathematics                                      | The teacher design teams (TDTs) |
|              | 1.00 pm - 3.00pm | Designing GeoGebra-based lesson mathematics                                      | TDTs |
| Day three 9/8/2017 | 9.00 am -12.00pm | Designing GeoGebra-based lesson mathematics                                      | TDTs |
|              | 1.00 pm - 3.00pm | Designing GeoGebra-based lesson in mathematics                                   | TDTs |
| Day four 10/8/2017 | 9.00 am –12.00pm | Designing GeoGebra-based lesson in mathematics                                   | TDTs |
|              | 1.00 pm - 3.00pm | 1. Micro-teaching by TDT  
|              |               | 2. Observation using TPACK evaluation checklist  
|              |               | 3. Discussion of micro-teaching                                                  | Researcher/participants |
| Day five 11/8/2017 | 9.00 am – 10.00am | 1. Micro-teaching by TDT                                                        | the design teams |
|              |               | 2. Discussion of micro-teaching                                                  | Researcher/the design teams |
|              | 10.00am –12.00pm | 1. Micro-teaching by TDT                                                        | the design teams |
|              |               | 2. Discussion of micro-teaching                                                  | Researcher/the design teams |
|              | 1.00 pm - 3.00pm | 1. Micro-teaching by TDT                                                        | the design teams |
|              |               | 2. Discussion of micro-teaching                                                  | Researcher/the design teams |
Appendix F: Interview Guide for Teachers after the PD

You have participated in technology professional development programme for the past one year. In this interview, you are kindly requested to provide your general impressions about the programme you participated. Please provide responses to each of the questions. Please be as candid as you can in your responses.

1. Do you think technology has some benefits at all in mathematics classroom?
2. Do you believe it is important to learn how to use technology?
3. Have you used technology in your mathematics teaching before?
4. How did you use the technology in mathematics teaching?
5. What motivated you to use technology in your mathematics teaching?
6. Describe how you feel when you use technology to teach mathematics?
7. Do you have sufficient knowledge and skills to use technology to teach mathematics?
8. In what way could the use of professional development influence your classroom practices?
9. Do you believe the use of technology could change your instructional approach?
   How?
10. How often do you use technology to teach mathematics?
    Once every day, once every week, once every month, once every year, or never at all
11. Would you encourage other teachers to use technology in their teaching? Why or Why not?
12. Do you need assistance before you can use technology in the classroom?
13. What has changed in your instructional practices since you got yourself involved in this professional development programme?
14. How did the professional development programme in which you participated influence your:
    (i) knowledge of technology integration in mathematics teaching?
    (ii) beliefs about the importance of using technology in mathematics?
    (iii) confidence and interest to use technology in mathematics?
    (iv) intention to use technology in mathematics?
15. Which of the activities in the professional development programme did you enjoyed most and how did it help you?
16. Which (if any) of the activities in the professional development programme were not helpful to you?

17. What challenges did you encounter in this professional development programme and how did you solve them?

18. What strategies will you suggest to improve this professional development programme?
Thank you for showing an interest in this project. Please read this information sheet carefully before deciding whether or not to participate. If you decide to participate we thank you. If you decide not to take part there will be no disadvantage to you and we thank you for considering our request.

What is the Aim of the Project?

This study will explore how a GeoGebra professional development programme will impact on Ghanaian senior high school mathematics teachers’ beliefs, attitudes, knowledge, and skills related to the use of technology in mathematics teaching. GeoGebra is a mathematical software package that embodies mathematical concepts such as geometry, algebra, calculus, and statistics. The researcher is interested in reporting how a GeoGebra professional development approach based on TPACK (technology, pedagogy, and content knowledge) framework is adapted for professional learning within the context of senior high school (SHS) mathematics curriculum in Ghana. This project is being undertaken as part of the requirements for Isaac Benning’s Doctor of Philosophy Degree in Education.

What Type of Participants are being sought?

Any teacher who is currently teaching the core and/or elective mathematics at Aburaman Senior High School in Ghana can participate in the study. The study is expected to involve 6 to 12 teachers. Though participation is voluntary, teachers who agree to take part in this project are helping to support the national call of improving mathematics teaching and learning in the country. The participant is free to withdraw from the study at any phase but he or she cannot do so after the thesis is completed on June 31, 2019. No payment will be made to the participants but they will be provided lunch on the days that the workshop training will be organised. The cost of the lunch will be borne by the researcher.

What will Participants be asked to Do?

The teachers who agree to participate in the study will be taken through a professional development programme involving three phases: (i) collection of preliminary data, (ii) workshop training, and (iii) enactment of GeoGebra-based mathematics lesson in their classrooms.
The first phase of the professional development programme will be carried out in July 2017 and it is expected to last for four weeks. The participating teachers’ background information and their experience on how they currently use technology in mathematics teaching will be collected through self-reported questionnaire (which will take no more than 45 minutes to complete) and individual interviews (one interview for each teacher, lasting no more than 60 minutes).

The second phase will be done in August 2017. A five-day professional development programme will be organised for teachers who agree to participate in the study during the school holidays (August, 2017). Each session is expected to last 6 hours approximately. The teachers will be introduced to the principles of teacher design team (TDT) professional development, the concept of the TPACK framework, and how to use GeoGebra. The researcher will facilitate the professional development programme. The teachers are expected to participate in the GeoGebra training and then they will identify a topic in the senior high school mathematics curriculum which they could use GeoGebra to teach. They will then design a GeoGebra-based mathematics lessons and teach them as micro-teaching. Focus group discussion will be held after each micro-teaching.

The third phase will start September 2017 through March, 2018. The teachers will be encouraged to use GeoGebra in their actual classroom while the researcher will conduct observations (a maximum of six lessons per teacher will be observed), coach, and facilitate. The teacher delivering the lesson has the opportunity to pause it and invites the researcher or the researcher could as well interject the lesson where necessary to give a quick demonstration to address issues regarding the use of the GeoGebra for the teachers to observe. Again, the researcher will meet the participating teachers fortnightly to discuss their experience about the use of the GeoGebra in the actual classroom and areas that need improvement. The researcher will provide the facilitatory and coaching roles to initiate discussion among the teachers about the pedagogical use of the GeoGebra.

What Data or Information will be collected and What Use will be made of it?

Various forms of data will be collected from the participating teachers before, during, and after the professional development programme. Individual interviews, focus group discussion, self-report questionnaire, micro-teaching observation, video recording, and lesson plans developed by the teacher design teams will be used to collect data about how participating teachers’ beliefs, attitudes, knowledge, and skills about the use technology in mathematics teaching are developed as they go through the GeoGebra professional development programme. The researcher’s logbook will be used to record day to day activities.

Both the interviews and focus group discussions will be audiotaped. The researcher will give basic discussion questions but the actual topic may vary as the discussion unfolds. Although, the University of Otago Human Ethics Committee is aware of the general areas to be explored in the focus group discussion, the Committee has not been able to review the precise questions to be used. In the event that the line of questioning does develop in such a way that you feel hesitant or uncomfortable you are reminded of your right to decline to answer any particular question(s) and also that you may withdraw from the project at any stage without any disadvantage to yourself of any kind.
The self-report questionnaire contains items to measure how the teachers develop their beliefs, attitudes, knowledge, and skills on the use of technology in mathematics teaching. The observation checklist and lesson plan will be evaluated to determine how technological, pedagogical, and content knowledge of the teachers are developed as they go through the GeoGebra professional programme. Any participant may choose which question to answer during the data collection process. Any data collected from the participants will be handled with care and it will be restricted to the principal researcher (Isaac Benning) and the two supervisors of this research (Dr. Chris Linsell and Dr. Naomi Ingram). Pseudonym will be used to ensure the anonymity of the participants. Electronic files will be saved with password and the hard copies will be saved in a locked cabinet. The participants can feel free and ask for any of the information collected during the research. The data will be stored for at least 5 years after which it will be destroyed.

The result of this study will be presented at seminars for staff and students, at conferences, in published articles, and in PhD Thesis that will be submitted to University Otago for the award of Doctor of Philosophy Degree in Education. The published articles and PhD Thesis produced from this research will be available at University of Otago’s Library. The participants have the opportunity to request for any of these materials for reading and it will be made available for them.

**What if Participants have any Questions?**

If you have any questions about our project, either now or in the future, please feel free to contact either:

Dr Chris Linsell (Primary Supervisor)  
University of Otago College of Education  
PO Box 56, Dunedin 9054, New Zealand  
Tel +64 3 479 4253  Mobile +64 27 344 8775  
Email chris.linsell@otago.ac.nz

Or  
Isaac Benning (Student Researcher)  
University of Otago College of Education  
145 Union Street East  
PO Box 56, Dunedin 9054, New Zealand  
Tel: +64 3 479 8804  
Email Address: benis512@student.otago.ac.nz

This study has been approved by the University of Otago Human Ethics Committee. If you have any concerns about the ethical conduct of the research you may contact the Committee through the Human Ethics Committee Administrator (ph +643 479 8256 or email gary.witte@otago.ac.nz). Any issues you raise will be treated in confidence and investigated and you will be informed of the outcome.
Appendix H: Consent Form for the Participants

[Reference Number:]
[Date]

Ghanaian In-service Mathematics Teachers’ Beliefs, Attitudes, Knowledge, and Skills on the Use of Technology in Mathematics Teaching: A Case of GeoGebra Professional Development programme

Consent Form for Participants

I have read the Information Sheet concerning this project and understand what it is about. All my questions have been answered to my satisfaction. I understand that I am free to request further information at any stage.

I know that:-
1. My participation in the project is entirely voluntary;
2. I am free to withdraw from the project at any time without any disadvantage;
3. Personal identifying information that will be collected through self-report questionnaire, observation checklist, and recorded interviews and focus group discussion may be destroyed at the conclusion of the project but any raw data on which the results of the project depend will be retained in secure storage for at least five years;
4. This project involves both closed and open-questioning techniques. The general line of questioning includes my beliefs, attitudes, knowledge, and skills about the use of technology in mathematics teaching. The precise nature of the questions that will be asked during the focus group discussion have not been determined in advance, but will depend on the way in which the discussion will go;
5. In the event that the line of questioning develops in such a way that I feel hesitant or uncomfortable I may decline to answer any particular question(s) and/or may withdraw from the project without any disadvantage of any kind;
6. I have also been made aware that my lunch will be provided by the researcher on the days that professional development meetings will be held;
7. The results of the project may be published and will be available in the University of Otago Library (Dunedin, New Zealand) but every attempt will be made to preserve my anonymity.

.................................................................................................................
(Signature of participant) .................................................................
(Date)

.................................................................................................................
(Printed Name)

This study has been approved by the University of Otago Human Ethics Committee. If you have any concerns about the ethical conduct of the research you may contact the Committee through the Human Ethics Committee Administrator (ph +643 479 8256 or email gary.witte@otago.ac.nz). Any issues you raise will be treated in confidence and investigated and you will be informed of the outcome.
### Appendix I: Sample Preliminary Coding in MS Word Document

<table>
<thead>
<tr>
<th>Questions</th>
<th>Bernard</th>
<th>Initial coding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Have you used technology in your mathematics teaching before?</td>
<td>Yes during my university practice. We were taught how to use it. But, I myself using it in the classroom, I have never done it.</td>
<td><strong>Isaac Benning</strong>&lt;br&gt;Frequency of use: During university practice, not used in classroom before.</td>
</tr>
<tr>
<td>What prevented you from using technology in mathematics teaching?</td>
<td>We don't have the <strong>facilities</strong>. Also, I don't have <strong>in-depth knowledge</strong> in it. The kind of softwares I will use and how to get them. I don't have enough <strong>knowledge to use in teaching mathematical concepts</strong> and also to use it in my department.</td>
<td><strong>Isaac Benning</strong>&lt;br&gt;Barriers of using technology: facilities not available, no in-depth knowledge of maths software</td>
</tr>
<tr>
<td>Do you think technology has some benefits at all in mathematics classroom?</td>
<td>Yes, it has. It helps <strong>bring mental picture of mathematical concepts</strong> to the children. The technology is able <strong>to bring pictures that explain certain concepts</strong> that help children to grasp. If you use it at least it helps them to explore. When you leave the technology with the children it makes them to <strong>explore and practice more examples</strong> and get <strong>sources of references from the technological basis</strong> which help them at least to hit their target.</td>
<td><strong>Isaac Benning</strong>&lt;br&gt;Pedagogical importance: it bring mental picture of maths concept to learner. Aids explaining maths concepts, It aids students understanding in mathematics, opportunity for students to explore and practice more examples in maths, get sources of different reference.</td>
</tr>
<tr>
<td>Do you believe it is important to learn how to use technology?</td>
<td>Yes, I do. It is very important because apart from the classroom work, <strong>every gadgets we use today is more or less like a computer</strong>. Everybody has a phone, it is one computer. Now we access and <strong>take money and transact businesses using ATM machines</strong>. That is what I said computer has become the order of the day. Everything we do is about data collection, processing data and analysing it and these are the tools that helps us to do it in a simpler way.</td>
<td><strong>Isaac Benning</strong>&lt;br&gt;Intention to use technology: It's important to learn</td>
</tr>
<tr>
<td>Describe how you feel about using technology in the classroom?</td>
<td>I feel <strong>great</strong> and I feel that it should be in all classrooms. When I see people using technology in the classroom I happy for them and myself. If I am using it yes, but I don't know much into it, at the few that I know I enjoy doing it. I enjoy doing it really I enjoy doing it really.</td>
<td><strong>Isaac Benning</strong>&lt;br&gt;May 21, 2018&lt;br&gt;Personal importance of technology: Today's gadgets are digital, business transaction, to meet computer age,</td>
</tr>
<tr>
<td>Do you believe the use of technology could change your instructional approach? How?</td>
<td>Yes, I do. Like I said, we <strong>have to integrate it</strong>. That one will let the lesson be <strong>more child-centred</strong> than that of the teacher. Because I will just demonstrate and after the demonstration the student will around the software if we have the ICT equipment the child will be able to do it by him or herself. It will let the lesson be practical and child-centred.</td>
<td><strong>Isaac Benning</strong>&lt;br&gt;Pedagogical importance of technology: It simplifies our work, for data processing and analysys.</td>
</tr>
</tbody>
</table>

278
### Appendix J: Sample Codes for Pre-interviews and Focus Group Discussion Data (HyperRESEARCH output)

<table>
<thead>
<tr>
<th>Code</th>
<th>Total</th>
<th>Mean</th>
<th>Bar Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD influences technology use in mathematics</td>
<td>19</td>
<td>0.905</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Plan to acquire technology tools</td>
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<td>0.048</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Prior on existing maths software</td>
<td>13</td>
<td>0.619</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Problem with maintenance</td>
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<td>0.095</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Professional competence</td>
<td>3</td>
<td>0.143</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Representational tool</td>
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<td>[Bar Graph]</td>
</tr>
<tr>
<td>Teacher use technology for social activities</td>
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<td>0.048</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Teaching mainly verbal</td>
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<td>0.048</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology aids students to explore maths concepts</td>
<td>2</td>
<td>0.095</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology brings cooperative learning</td>
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<td>0.19</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology broadens teachers’ subject matter knowledge</td>
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<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology can change instructional approach</td>
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<td>[Bar Graph]</td>
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<tr>
<td>Technology can help student to make mathematics</td>
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<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology can help students problem skills</td>
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<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology can help teacher teach from known to unknown</td>
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<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology can make mathematics teaching real</td>
<td>8</td>
<td>0.381</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology can make students understand mathematics</td>
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<td>0.952</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology can provide visual representation</td>
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<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology can reduce abstract teaching</td>
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<td>0.381</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology catches the attention of the students</td>
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<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology changes teacher’s role to become facilitator</td>
<td>2</td>
<td>0.095</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology enhances business activities</td>
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</tr>
<tr>
<td>Technology enhances independent learning</td>
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<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology enhances lesson delivery</td>
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<tr>
<td>Technology enhances school administrative exercises</td>
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<tr>
<td>Technology enhances students interest in learning</td>
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<td>Technology enhances students participation in learning</td>
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<tr>
<td>Technology enhances students to practice more in mathematics</td>
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<tr>
<td>Technology help teachers to organise themselves</td>
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<tr>
<td>Technology helps learners to be creative</td>
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<tr>
<td>Technology helps teachers to assess their instruction</td>
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<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology helps to assess students’ learning</td>
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<td>0.143</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology improves students’ performance</td>
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<tr>
<td>Technology improves teachers pedagogical skills</td>
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<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology increases teachers’ confidence</td>
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<td>[Bar Graph]</td>
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<tr>
<td>Technology makes mathematics teaching easier</td>
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<td>Technology makes students feel comfortable in learning</td>
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</tr>
<tr>
<td>Technology makes students open in their thinking</td>
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<td>0.095</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology not used in mathematics classrooms</td>
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<td>0.095</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology outside physical school</td>
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<td>0.143</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology reduces teacher’s workload</td>
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<tr>
<td>Technology reduces time we copy notes on the board</td>
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<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology reduces word-only discretion of concepts</td>
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<td>0.095</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology saves time in teaching and learning</td>
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<td>[Bar Graph]</td>
</tr>
<tr>
<td>Technology will help student to fit into work place</td>
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<td>0.095</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Used technology during teaching practice</td>
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<td>[Bar Graph]</td>
</tr>
<tr>
<td>Used technology in the mathematics classroom</td>
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<td>0.143</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Using technology makes you feel pride</td>
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<td>0.143</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Using technology makes you feel comfortable</td>
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<td>0.048</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Using technology makes you feel good and better</td>
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<td>[Bar Graph]</td>
</tr>
<tr>
<td>Using technology makes you feel great</td>
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<td>0.143</td>
<td>[Bar Graph]</td>
</tr>
<tr>
<td>Using technology makes you feel happy</td>
<td>2</td>
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<tr>
<td>Willingness to use technology in the classroom</td>
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<td>1.238</td>
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Total: 94
Appendix K: Codes clouded for Identifying Themes (HyperRESEARCH output)
### Appendix L: Teachers perceived Belief about the Usefulness of Technology in Mathematics (themes extracted and organised from HyperRESEARCH)

<table>
<thead>
<tr>
<th>Sub-themes</th>
<th>Sample Codes</th>
<th>Sample quotes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pedagogical tool</strong></td>
<td>Explorative instruction</td>
<td>Using the technology will assist students to learn cooperatively because when a student press and he or she does not get the answer he will quickly go to a friend for a support through that they will be sharing ideas. … My role only becomes a facilitator rather than imposing formulas and other things on the students (Peter).</td>
</tr>
<tr>
<td>Socio-cognitive learning</td>
<td>Facilitatory instruction</td>
<td></td>
</tr>
<tr>
<td>Mathematical connection</td>
<td></td>
<td>If we are able to use technology in our instruction, students can easily translate whatever they learnt in school in solving problem outside the school (Martey).</td>
</tr>
<tr>
<td><strong>Assessment practice tool</strong></td>
<td>Monitor students learning</td>
<td>With the projector and laptop you can show whatever you have for the students to have a feel or to see it real. That one will stick [in their minds] because anything seen with the eyes goes into the mind. That one will help them see it clearer and they will understand it better. They will get it real than the abstract teaching (Michael).</td>
</tr>
<tr>
<td>Prompt feedback</td>
<td></td>
<td>Even in the classroom where there is internet connectivity, any question that comes in, the teacher can easily google to get a pictorial view of the question for the students. So from introduction down to the assessment, they are well aware of what is happening (Joshua).</td>
</tr>
<tr>
<td><strong>Development of Professional competence</strong></td>
<td>Knowledge of subject matter</td>
<td>The teacher can have his or her continuous assessment sheet on the computer. It can assist you to add the figures easily. You just have to enter the figure, and it gives you the total. Unlike sitting down and doing it manually, trying to calculate everything. (Bernard)</td>
</tr>
<tr>
<td>Knowledge of teaching skills</td>
<td></td>
<td>It helps the teachers to broaden their horizon. That is they become more knowledgeable about their subject matter and other areas of teaching. With that the teachers can easily pass it on to the students (Martey).</td>
</tr>
<tr>
<td><strong>General productive tool</strong></td>
<td>Confidence in teaching</td>
<td>Using technology comes with some sort of pride… if you are abreast with the use of technology then you are on top of the world. It will also boost my confidence (Martey).</td>
</tr>
<tr>
<td>Performance</td>
<td></td>
<td>To me, not until we begin to embrace the use of technology and its associated benefits we shall be continuing to record poor performance in maths. We’ve been placing emphasis on the theoretical aspect of maths instead of the practical aspect and our students aren’t able to apply what they learnt in industries and other fields of endeavour (Martey).</td>
</tr>
<tr>
<td><strong>Saves Time</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reduce workload</td>
<td></td>
<td>It [technology] will give us the opportunity to teach a lot of concepts within a short possible time. Using ICT, notes, diagrams and other things will be clear to students than writing them on the board (Cynthia).</td>
</tr>
<tr>
<td><strong>Interest in learning</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Motivation</td>
<td></td>
<td>Using technology in the class makes it easier and interesting for you the teacher and students. So at the end the teacher will feel happy because the students will understand whatever you taught them (Mary).</td>
</tr>
</tbody>
</table>
Appendix M: Proposed Framework for Analysing the Core Practice of Effective Mathematics Pedagogy (Based on the literature reviewed for the study)

<table>
<thead>
<tr>
<th>Effective mathematics pedagogy</th>
<th>Sample Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Creation of mathematical setting</td>
<td>Prepares and rehearses mathematical task.</td>
</tr>
<tr>
<td></td>
<td>Uses GeoGebra to create equitable learning environments for students.</td>
</tr>
<tr>
<td></td>
<td>Provides useful mathematical task to students.</td>
</tr>
<tr>
<td></td>
<td>Shows flexibility of navigating through the GeoGebra window.</td>
</tr>
<tr>
<td></td>
<td>Arranges the learning activities sequentially to achieve instructional objectives.</td>
</tr>
<tr>
<td></td>
<td>Assigns roles to students working in group.</td>
</tr>
<tr>
<td></td>
<td>Sets norms for students to work within.</td>
</tr>
<tr>
<td></td>
<td>States objectives of the topic to the students.</td>
</tr>
<tr>
<td></td>
<td>Uses GeoGebra to provide tasks that bring connections between past and present learning experiences of students.</td>
</tr>
<tr>
<td></td>
<td>Uses GeoGebra to provide students with a common base of experiences to identify and develop concepts, processes, and skills.</td>
</tr>
<tr>
<td></td>
<td>Uses GeoGebra to provide tasks which make students actively work in groups to explore mathematics concept.</td>
</tr>
<tr>
<td></td>
<td>Creates opportunities for students to use technology (e.g., GeoGebra) to develop and confirm mathematical conjectures.</td>
</tr>
<tr>
<td>Useful mathematical task</td>
<td>Engages students in learning activities that arouse students’ mathematical thinking.</td>
</tr>
<tr>
<td></td>
<td>Responds adequately to students critical questions.</td>
</tr>
<tr>
<td></td>
<td>Ask questions to elicit further clarification, elaboration and justification of students thinking.</td>
</tr>
<tr>
<td></td>
<td>Revoices students’ responses to highlight key concept.</td>
</tr>
<tr>
<td></td>
<td>Helps students to explain the concepts they have been exploring in verbal, written and graphical form.</td>
</tr>
<tr>
<td></td>
<td>Introduces official definitions/terminologies and further engages students in discussion.</td>
</tr>
<tr>
<td></td>
<td>Identifies and corrects students’ misconceptions.</td>
</tr>
<tr>
<td></td>
<td>Encourages group presentation.</td>
</tr>
<tr>
<td></td>
<td>Provides opportunities for students to ask open-ended questions.</td>
</tr>
<tr>
<td>Mathematical discussion</td>
<td>Assists students to apply concepts in new ways through problem-solving.</td>
</tr>
<tr>
<td></td>
<td>Guides students to relate concepts to different context.</td>
</tr>
<tr>
<td></td>
<td>Encourages students to persevere when mathematical task seem challenging.</td>
</tr>
<tr>
<td></td>
<td>Uses problem-solving to arouse students’ curiosity.</td>
</tr>
<tr>
<td></td>
<td>Provides opportunities for students to pose problem.</td>
</tr>
<tr>
<td>Mathematical Connection</td>
<td></td>
</tr>
<tr>
<td>Assessment of student’s learning</td>
<td></td>
</tr>
</tbody>
</table>
### Appendix N: Sample Coding of Core Practices of Effective Mathematics Pedagogy (HyperRESEARCH output)

<table>
<thead>
<tr>
<th>Link Case</th>
<th>Code</th>
<th>Position</th>
<th>Content</th>
</tr>
</thead>
</table>
| Creating mathematical setting | equipment setups | 1227,1781 | **Setting up GeoGebra Window**  
This guide gives you opportunity and support to utilise the GeoGebra for the whole class teaching in exploring the trigonometric ratios: sine, cosine and tangent. As the instructor, your core task in the lesson execution is to set up the lesson environment and facilitate activities. The following instructions will give you a step-by-step direction of drawing right-angled triangles in a GeoGebra window. You may want to bookmark the activity pages for your students. If you like, make copies of the worksheet for each student. |
| Creating mathematical setting | Preplanned documents | 851,1376 | **Advance preparation**  
- Teacher downloaded and installed the GeoGebra software on the computers the students will use.  
- Teacher drew various polygons in the GeoGebra window for exploring of rotating an object/a point when the angle and point of rotation are given.  
- Teacher searched for information on the internet to augment his materials.  
- Teachers prepares the slides for demonstration using GeoGebra, MS word and MS PowerPoint.  
- Teacher prepares and tests the computer and projector before class begins. |
| Creating mathematical setting | Rehearsing | 851,1376 | **Advance preparation**  
- Teacher downloaded and installed the GeoGebra software on the computers the students will use.  
- Teacher drew various polygons in the GeoGebra window for exploring of rotating an object/a point when the angle and point of rotation are given.  
- Teacher searched for information on the internet to augment his materials.  
- Teachers prepares the slides for demonstration using GeoGebra, MS word and MS PowerPoint.  
- Teacher prepares and tests the computer and projector before class begins. |
| Creating mathematical setting | Clear instruction on worksheet | 9082,9380 | **Activity 2: Deduction of the volume of a cylinder**  
i. Using the illustrations on the GeoGebra window, identify the circular base area (A) of a height (h) of the cylinder.  
ii. In small groups observe the animation of the cylinder shown on the GeoGebra window and record your observations in the table below. |
<p>| Creating mathematical setting | Shared instructional objectives | 7082,7425 | In this worksheet, you will be provided with activities that will lead you in exploring the properties, perimeters, and area of a rectangle. You will be expected to work in pairs where you will observe the animations in GeoGebra window. You will again be expected to record your observation and write your conclusions for whole class discussion. |</p>
<table>
<thead>
<tr>
<th>Link</th>
<th>Case Description</th>
<th>Code</th>
<th>Position</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Useful mathematical task</td>
<td>calculation</td>
<td></td>
<td>13934,14676</td>
<td>After each drag, he waited for the students to do their computation which he then asked them to compare their answers to the one shown on the screen. After using the Geogebra to confirm the students’ responses, he proceeded with the derivation of the volume and surface area of a cylinder. He first showed the animation of the development of a cylinder from its nets.</td>
</tr>
<tr>
<td>Useful mathematical task</td>
<td>Recording</td>
<td></td>
<td>13642,13920</td>
<td>Setho: To find the circumference and area of a circle, you need the radius [he pointed the radius of the circle on the circle]. Setho: Look on the screen, I am going to show you circle. You should calculate the area and circumference of the circle anytime I change the radius.</td>
</tr>
</tbody>
</table>
| Useful mathematical task | constructing new idea | | 16193,17292 | Setho: Now look on the worksheet and provide responses to the questions. 
What is the total surface area of a closed cylinder? 
What is the surface area of an opened cylinder? 
What is the surface area of a cylinder when the top is opened? In my delivery, it make my work very easy. Because I was not doing anything. I was just guiding them as to what to do and they were doing more work than I was doing. Because in my case, I was just overseeing like a supervisor going round to see what they are doing is the right thing. Unlike previously where the teachers will be a lot of calculation, working, explaining things. This time around because of the activity-based worksheet, they are rather doing the introduction. In this worksheet, you are provided with activities that will lead you to explore the properties of a point or an object when it is rotated through a given angle of rotation about the origin. It expected that by the time you go through the activities, you will be able to: 
1. Identify a rotation of an object about a centre (origin) through |
| Useful mathematical task | constructing new idea | | 20646,21159 | Predicting | 9738,9943 | Based on the observations in Activity 4 recorded in the table above, the students are guided in their groups to write their conclusions when: 
1. An object is rotated through $90^\circ$ anticlockwise about the origin |
| Useful mathematical task | constructing new idea | | 9738,9943 | Activity 6 | 10335,10673 | Based on the observations in Activity 4 recorded in the table above, the students are guided in their groups to write their conclusions when: 
1. An object is rotated through $90^\circ$ anticlockwise about the origin |
<p>| Useful mathematical task | Recording | | 10335,10673 | Copy and complete this table for whole class discussion. The students can confirm their response using the GeoGebra window. |</p>
<table>
<thead>
<tr>
<th>Link</th>
<th>Case</th>
<th>Code</th>
<th>Position</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mathematical</td>
<td>confirm students'</td>
<td>8453,8869</td>
<td>Joseph reset the animation then at each click he asked the students to discuss their observations. He drilled the students on the distance-time relationship. For example in the diagram above, Joseph let the students so realised that the speed of the car is 50km/h which he interpreted as the car travelled 50 km for every hour.</td>
</tr>
<tr>
<td></td>
<td>Discussion</td>
<td>answers with GeoGebra</td>
<td></td>
<td>Joseph: Let's look at this time graph. Describe the journey for each mode of transport?</td>
</tr>
<tr>
<td></td>
<td>Mathematical</td>
<td>positive reward</td>
<td>8879,9164</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Discussion</td>
<td>group/individual</td>
<td>8879,9164</td>
<td>For one and hours, the car travelled 58 km.</td>
</tr>
<tr>
<td></td>
<td>representation</td>
<td></td>
<td></td>
<td>S6: The cyclist covered 35 km in 1.5 hours.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>S7: The runner used one and half hours to cover 10 km.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>S8: The one walking covered only one kilometre within one and half hours.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Joseph: Good, so what will be the speed of each mode of transport?</td>
</tr>
<tr>
<td></td>
<td>Mathematical</td>
<td>posing question</td>
<td>9457,9707</td>
<td>oseph: I would want you to provide responses to the following questions:</td>
</tr>
<tr>
<td></td>
<td>Discussion</td>
<td></td>
<td></td>
<td>How long did the car travel before it stopped?</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>How long did it rest?</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>How long did the car take to return to its home?</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>What is total distance the car covered so far?</td>
</tr>
<tr>
<td></td>
<td>Mathematical</td>
<td>autonomy to apply</td>
<td>9457,9707</td>
<td>oseph: I would want you to provide responses to the following questions:</td>
</tr>
<tr>
<td></td>
<td>Discussion</td>
<td>concept</td>
<td></td>
<td>How long did the car travel before it stopped?</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>How long did it rest?</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>How long did the car take to return to its home?</td>
</tr>
<tr>
<td>Mathematical connection</td>
<td>Using GeoGebra to emphasise real life phenomenon</td>
<td>4145,4513</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------</td>
<td>-----------------------------------------------</td>
<td>----------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Math activity:</td>
<td>Assist students to apply the formula they have deduce to solve problems involving trigonometry ratios. Examples: 1. If $0^\circ \leq \theta \leq 90^\circ$ and $\tan \theta = \frac{3}{4}$. Find $\sin \theta$ and $\cos \theta$. 2. A wooden ladder leans against a vertical wall. If the foot of the ladder is 8 m away from the wall and the height of the wall is 10m, find what we are coming to do today. From there we continue by solving some questions. So pay attention to whatever we are going to do. But before I continue, you remember I told you I will show some diagrams of polygons. These are examples of real life applications of polygons.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Math activity:</td>
<td>Activity 6: Real Life Application (hollow cylinder or tube or pipe)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Math activity:</td>
<td>4894,5151</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Math activity:</td>
<td>Anson: I have three pictures for you [he projected it on the screen]. What can you say about each of them?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Math activity:</td>
<td>9155,10516</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Math activity:</td>
<td>Joseph: Reset the animation then at each click he asked the students to discuss their observations. He drilled the students on the distance-time relationship. For example in the diagram above, Joseph let the students to realized that the speed of the car is 50km/h which he interpreted as the car travelled 50 km for every hour. Joseph: Let's look at this time graph. Describe the journey for each mode of transport?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Math activity:</td>
<td>8434,8874</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Al: What is it?
S1: Here (pointing the side AC in relation to angle θ)
Al: Yes, I know you are talking about this [point at side AC] but what is it? [he wanted the student to say that the opposite is AC?]
S1: [got stuck and confused, staring at Alber]
Al: I moved his finger pointed at θ and then AC1. This is the angle and the students in groups are guided to copy and complete this table for whole class discussion. The students can confirm their responses using the GeoGebra window.

<table>
<thead>
<tr>
<th>Point</th>
<th>Rota</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(11, 3)</td>
<td>90</td>
</tr>
<tr>
<td>B(0, -5)</td>
<td>270</td>
</tr>
</tbody>
</table>

Observe the Figure ABCDEFG on the graph below and use it to answer the questions that follow. The students or pairs work on this activity and present their solution for whole class discussion. They can also use the GeoGebra to confirm their responses to each of the questions.

**Question:** How many sides does a polygon have?

**Answer:**

1. A regular polygon has eight sides, find the sum of the interior angles.

   \[ \text{The sum of interior angles} = (n-2) \times 180^\circ \]

   \[ = (8-2) \times 180^\circ = 6 \times 180^\circ = 1080^\circ \]

2. The interior angles of a pentagon are 100^\circ, m^\circ, (3m + 30)^\circ, (m - 10)^\circ, and 7: The number of triangle is a polygon is two minus the number of sides of the polygon.

   S8: We wrote \(n = 2\).

   Emma: So suppose we have a polygon which has 13 sides. How many triangles are we going to get from it?

   S9: That will be thirteen minus two. So we will get 11.

   Emma: That is good. What about a polygon with 30 sides?

   Students: 28.

He showed the nature of the curve \( y = 3x^2 - 2x + 5 \) on the GeoGebra window to the students.

Using the illustration on the GeoGebra window, let students in small groups observe the values of angles subtended by chord AB as the points C, D or E are dragged and let them write down their observations.

**Expected answer:** Angles remain the same because they are produced by the same chord AB.
## Appendix O: Sample Lesson Plan

<table>
<thead>
<tr>
<th>Subject: Core Mathematics</th>
<th>Class: SHS1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topic: Properties, Perimeter, and Area of Rectangle</td>
<td></td>
</tr>
<tr>
<td>Date: 5th June 2017</td>
<td></td>
</tr>
<tr>
<td>Duration: 80 minutes</td>
<td></td>
</tr>
<tr>
<td>Number of students: 50</td>
<td></td>
</tr>
</tbody>
</table>

### Objectives
By the end of the lesson, student will be able to
1. State at least three properties of rectangle.
2. Deduce the formula for finding the area a rectangle.
3. Deduce the formula for finding the perimeter of a rectangle.
4. Apply the concept learned in solving problems related to real life.

### Relevant Previous Knowledge
Students can draw given polygon.

### Resources
1. Mathematics syllabus for senior high school
4. MS word and PowerPoint
5. GeoGebra software

### Advance preparation
a) Teacher downloaded and installed the GeoGebra software on the computers the students will use. ([http://www.geogebra.org](http://www.geogebra.org))
b) Teacher drew various rectangles in the GeoGebra window for exploring of the properties, perimeter and area of a rectangle.
c) Teacher searched for information on the internet to augment his materials.
i. Teachers prepares the slides for demonstration using GeoGebra, MS word and MS PowerPoint.

ii. Teacher prepares and tests the computer and projector before class begins.

Setting up GeoGebra Window
This guide gives you opportunity and support to utilise the GeoGebra for the whole class teaching in exploring the properties, perimeter and area of a rectangle. As the instructor, your core task in the lesson execution is to set up the lesson environment and facilitate activities. The following instructions will give you a step-by-step direction of drawing an object for demonstration in a GeoGebra environment. You may want to bookmark the activity pages for your students. If you like, make copies of the worksheet for each student.

Instructions for the teacher
1. Before you conduct this lesson in a GeoGebra, it is important that you know some basic use of the GeoGebra (eg. Creating line segment, perpendicular line, parallel line, intersecting two objects, polygon).

2. Follow these steps to open the GeoGebra:
   a) Click the Start menu.
   b) Point your mouse to All Programs.
   c) In the dialog that opens, click GeoGebra.
   d) This opens the GeoGebra window.
   e) Create line segment AB.
   f) Create perpendicular line to AB through B.
   g) Create point C on the perpendicular line.
   h) Create parallel line (h) to AB through point C.
   i) Create perpendicular line (i) to AB through A.
   j) Construct intersection of object h and i at D.
   k) Create the polygon ABCD.
   l) Measure each side and each interior angle of the rectangle.
   m) Measure the area of the rectangle.
   n) Type in the textbox the instructions you want your students to follow in exploring the properties and area of a rectangle.
LESSON EXECUTION

Introduction
Let students draw on sheet of paper polygons of give sides. Example triangle, rectangle.

Main lesson

Activity 1
As any of the vertices of rectangle $ABCD$ is dragged, students in their small groups (2-3 students) observe the changes in each side and interior angle of the rectangle $ABCD$. Have students in their groups to write down their observations and conclusions about rectangle for whole class discussion.

Activity 2
Students apply their observations and conclusions to answer the following questions.

a) What is the value of each interior angle of a rectangle?
b) What is the sum of the interior angles of a rectangle?
c) The opposite sides of a rectangle are (i).......................... (ii)..................
d) The sum of two interior opposite angles of rectangle is complementary. True/False
Activity 3
Have students in small groups (2 or 3 students) to observe the lengths AB and BC in relation to area $ABCD$ of the rectangle, as the vertices ($A$, $B$, or $C$) is dragged. Let them record the values of the length($AB$), breadth ($BC$), and the area($A$) of rectangle $ABCD$. Guide them to write mathematical relation connecting length, breadth, and area of the rectangle.

Activity 4
Drag the vertices ($A$, $B$, or $C$) and have students in their groups to sum the values of the sides and compare their result with the value of the perimeter. Ask students to write their conclusion for whole class discussion.

Activity 5
Students apply the concepts learnt to answer the following questions.
1. Calculate the area and the perimeter of each of the following rectangles.
2. Draw on the grid below three different rectangles which all have an area of 24 cm².

3. Draw on the grid below three different rectangles which all have a perimeter of 20cm.
4. The area and perimeter of a rectangle are $32 \text{ cm}^2$ and $24 \text{ cm}$ respectively. Determine the length and breadth of the rectangle.
5. In the rectangle below, the length, breadth, diagonal is as follow: $(3x + 1) \text{ cm}$, $x \text{ cm}$ and $\sqrt{109} \text{ cm}$. Calculate the perimeter and area of the rectangle.

\[ \text{Area} = (3x + 1) \times x = 32 \text{ cm}^2 \]
\[ \text{Perimeter} = 2 \times (3x + 1) + 2x = 24 \text{ cm} \]

\[ x = 1 \]
\[ (3x + 1) = 4 \]

\[ \text{Perimeter} = 2 \times 4 + 2 = 10 \text{ cm} \]
\[ \text{Area} = 4 \times 1 = 4 \text{ cm}^2 \]

**Activity 6: Real life application**

A gardener is using a moss killer on his lawn. The instructions say that 4 measures of the moss killer, in water, will treat 10 $\text{m}^2$ of lawn. The box contains 250 measures and costs £12.50. Find the area of the lawn and hence the cost of the moss killer required (source: Rayner, 2001, p. 45).
Conclusion
Rectangle has four sides. The opposite sides of a rectangle are equal and parallel. Each interior angle of a rectangle is right-angle (90°). The sum of interior angles of rectangle is 360°. The area of rectangle is the product of its length and breadth. The perimeter is twice the sum of its length and breadth.

CLASS EXERCISE
1. What do we mean when we talk about Area?
   A. The amount of surface/space that is covered
   B. The type of object it is
   C. The space around an object
   D. How many sides an object has
2. What is a Rectangle?
   A. Like a toothpaste box
   B. Anything with four sides
   C. The fence around your yard
   D. A quadrilateral with opposite sides that are equal and measures 90°.
3. A square has ____________ sides that are equal.
4. One difference between a square & a Rectangle is that a square must have all _______ sides __________ while a rectangle must have at least _______ sides __________ .
5. One similarity between a rectangle and a square is that they both sides that measure _____ degrees
6. Calculate the area of a rectangle whose length and breadth are 12 cm and 4 cm respectively
   A. 16cm²
   B. 28cm²
   C. 36m²
   D. 48cm²
7. A waterproofing spray is applied to the outside of the 4 walls, including the door, and the roof of the garage shown.
a. Calculate the total area to be sprayed.
b. The spray comes in cans costing £1.95 and each can is enough to cover 4 m\(^2\). How much will it cost to spray this garage? (Assume you have to buy full cans).
Appendix P: Sample students’ worksheet

In this worksheet, you will be provided with activities that will lead you in exploring the properties, perimeters, and area of a rectangle. You will be expected to work in pairs where you will observe the animations in GeoGebra window. You will again be expected to record your observation and write your conclusions for whole class discussion.

Introductory Activity
Draw on the grid sheet provided, the rectangle with the following dimensions.
- a. 3 cm by 4 cm
- b. 1 cm by 5 cm
- c. 6 cm by 3 cm
- d. 3.5 cm by 4.5

Activity 1: Exploring the Properties of Rectangle
1.1 As any of the vertices of rectangle $ABCD$ is dragged, observe the changes in each side and interior angle of the rectangle $ABCD$.
Write down your observations and conclusions about rectangle for whole class discussion.

1.2 Apply their observations and conclusions to answer the following questions.
   - e) What is the value of each interior angle of a rectangle?.................................
   - f) What is the sum of the interior angles of a rectangle?.................................
   - g) The opposite sides of a rectangle are (i).................................. (ii)..................
h) The sum of two interior opposite angles of rectangle is complementary. True/False

**Activity 2: Exploring the Area of a Rectangle**

2.1 Observe the length (l), breadth (b) and the area of rectangle (A), as the slider of the rectangle is dragged. Record the values of the length (l), breadth (b), and the area(A) of rectangle in the table below.

<table>
<thead>
<tr>
<th>Length (l)</th>
<th>Breadth (b)</th>
<th>Area of rectangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.2 Observe the values in the table above and write mathematical relation connecting length (l), breadth (b), and area (A) of the rectangle.

……………………………………………………………………………………………
……………………………………………………………………………………………

**Activity 3: Exploring the perimeter of a rectangle**

3.1 Observe the sides and the perimeter of rectangle as the slider is dragged. Record your observations in the Table below.

<table>
<thead>
<tr>
<th>Length (l)</th>
<th>Breadth (b)</th>
<th>Perimeter of rectangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
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3.2 Based on your observation, write down a formula connecting the perimeter (P), length (l) and breadth (b).

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Activity 4: Application of the Formula Deduced

4.1 Calculate the area and the perimeter of each of the following rectangles.

4.2 Draw on the grid below three different rectangles with same area 24 cm\(^2\).
4.3 Draw on the grid sheet provided, three different rectangles with perimeter 20cm.

4.4 The area and perimeter of a rectangle are $32 \text{ cm}^2$ and $24 \text{ cm}$. Determine the length and breadth of the rectangle.

4.5 In the rectangle below, the length, breadth, diagonal is as follow: $(3x + 1) \text{ cm}$, $x \text{ cm}$ and $\sqrt{109} \text{ cm}$. Calculate the perimeter and area of the rectangle.
4.6 A gardener is using a moss killer on his lawn. The instructions say that 4 measures of the moss killer, in water, will treat 10 m$^2$ of lawn. The box contains 250 measures and costs £12.50. Find the area of the lawn and hence the cost of the moss killer required.